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THE KAUFFMAN BRACKET SKEIN AS AN ALGEBRA OF OBSERVABLES

DOUG BULLOCK, CHARLES FROHMAN, AND JOANNA KANIA-BARTOSZYŃSKA

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ABSTRACT. We prove that the Kauffman bracket skein algebra of a cylinder over a surface with boundary, defined over complex numbers, is isomorphic to the observables of an appropriate lattice gauge field theory.

1. INTRODUCTION

Lattice gauge field theory brings the representation theory of an underlying manifold and its quantum invariants into the same setting. Consider the case of a cylinder over a compact, oriented surface with boundary. A lattice model of the surface determines an algebra of gauge invariant fields (i.e. observables). In the classical case, based on a connected, simply connected Lie group G, observables are the characters of the fundamental group of the lattice represented in G. Wilson loops can be understood as traces of conjugacy classes in the fundamental group of the lattice. For the theory based on a Drinfeld-Jimbo deformation of a simple Lie algebra $\mathfrak{g}$, the observables are a deformation quantization of the G-characters of the surface with respect to the standard Poisson structure [2]. In the case of $U_h(sl_2)$ this, together with the classical isomorphism [1, 6], allowed us to prove that the algebra of observables is the Kauffman bracket skein algebra of the surface, completed as an algebra over formal power series.

In this paper we return to an analytic setting in which the deformation parameter is any complex number other than a root of unity. The analogous theorem relating observables and the Kauffman bracket skein algebra is again true. The proof is based on the combinatorial equivalence between the Temperley-Lieb algebra and the quantized invariant theory of SL_2; it does not explicitly use the relationship with surface characters.

The paper is organized as follows. Section 2 recalls definitions and associated formulas of the Kauffman bracket skein algebra. Section 3 summarizes, for the generic parameter, the construction of a quasi-triangular matrix model of quantum SL_2. Section 4 outlines basic definitions and constructions of lattice gauge field theory. Section 5 describes the correspondence between skeins and intertwiners in the Verlinde algebra for quantum SL_2. Finally, Section 6 contains a proof of the main theorem.

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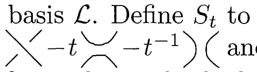
2. Kauffman bracket skein algebra

Let t be a complex number that is neither 0 nor a root of unity. Suppose that F is a compact, oriented surface with boundary and I is a closed interval. Denote by $\mathcal{L}$ the set of isotopy classes of framed links in $F \times I$, including the empty link. Let $\mathbb{C}\mathcal{L}$ be the vector space with basis $\mathcal{L}$. Define S_t to be the subspace of $\mathbb{C}\mathcal{L}$ spanned by all expressions of the form  and $+t^2+t^{-2}$, where the framed links in each expression are identical outside the balls pictured in the diagrams.

The Kauffman bracket skein algebra $K_t(F \times I)$ is the quotient $K_t(F \times I) = \mathbb{C}\mathcal{L}/S_t$. Multiplication is given by laying one link over the other. More precisely, if α and β are in $\mathcal{L}$, isotop them so that α lies in $F \times [0, \frac{1}{2})$, and β in $F \times (\frac{1}{2}, 1]$. Then $\alpha * \beta$ is the union of these two links in $F \times [0, 1]$. Extend linearly to a product on $\mathbb{C}\mathcal{L}$. Since S_t is an ideal, the product descends, making the skein module into a skein algebra. Since the algebra structure depends on the specific product structure of $F \times I$, rather than its topological type, we use the notation $K_t(F)$.

We use the standard convention of modeling a skein in $K_t(F)$ on a framed, admissibly colored, trivalent graph. An admissible coloring is an assignment of a nonnegative integer to each edge so that the colors at each vertex form admissible triples. A triple (a, b, c) is admissible if $a \le b+c$, $b \le a+c$, $c \le a+b$ and $a+b+c$ is even. The corresponding skein in $K_t(F)$ is obtained by replacing each edge labeled with the letter m by the m-th Jones–Wenzl idempotent (see [7], or [5, p. 136]), and replacing trivalent vertices with Kauffman triads (see [5, Fig. 14.7]). If s is a trivalent spine of F, then the set of skeins carried by admissible colorings of s forms a basis $\mathcal{B}_1$ for $K_t(F)$. If F is an annulus, $\mathcal{B}_1$ consists of skeins obtained by labeling the core with a Jones–Wenzl idempotent. One may think of the core as a "trivalent" spine with one vertex, whose admissible labels are $\{(n, n, 0)\}$. The space $K_t(F)$ also has a basis $\mathcal{B}_2$ consisting of all links with simple diagrams on F, i.e. with no crossings and no trivial components.

3. Representations

The details of the following can be found in [3]. Let $\mathcal{A}_t$ be the unital Hopf algebra on X, Y, K, K^{-1}, with relations:

$$KX = t^2XK, \quad KY = t^{-2}YK,$$
$$XY - YX = \frac{K^2 - K^{-2}}{t^2 - t^{-2}}, \quad KK^{-1} = 1.$$

Let $\underline{m}$ denote the irreducible $(m+1)$-dimensional representation of $\mathcal{A}_t$. Fixing an ordered basis for $\underline{m}$ we define linear functionals ${}^m c^i_j : A_t \to \mathbb{C}$ to be the coefficient in the i-th row and j-th column in the representation $\underline{m}$. The ${}^m c^i_j$ form a basis for the stable subalgebra ${}_qSL_2$ of the Hopf algebra dual A^o_t. (Here $q = t^4$.) Define

$$\overline{\mathcal{A}}_t = \prod_{m=0}^{\infty} M_{m+1}(\mathbb{C})$$

and give it the product topology. A typical element of $\overline{\mathcal{A}}_t$ is a sequence of arbitrarily chosen matrices in which the i-th term is an $(i+1) \times (i+1)$ matrix.

Let $\rho_m : \mathcal{A}_t \to M_{m+1}(\mathbb{C})$ be the homomorphism corresponding to the representation $\underline{m}$. The homomorphism

$$\Theta : \mathcal{A}_t \to \overline{\mathcal{A}}_t, \tag{1}$$

given by $\Theta(Z) = (\rho_0(Z), \rho_1(Z), \rho_2(Z), \dots)$, is injective and its image is dense in $\overline{\mathcal{A}}_t$ (see [3]). The algebra $\overline{\mathcal{A}}_t$ is the completion of $\mathcal{A}_t$ by equivalence classes of Cauchy sequences in the weak topology from ${}_qSL_2$. It has the structure of a topological ribbon Hopf algebra. The projection of $\overline{\mathcal{A}}_t$ onto its $(m+1)$-st factor is an irreducible representation of $\overline{\mathcal{A}}_t$, also denoted by $\underline{m}$. Composing ${}^mc^i_j$ with this projection yields a function on $\overline{\mathcal{A}}_t$, also called ${}^mc^i_j$. Thus ${}_qSL_2$ is understood to lie in $(\overline{\mathcal{A}}_t)^o$.

4. Lattice gauge field theory

In this section we recall basic definitions and constructions of lattice gauge field theory. Details can be found in [2].

Let Γ be an oriented, ciliated graph; i.e. the edges are oriented and the vertices carry a linear ordering of the adjacent edges. One can think of this as instructions for building a strip and disk model of an oriented surface F having Γ as a strong deformation retract. Vertices correspond to the disks, edges to strips, and the ciliation determines how to glue the strips to the disks. The surface F is called the envelope of Γ. Define a space of connections

$$\mathbb{A}(\Gamma) = \bigotimes_{\text{edges } e} (\overline{\mathcal{A}}_t)_e,$$

and define an algebra of fields

$$C[\mathbb{A}(\Gamma)] = \bigotimes_{\text{edges } e} ({}_qSL_2)_e.$$

Note that fields are functions on connections in the obvious way. The connections form a coalgebra with comultiplication as defined in [2]. Multiplication of fields is the convolution product dual to comultiplication of connections. There is an action of the gauge algebra,

$$\mathcal{G}(\Gamma) = \bigotimes_{\text{vertices } v} (\overline{\mathcal{A}}_t)_v,$$

on the space of connections, and adjointly on fields. The invariant part of the gauge fields under this action is called the observables, $\mathcal{O}(\Gamma)$. The multiplication of fields restricts to make $\mathcal{O}(\Gamma)$ into an algebra, which is a deformation of the $SL_2(\mathbb{C})$-characters of $\pi_1(F)$.

Let V be a representation of the Hopf algebra $\overline{\mathcal{A}}_t$, that is, V is a finite dimensional left module over $\overline{\mathcal{A}}_t$.

The dual vector space to V carries two distinct $\overline{\mathcal{A}}_t$-module structures. When $\overline{\mathcal{A}}_t$ acts on the left, the dual module is denoted by V^*. The action is

$$Z \cdot \phi(v) = \phi(S(Z) \cdot v)$$

for any $Z \in \overline{\mathcal{A}}_t$, $v \in V$ and ϕ in the dual vector space to V. When $\overline{\mathcal{A}}_t$ acts on the right, denote the dual by V' with

$$\phi(v) \cdot Z = \phi(Z \cdot v).$$

There is an alternate description of observables in terms of "colorings" of the lattice. Assigning a representation V_e to each positively oriented edge e of the lattice Γ determines a map

$$\mathbb{A}(\Gamma) \to \bigotimes_e (V_e^* \otimes V_e).$$

This yields, at each vertex, a tensor product of representations coming from the edges adjoining that vertex taken in the order given by the ciliation. Use the representation V_e for the edges e starting at a vertex and the dual V_e^* for the edges terminating there. The resulting representation at a vertex v is denoted by V_v. Finally, choose $\phi_v \in \mathrm{Inv}(V_v')$ for each vertex v. The element $\bigotimes_v(\phi_v)$ is evaluated on a connection $\bigotimes_e x_e$ by mapping $\bigotimes_e x_e$ to $\bigotimes_e(V_e^* \otimes V_e)$ and then re-parsing to an element of $\bigotimes_v V_v$. By [2, Corollary 1] every observable is a linear combination of observables of this form.

Now assume that Γ is a trivalent lattice colored by irreducible representations. The coloring is admissible if, at each vertex, the integers corresponding to the colorings of the incident edges form an admissible triple.

Proposition 1. *Suppose that Γ is an admissibly colored trivalent lattice. For each V_v there exists a non-zero dual element invariant under the right action of $\overline{\mathcal{A}}_t$. The tensor product of these invariants over all vertices defines an observable.*

The set of such observables, one for each admissible coloring, is a basis for $\mathcal{O}(\Gamma)$.

Proof. Let $c = \{\underline{m}_e \mid e \text{ is an edge of } \Gamma\}$ be an admissible coloring. Note that admissibility implies a 1-dimensional invariant subspace in each V_v'. Hence there is a non-zero observable $o_c = \bigotimes_v \phi_v$. Since o_c is nonzero, there exists $X_c \in \bigotimes_v V_v$ so that $o_c(X_c) \neq 0$.

Let $P : \bigotimes_v V_v \to \bigotimes_e (\underline{m}_e^* \otimes \underline{m}_e)$ be the "parsing" map. Let ι_m be the map

$$\underline{m}^* \otimes \underline{m} \cong M_{m+1}(\mathbb{C}) \to \prod_n M_n(\mathbb{C}) = \overline{\mathcal{A}}_t,$$

where the inclusion is given by forming a sequence that has all zero matrices except for the $(m+1)$-st entry corresponding to $\underline{m}$.

Define x_c to be the connection $(\bigotimes_e \iota_{m_e})(P(X_c))$. Clearly $o_c(x_c) \neq 0$. Since $\rho_m \circ \iota_n = 0$ unless $m = n$, x_c is annihilated by all observables constructed from colorings different than c.

From this we conclude that any set of observables constructed from distinct colorings is independent. By [2] they span. □

There is a map

$$\Phi : K_t(F) \to \mathcal{O}(\Gamma) \tag{2}$$

that assigns to each framed link a Wilson operator. Classically, a Wilson operator is the trace of the holonomy of a connection along some fixed loop. The construction in the quantum setting is described in [2]. In a theory based on the Drinfeld-Jimbo $U_h(sl_2)$ and its Hopf algebra dual, the observables are isomorphic to the Kauffman bracket skein algebra of the lattice envelope. Since $U_h(sl_2)$ is a complete topological algebra over $\mathbb{C}[[h]]$, it was necessary to complete the skein algebra as well. The isomorphism was inferred from the classical isomorphism, the agreement of Poisson brackets, and the h-adic completion. In Section 6 we will show directly that the map Φ is an isomorphism.

5. Temperley-Lieb theory

In this section we recall the correspondence between skeins and intertwiners in the category of representations $\underline{m}$ of $\overline{\mathcal{A}}_t$.

Consider a rectangle $R = I \times I$ with $2n$ distinguished points: n of them on $I \times \{0\}$ and n on $I \times \{1\}$. Take the space of blackboard framed tangles with n

arc components ending at the distinguished points. Its quotient by the Kauffman bracket skein relations is denoted by $K_t(R,n)$. This quotient has an algebra structure given by placing the bottom of a rectangle on the top of another in such a way that the distinguished points meet.

Fact 1. *The algebra $K_t(R,n)$ is isomorphic to $\mathrm{End}_{\overline{\mathcal{A}}_t}(\underline{1}^{\otimes n})$, the space of $\overline{\mathcal{A}}_t$-linear maps of $\underline{1}^{\otimes n}$ to itself.*

In the basis $\{e_{1/2}, e_{-1/2}\}$ of $\underline{1}$, the isomorphism is given by a tangle functor which makes the following assignments. A local maximum is sent to the morphism $\mu : \underline{1} \otimes \underline{1} \to \underline{0}$ defined by

$$\mu(e_{1/2} \otimes e_{-1/2}) = it, \quad \mu(e_{-1/2} \otimes e_{1/2}) = -it^{-1}, \tag{3}$$

$$\mu(e_{-1/2} \otimes e_{-1/2}) = \mu(e_{1/2} \otimes e_{1/2}) = 0.$$

A local minimum is associated to the morphism $\eta : \underline{0} \to \underline{1} \otimes \underline{1}$ given by

$$\eta(1) = ite_{1/2} \otimes e_{-1/2} - it^{-1}e_{-1/2} \otimes e_{1/2}.$$

Fact 2. *The isomorphism takes the n-th Jones-Wenzl idempotent to the intertwiner that projects $\underline{1}^{\otimes n}$ onto its highest weight invariant subspace.*

Let $\mathbb{H} = \{(x,y) \mid y \geq 0\}$ be the closed upper half plane. For any n, choose $2n$ distinguished points on the x-axis, $\{(1,2,\ldots,2n)\}$, and form a space of blackboard-framed tangles with n arc components ending at the distinguished points. The quotient of this space by the Kauffman bracket skein relations is denoted $K_t(\mathbb{H}, 2n)$.

Fact 3. $K_t(\mathbb{H}, 2n) \cong \mathrm{Inv}\left(((\underline{1} \otimes \underline{1})^{\otimes n})'\right)$.

The isomorphism is given by the same tangle functor as for Fact 1.

An admissible triple (m,n,p) determines a skein in $K_t(\mathbb{H}, m+n+p)$ consisting of a Kauffman triad with all three legs attached to the x-axis. Fact 2 gives a canonical inclusion of $\underline{m} \otimes \underline{n} \otimes \underline{p}$ into $\underline{1}^{\otimes m} \otimes \underline{1}^{\otimes n} \otimes \underline{1}^{\otimes p} \cong (\underline{1} \otimes \underline{1})^{\otimes (m+n+p)/2}$.

Fact 4. *An admissible triple (m,n,p) — equivalently, a Kauffman triad — corresponds to a nonzero vector in the 1-dimensional space $\mathrm{Inv}((\underline{m} \otimes \underline{n} \otimes \underline{p})')$.*

6. Observables and the Kauffman bracket skein algebra

Our goal is to prove a theorem analogous to [2, Theorem 10], but replacing power series by complex numbers.

Theorem 1. *Let Γ be a lattice and let F be its envelope. Assume that $t \in \mathbb{C} \setminus \{0\}$ is not a root of unity. The algebra of observables of lattice gauge field theory on Γ based on $(\overline{\mathcal{A}}_t, {}_qSL_2)$ is isomorphic to $K_t(F)$.*

Proof. From [2] we have an algebra map from $\mathbb{C}\mathcal{L}$ to $\mathcal{O}(\Gamma)$ taking a link to the corresponding Wilson operator. As in [2, Theorem 10] this map descends to $\Phi : K_t(F) \to \mathcal{O}(\Gamma)$.

The following description of Φ is implicit in [2]. Since $\mathcal{O}(\Gamma)$ is a homeomorphism invariant of F, we can assume that the lattice Γ comes from giving orientation and ciliation to a trivalent spine γ of F. Let L be an element of the basis $\mathcal{B}_2$ of $K_t(F)$ (i.e., a link with a simple diagram). Choose an orientation of L. To compute the image of L under Φ, first perform the composition

$$\bigotimes_e \overline{\mathcal{A}}_t \xrightarrow{\ \Delta\ } \bigotimes_e \overline{\mathcal{A}}_t^{\otimes n(e)} \xrightarrow{\ \otimes_e \rho_1^{\otimes n(e)}\ } \bigotimes_e (\underline{1}^* \otimes \underline{1})^{\otimes n(e)}. \tag{4}$$

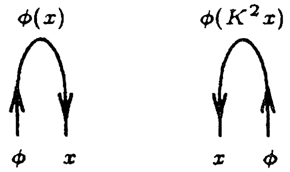


FIGURE 1. Cap tangles

Here $n(e)$ is the number of strands of L running along the edge e of Γ, and Δ means comultiplying $n(e)-1$ times in the factor corresponding to e.

Second, in each factor where the corresponding segment of L runs against the orientation of the edge of Γ, apply $iD\otimes iD^{-1}:\underline{1}^*\otimes\underline{1}\to\underline{1}\otimes\underline{1}^*$. Here, the morphism $D:\underline{1}^*\to\underline{1}$ is defined by

$$D\left(e^{1/2}\right)=ite_{-1/2},\quad D\left(e^{-1/2}\right)=-it^{-1}e_{1/2}. \tag{5}$$

Third, treat each ciliated vertex as a half plane with the cilium at infinity. Up to isotopy, the link L now appears as a collection of oriented caps in each half plane. The cap pictured on the left of Figure 1 is associated with the map $\underline{1}^*\otimes\underline{1}\to\mathbb{C}$ where $\phi\otimes x\mapsto\phi(x)$, and $\underline{1}\otimes\underline{1}^*\to\mathbb{C}$ is given by $x\otimes\phi\mapsto\phi(K^2x)$ for the cap on the right.

Finally, to obtain $\Phi(L)$, multiply the result by $(-1)^{|L|}$, where $|L|$ denotes the number of components of L.

Notice that ρ_1 sends the switch map of [2] to the map $iD\otimes iD^{-1}$ and sends multiplications to contractions. Hence our description of $\Phi(L)$ for $L\in\mathcal{B}_2$ coincides with the Wilson operator. It follows from [2] that Φ does not depend on the choice of orientation of L. Extend it linearly to all of $K_t(F)$. It is a homomorphism of algebras $K_t(F)$ and $\mathcal{O}(\Gamma)$. In order to prove that Φ is an isomorphism we factor it into two maps which are isomorphisms on the level of vector spaces.

The first map, expressed in the basis $\mathcal{B}_2$, is given by a diagonal matrix with 1's and -1's on the diagonal. The second map,

$$\Phi_u:K_t(F)\to\mathcal{O}(\Gamma),$$

does not require a choice of orientation of a link L. To compute the image of $L\in\mathcal{B}_2$ under Φ_u first perform the composition (4). Second, apply the map D to each copy of $\underline{1}^*$. Third, treat each vertex as a half plane and associate the map μ from equation (3) to the (unoriented) caps.

Checking all possible orientations of L and Γ shows that $\Phi(L)=\pm\Phi_u(L)$.

By Fact 4, the map Φ_u takes an element of the basis $\mathcal{B}_1$ (i.e., a skein obtained by an admissible coloring of γ) to an observable coming from coloring the edges of Γ with corresponding irreducible representations of $\overline{\mathcal{A}}_t$. Thus, by Proposition 1, the map Φ_u takes the basis $\mathcal{B}_1$ of $K_t(F)$ to a basis of $\mathcal{O}(\Gamma)$.

As Φ and Φ_u differ by a composition with an isomorphism, both maps are isomorphisms. □

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THE YANG-MILLS MEASURE IN THE KAUFFMAN BRACKET SKEIN MODULE

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ABSTRACT. For each closed, orientable surface Σ_g, we construct a local, diffeomorphism invariant trace on the Kauffman bracket skein module $K_t(\Sigma_g \times I)$. The trace is defined when $|t|$ is neither 0 nor 1, and at certain roots of unity. At $t = -1$, the trace is integration against the symplectic measure on the $SU(2)$ character variety of the fundamental group of Σ_g.

1. Introduction

Since the introduction of quantum invariants of 3-manifolds [20, 25] the fact that they are only defined at roots of unity has been an obstruction to analyzing their properties. One approach has been to study the perturbative theory of quantum invariants [17]. However, there is ample evidence quantum invariants of three manifolds exist as holomorphic functions on the unit disk, that diverge everywhere on the unit circle but at roots of unity [14]. This paper takes a step towards seeing that this holds in general. The Yang-Mills measure is the path integral on a topological quantization [3] of the $SU(2)$-characters of the fundamental group of a closed surface. The measure displays the same convergence properties as are expected of quantum invariants of 3-manifolds.

The Yang-Mills measure in the Kauffman bracket skein algebra of a cylinder over a closed surface Σ_g is a local, diffeomorphism invariant trace. It quantizes the symplectic measure on the space $\mathcal{M}(\Sigma_g)$ of conjugacy classes of representations of the fundamental group of Σ_g into $SU(2)$. The definition of the symplectic structure and formulas for its computation are in [10, 11]. The volume of $\mathcal{M}(\Sigma_g)$ was computed by Witten in [26] in two ways: via the equivalence of two computations in quantum field theory, and by noting that the symplectic measure is equal to the measure coming from Reidemeister torsion. In Witten's setting the Yang-Mills measure is a path integral in a lattice model of field theory that depends on area. Forman [7] gave a direct proof that Witten's measure converges to the symplectic measure as the area goes to zero.

Alekseev, Grosse and Schomerus [1] conceived of a method of constructing lattice gauge field theory based on a quantum group. This idea was further developed by Buffenoir and Roche [6] who gave a construction of the algebra, its Wilson loops and a trace called the Yang-Mills measure that were completely analogous to Witten's construction. Their theory is topological when the area is set to zero.

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The method of constructing the algebras in [1, 6] is combinatorial and based on generators and relations. We gave a new construction of lattice gauge field theory in [4] that is "coordinate free". The connections form a co-algebra and the product on the gauge fields is a convolution with respect to the co-multiplication of connections. This allows the structure of the observables to be elucidated. We found working over formal power series, basing the theory on quantum sl_2, that the observables are the Kauffman bracket skein algebra of a cylinder over a regular neighborhood of the 1-skeleton. In [5] we recover the same result working over the complex numbers.

These considerations lead one to expect that the Yang-Mills measure exists as a trace on the Kauffman bracket skein algebra of a closed surface. In this paper we affirm this fact, with the only reservation that if the deformation parameter t is a generic point on the unit circle, then the measure does not converge. However, at roots of unity the trace exists and is well known. Furthermore, at $t = -1$ the Yang-Mills measure is the symplectic measure on $\mathcal{M}(\Sigma_g)$.

This paper is organized as follows. Section 2 recalls definitions, associated formulas and the algebra structure of the Kauffman bracket skein module of a cylinder over a surface. In section 3 the Yang-Mills measure is defined for compact surfaces with boundary, and is proved to be a trace. In section 4, working with the parameter t such that $|t| \neq 1$, we obtain estimates for the absolute value of the tetrahedral coefficients and use these to show that the Yang-Mills measure can be defined for closed surfaces. In section 5 we define and investigate the measure when t is a root of unity.

2. Preliminaries

Let M be an orientable 3-manifold. A framed link in M is an embedding of a disjoint union of annuli into M. Framed links are depicted by showing the core of an annulus lying parallel to the plane of the paper (i.e. with blackboard framing). Two framed links in M are equivalent if there is an isotopy of M taking one to the other. Let $\mathcal{L}$ denote the set of equivalence classes of framed links in M, including the empty link. Fix a complex number $t \neq 0$. Consider the vector space $\mathbb{C}\mathcal{L}$ with basis $\mathcal{L}$. Define $S(M)$ to be the smallest subspace of $\mathbb{C}\mathcal{L}$ containing all expressions of the form  $-t$  $-t^{-1}$  and  $+t^2+t^{-2}$, where the framed links in each expression are identical outside balls pictured in the diagrams. The Kauffman bracket skein module $K_t(M)$ is the quotient

$$\mathbb{C}\mathcal{L}/S(M).$$

Let F be a compact orientable surface and let $I = [0, 1]$. There is an algebra structure on $K_t(F \times I)$ that comes from laying one link over the other. Suppose that $\alpha, \beta \in K_t(F \times I)$ are skeins represented by links L_α and L_β. After isotopic deformations, to "raise" the first link and "lower" the second, $L_\alpha \subset F \times (\frac{1}{2}, 1]$ and $L_\beta \subset F \times [0, \frac{1}{2})$. The skein $\alpha * \beta$ is represented by $L_\alpha \cup L_\beta$. This product extends to a product on $K_t(F \times I)$. We denote the resulting algebra by $K_t(F)$ to emphasize that it comes from viewing the underlying three manifold as a cylinder over F.

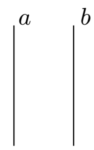
The notation and the formulas in this paper are taken from [13]. However, the variable t replaces A, and we use quantum integers

$$[n] = \frac{t^{2n} - t^{-2n}}{t^2 - t^{-2}}.$$

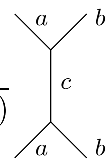
When $t = \pm 1$, $[n] = n$. Note that Δ_n from [13] is equal to $(-1)^n[n+1]$.

There is a standard convention for modeling a skein in $K_t(M)$ on a framed trivalent graph $\Gamma \subset M$. When Γ is represented by a diagram we assume blackboard framing. An *admissible coloring* of Γ is an assignment of a nonnegative integer to each edge so that the colors at trivalent vertices form admissible triples (defined below). The corresponding skein in $K_t(M)$ is obtained by replacing each edge labeled with the letter m by the m-th Jones–Wenzl idempotent (see [24], or [15], p.136), and replacing trivalent vertices with Kauffman triads (see [15, Fig. 14.7]).

Recall the fusion identity:



$$= \sum_c (-1)^c \frac{[c+1]}{\theta(a,b,c)}$$



where the sum is over all c so that the triples (a, b, c) are admissible, i.e. $a + b + c$ is even, $a \leq b + c$, $b \leq a + c$, and $c \leq a + b$. Value of $\theta(a, b, c)$ is given by equation (4) below. The fusion relation is satisfied in $K_t(M)$ unless t is a root of unity other than ± 1.

3. The Yang-Mills Measure in a Handlebody

Throughout this section we assume that t is not a root of unity. The first result is well known and comes from Przytycki's [18] construction of examples of torsion in skein modules.

Lemma 1 (The Sphere Lemma). *Let s_c be a skein represented by coloring a trivalent framed graph in the manifold M. Suppose further that there is a sphere embedded in M which intersects the underlying graph transversely in a single point in the interior of an edge, and the color of that edge is not zero. Then $s_c = 0$.*

Proof. Using the "light bulb trick" isotope the framed graph s_c so that it is the same graph, but the framing on the edge intersecting the sphere has been changed by adding two kinks. Using the formula for eliminating a kink, notice that s_c is a nontrivial complex multiple of itself. Ergo, s_c represents zero in $K_t(M)$. □

Consider now $K_t(\#_g S^1 \times S^2)$, the Kauffman bracket skein module of the connected sum of g copies of $S^1 \times S^2$.

Proposition 1. *The skein module $K_t(\#_g S^1 \times S^2)$ is canonically isomorphic to $\mathbb{C}$. The isomorphism is given by writing each skein as a complex multiple of the empty skein.*

Proof. This follows easily from theorems of Hoste and Przytycki [12, 18, 19]. In [12] the Kauffman bracket skein module of $S^1 \times S^2$ is computed over $\mathbb{Z}[t, t^{-1}]$. This along with the results in [19] on the Kauffman bracket skein module of a connected sum over rational functions in t, combined with the universal coefficient theorem stated in [18], proves the desired result.

We outline the actual isomorphism with the complex numbers. Choose a system of spheres in $\#_g S^1 \times S^2$ that cut it down to a punctured ball. Given a skein in $\#_g S^1 \times S^2$, represent it as a linear combination of colored, framed, trivalent graphs intersecting the spheres transversely in interior of edges, and so that each graph intersects any sphere at most once. This is done by fusing multiple edges passing through the same sphere. By the sphere lemma, we can assume the graphs miss the spheres. Now take the Kauffman bracket of the skein in the punctured ball to write it as a complex multiple of the empty skein. □

Given a handlebody H of genus g its double is $\#_g S^1 \times S^2$. There is a linear functional $\mathcal{YM} : K_t(H) \to \mathbb{C}$ computed by taking the inclusion of H into $\#_g S^1 \times S^2$ followed by taking the " Kauffman bracket" as above. Let F be a compact, oriented surface with boundary. Since $F \times I$ is a handlebody the linear functional

$$\mathcal{YM} : K_t(F) \to \mathbb{C},$$

is defined. We call this the *Yang-Mills measure.*

Choose a trivalent spine of F. The admissible colorings of that spine form a basis for $K_t(F)$. The skein modules of the disk and annulus are exceptions; the first is spanned by the empty skein and the latter is described in Section 4. In terms of this basis the Yang-Mills measure is just the coefficient of the skein coming from labeling all the edges of the spine with 0.

Proposition 2. *The Yang-Mills measure is a trace, that is*

$$\mathcal{YM}(\alpha * \beta) = \mathcal{YM}(\beta * \alpha).$$

Furthermore, the trace is invariant under the action of the diffeomorphisms of $F \times I$ on $K_t(F)$.

Proof. Let L be the link $\partial F \times \{1/2\}$. The result of removing L from the double of $F \times I$ is homeomorphic to the Cartesian product of the interior of F with a circle. Given any skein in $F \times I$ we can represent it by a linear combination of framed links that miss L. Hence, the Yang-Mills measure factors through the skein module of $F \times S^1$. In $F \times S^1$ the skeins $\alpha * \beta$ and $\beta * \alpha$ are the same.

The group of diffeomorphisms of the handlebody $F \times I$ acts on $K_t(F)$ in the obvious way. If $f : F \times I \to F \times I$ is a diffeomorphism then it can be extended to Df : $\#_g S^1 \times S^2 \to \#_g S^1 \times S^2$. Since the image of the empty skein under a diffeomorphism is the empty skein, the action of Df on $K_t(\#_g S^1 \times S^2)$ is trivial. Therefore, $\mathcal{YM}(f(\alpha)) = \mathcal{YM}(\alpha)$. □

The final commonly used property of the Yang-Mills measure is that it is *local.* Suppose that k is a proper arc in F. Cut F along k to get a surface F'. It is evident that if we write a skein α as a linear combination of admissibly colored graphs, each one intersecting k transversely in at most a single point, then we can throw out any graph such that the edge intersecting k carries a nonzero label. This yields a skein in F', denoted by α_k. Then $\mathcal{YM}(\alpha) = \mathcal{YM}(\alpha_k)$.

4. The Yang-Mills measure on a closed surface

Throughout this section assume that $|t| \neq 1$. In fact, we only work with $0 < t < 1$. However, it is evident that the same proofs are valid when $1 < t$ since the formulas are symmetric in t and t^{-1}. Finally, the arguments extend to the case where t is not real by replacing the estimates for $t \in \mathbb{R}$ by estimates of the absolute value of $t \in \mathbb{C}$.

Recall the Kauffman bracket skein algebra of a cylinder over an annulus A. The central core of the annulus can be seen as a link by giving it the blackboard framing. Let s_i be the skein in the annulus which is the result of plugging the i-th Jones-Wenzl idempotent into the core. The skein module $K_t(A)$ is the vector space with basis $\{s_i\}$, where i runs from zero to infinity. The product with respect to this basis is given by

$$s_i * s_j = \sum_{q \geq |i-j|, \text{by 2's}}^{i+j} s_q. \tag{1}$$

Use the Yang-Mills measure on $K_t(A)$ to define a pairing:

$$\langle \alpha, \beta \rangle = \mathcal{YM}(\alpha * \beta). \tag{2}$$

The s_i form an orthonormal basis with respect to (2). This pairing identifies the linear dual of $K_t(A)$ with series of the form $\sum_i \alpha_i s_i$, where the α_i are complex numbers. Note that:

$$\langle \sum_{i=0}^{\infty} \alpha_i s_i, \sum_{j=0}^{n} \beta_j s_j \rangle = \sum_{i=0}^{n} \alpha_i \beta_i.$$

Let $\Sigma_{g,1}$ denote the compact orientable surface of genus g with one boundary component. There is a pairing,

$$K_t(A) \otimes K_t(\Sigma_{g,1}) \to K_t(\Sigma_{g,1})$$

given by representing the skein in $K_t(\Sigma_{g,1})$ by a linear combination of links disjoint from some collar of the boundary, and plugging the skein in $K_t(A)$ into the collar. The Yang-Mills measure can then be applied to give a pairing,

$$K_t(A) \otimes K_t(\Sigma_{g,1}) \to \mathbb{C}. \tag{3}$$

This means there is a well defined map,

$$Y : K_t(\Sigma_{g,1}) \to K_t(A)^*.$$

Topologize $K_t(A)$ by giving it the weak topology from Y. That is a sequence $\sigma_n \in K_t(A)$ is Cauchy if for every skein $\alpha \in K_t(\Sigma_{g,1})$, the sequence of complex numbers $Y(\alpha)(\sigma_n)$ is Cauchy. A linear functional on $K_t(\Sigma_{g,1})$ that comes from an element of this completion via the pairing (3) is called a *distribution*. It is interesting to note that the weak topology from Y on $K_t(A)$ depends on the genus of the surface.

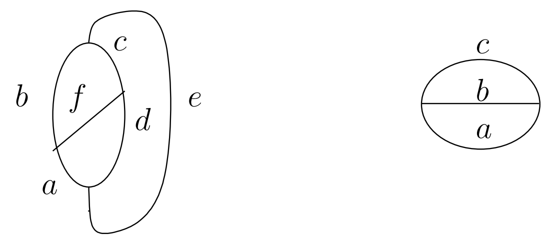


FIGURE 1. Tet and theta

If $g > 1$ there is a distribution on $K_t(\Sigma_{g,1})$ which annihilates all "handle-slides" (*Skeins that are represented by the difference of two links such that one can be obtained from the other by a slide across an imagined disk filling the boundary of* $\Sigma_{g,1}$). This linear functional descends to the skein module of the closed surface. Yang-Mills measure on a closed surface is the result of evaluating this distribution followed by a normalization.

Let's think about what a skein in $K_t(A)$ would be like if it annihilated all handle-slides. Begin by writing it as $\sum_i \alpha_i s_i$ and solve for the α_i. A simple computation shows that if α_0 is zero then all α_i are zero. Normalize so that $\alpha_0 = 1$. Notice that if our skein annihilates handle-slides then the skein $s_1 + [2]s_0$ must be annihilated. Using the rules for multiplication (1) we see that the coefficient α_1 is equal to $-[2]$. Continuing on this way we see that this skein has to be

$$\sum_i (-1)^i [i+1] s_i,$$

which is of course not in $K_t(A)$.

The first goal is to show that for $g > 1$ the sequence of partials sums $\sum_{i=0}^n (-1)^i [i+1] s_i$ is Cauchy in the weak topology from Y, and so defines a distribution.

The notation $\mathrm{Tet}\begin{pmatrix} a & b & e \\ c & d & f \end{pmatrix}$ stands for the Kauffman bracket of the skein pictured in Figure 1 on the left. The explicit formula is given in [13]. We also need the quantity $\theta(a, b, c)$ which is the Kauffman bracket of the colored graph on the right in Figure 1. In terms of quantum integers

$$\theta(a,b,c) = (-1)^{\frac{a+b+c}{2}} \frac{[\frac{a+b+c}{2}+1]![\frac{a+b-c}{2}]![\frac{b+c-a}{2}]![\frac{c+a-b}{2}]!}{[a]![b]![c]!}. \tag{4}$$

Another quantity, called a $6j$ symbol, is derived from the tetrahedral evaluation. Specifically,

$$\begin{Bmatrix} a & b & e \\ c & d & f \end{Bmatrix} = \frac{\mathrm{Tet}\begin{pmatrix} a & b & e \\ c & d & f \end{pmatrix} (-1)^e [e+1]}{\theta(a,d,e)\theta(c,b,e)}. \tag{5}$$

The $6j$ symbols can be woven together to give a change of basis matrix for the Whitehead move on graphs. As a consequence they satisfy an orthogonality equation:

$$\sum_e \begin{Bmatrix} a & b & e \\ c & d & f \end{Bmatrix} \begin{Bmatrix} d & a & g \\ b & c & e \end{Bmatrix} = \delta_f^g, \tag{6}$$

where δ_f^g is the Kronecker delta.

The following proposition seems quite weak, but turns out to be a powerful tool for gauging the convergence of series of Kauffman brackets.

Proposition 3.

$$\left| Tet \begin{pmatrix} a & b & e \\ c & d & f \end{pmatrix} \right| \leq \sqrt{\frac{\theta(b,c,e)\theta(a,d,e)\theta(a,b,f)\theta(c,d,f)}{(-1)^{e+f}[e+1][f+1]}}$$

Proof. In order for all the triples at the vertices of a tetrahedron to be admissible , the parity of the sum of the entries in any two columns of

$$\text{Tet} \begin{pmatrix} a & b & e \\ c & d & f \end{pmatrix}$$

has to be the same. Use (5) to expand the formulas for the $6j$ symbols in the orthogonality relation (6), with $g = f$. The tetrahedral evaluations are equal and the signs of the θ's and the $(-1)^{e+f}$ cancel so that each term in the sum is positive. Hence every term in the sum is less than 1. Fixing e and putting everything except for the tetrahedral evaluations on the right hand side, and taking square roots yields the desired result. □

Corollary 1. *There is a real valued function $C(k_1, k_2, k_3)$ so that*

$$\frac{| Tet \begin{pmatrix} i & i & i \\ k_1 & k_2 & k_3 \end{pmatrix} |}{\sqrt{|\theta(i,i,k_1)\theta(i,i,k_2)\theta(i,i,k_3)|}} \tag{7}$$

is less than $t^i C(k_1, k_2, k_3)$ whenever the graphs corresponding to the functions in the formula are admissibly labeled.

Proof. Substitute into the inequality from Proposition 3 to get,

$$\left| \text{Tet} \begin{pmatrix} i & i & i \\ k_1 & k_2 & k_3 \end{pmatrix} \right| \leq \sqrt{\frac{\theta(k_1,k_2,k_3)\theta(i,i,k_1)\theta(i,i,k_2)\theta(i,i,k_3)}{(-1)^{i+k_3}[k_3+1][i+1]}}. \tag{8}$$

Shift $\sqrt{\theta(i,i,k_1)\theta(i,i,k_2)\theta(i,i,k_3)}$ to the left hand side. Use the fact that $\frac{1}{[i+1]} \leq t^{2i}$ to make the right hand side bigger. Finally, note that the remaining factor on the right hand side is a function of k_1, k_2 and k_3. □

Theorem 1. *The sequence $\sum_{i=0}^n (-1)^i [i+1] s_i$ defines a distribution for $g > 1$. That is, the limit*

$$\mathcal{YM}_D(\alpha) = \lim_{n \to \infty} \mathcal{YM}(\alpha * \sum_{i=0}^n (-1)^i [i+1] s_i)$$

exists and gives a well defined trace on $K_t(\Sigma_{g,1})$ when $g > 1$.

Proof. Choose a trivalent spine for $\Sigma_{g,1}$ with $4g-2$ vertices and $6g-3$ edges. Basis elements s_c for $K_t(\Sigma_{g,1})$ correspond to labeling the edges admissibly with integers k_j, where j runs from 1 to $6g-3$. Let s_i denote the core of an annulus that runs parallel to the boundary, labeled with the ith Jones-Wenzl idempotent. In order to compute $\mathcal{YM}(s_c * s_i)$ place both skeins in the same diagram. Choose a system of arcs, each intersecting this configuration transversely in three points, that isolate the vertices from one another. The transverse points of intersection are labeled i, k_j, i as you traverse each arc. Fuse along these arcs, until the resulting graphs intersect each arc in at most one point. Discard any term where the label on an edge intersecting an arc is not zero. Given a vertex v, let (k_{v1}, k_{v2}, k_{v3}) be the triple of colors appearing there. The resulting answer is:

$$\mathcal{YM}(s_c * s_i) = \prod_{j=1}^{6g-3} \frac{1}{\theta(i,i,k_j)} \prod_v \mathrm{Tet}\begin{pmatrix} i & i & i \\ k_{v1} & k_{v2} & k_{v3} \end{pmatrix}. \tag{9}$$

Each edge appears at exactly two vertices, so (9) can be written as a product of $4g-2$ factors like (7). By Corollary 1 the absolute value of $\mathcal{YM}(s_c * s_i)$ is less than $C(k_j)t^{i(4g-2)}$, where $C(k_j)$ is a number depending only on the k_j. The nth partial sum for $\mathcal{YM}_D(s_c)$ is

$$\sum_{i=0}^{n} (-1)^i [i+1] \prod_{j=1}^{6g-3} \frac{1}{\theta(i,i,k_j)} \prod_v \mathrm{Tet}\begin{pmatrix} i & i & i \\ k_{v1} & k_{v2} & k_{v3} \end{pmatrix}.$$

Note that $[i+1]$ is less than $(i+1)t^{-2i}$. Hence the i-th summand is less than $(i+1)(-1)^i C(k_j)t^{i(4g-4)}$. The ratio test implies that the sequence of partial sums is absolutely convergent for $0 < t < 1$.

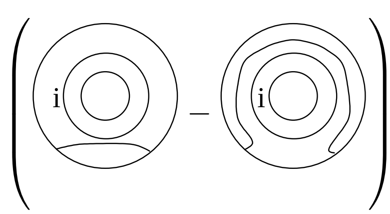
Finally, $\mathcal{YM}_\mathcal{D}$ is a trace since the partial sums $\sum_{i=0}^{n}(-1)^i[i+1]s_i$ can be seen as lying in the center of $K_t(\Sigma_{g,1})$. □

Theorem 2. *$\mathcal{YM}_D$ descends to give a well defined trace*

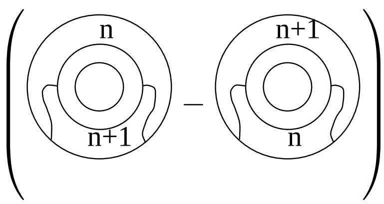
$$\mathcal{YM} : K_t(\Sigma_g) \to \mathbb{C}.$$

Proof. There is a homomorphism $K_t(\Sigma_{g,1}) \to K_t(\Sigma_g)$ induced by inclusion. The surface $K_t(\Sigma_g)$ is the result of adding a disk to the boundary of surface $K_t(\Sigma_{g,1})$. The kernel of this homomorphism consists of all skeins that can be written as a linear combination of handle-slides. The next step is to show that the linear functional $\mathcal{YM}_D$ annihilates all handle-slides. To this end we analyze the difference of the two skeins in the annulus (relative to a pair of points in the boundary).

(10) $\displaystyle\sum_{i=0}^{n}(-1)^i[i+1]$



The analysis of the diagram (10) diagram is due to Lickorish, [15]. It is equal to:

(11) $(-1)^n[n+1]$ 

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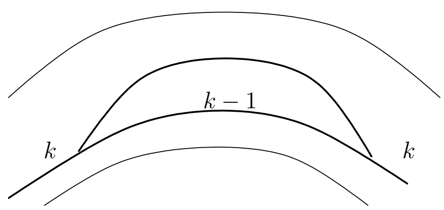
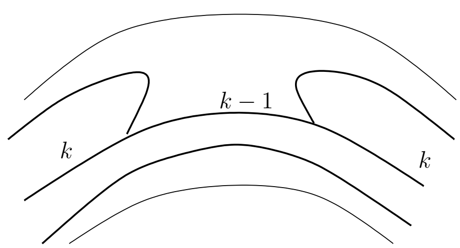
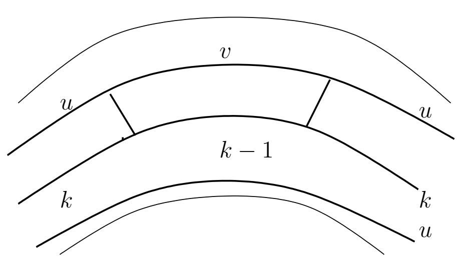
This diagram needs to be set in place. Using standard arguments as in [2] yields that we only need to check handle-slides of the following form. Take a skein corresponding to a colored spine, and separate one strand along an edge.



Now slide the strand over the added disk, locally the diagram looks like:



Multiplying each of the diagrams above by $\sum_{i=0}^{n}(-1)^i[i+1]s_i$, taking their difference, and using the identity (10)=(11), we get a difference of two terms like the one below. In the first one the label $u = n$ and the label $v = n+1$, and in the second one $u = n+1$ and $v = n$.



Fusing to isolate the vertices of this diagram requires two more cross cuts than the diagrams we have been working with up till now. We get the product of

$$(-1)^n[n+1]\frac{1}{\theta(u,k,u)\theta(u,k-1,v)}\mathrm{Tet}\begin{pmatrix} u & u & v \\ 1 & k-1 & k \end{pmatrix}\mathrm{Tet}\begin{pmatrix} u & v & u \\ 1 & k & k-1 \end{pmatrix} \tag{12}$$

with the standard product,

$$\prod_{j=1}^{6g-3} \frac{1}{\theta(u,u,k_j)} \prod_v \mathrm{Tet}\begin{pmatrix} u & u & u \\ k_{v1} & k_{v2} & k_{v3} \end{pmatrix}. \tag{13}$$

The product (13) is smaller than a global constant, depending on the k_j, times $t^{n(4g-2)}$. It remains to ascertain that the term (12) is not too large. Using the inequality from Proposition 3 we get that, regardless of whether $u = n$ and $u = n+1$, or $u = n+1$ and $u = n$, the absolute value of (12) is less than $[n+2]$, which is a universal constant times t^{-2n}. As long as the genus of the surface is greater than 1, the full product goes to zero as n goes to infinity. So, in the limit, all handle-slides are annihilated. □

The case of a surface of genus 1 is slightly different. To get a convergent distribution we need to divide the partial sum $\sum_{i=0}^{n}(-1)^i[i+1]s_i$ by n. The sequence is then Cauchy and defines a distribution on $K_t(T^2)$.

The algebra $K_t(T^2)$ is very nice for working examples. If (p,q) is a pair of integers that are relatively prime there is an obvious skein $s_{(p,q)}$ in $K_t(T^2)$ corresponding to the (p,q) curve on the torus . Define a family of skeins based on (p,q) by using the following iterative scheme: $s_{(p,q)_0} = 2s_{(0,0)}$, that is, twice the empty skein, and $s_{(p,q)_1} = s_{(p,q)}$. For $d > 1$ define:

$$s_{(p,q)_d} = s_{(p,q)} * s_{(p,q)_{d-1}} - s_{(p,q)_{d-2}}.$$

Finally, if $d = \gcd\{p,q\}$, let

$$s_{(p,q)} = s_{(p/d,q/d)_d}.$$

Using this notation the product in $K_t(T^2)$ is given by

$$s_{(p,q)} * s_{(u,v)} = t^{\begin{vmatrix} p & q \\ u & v \end{vmatrix}} s_{(p+u,q+v)} + t^{-\begin{vmatrix} p & q \\ u & v \end{vmatrix}} s_{(p-u,q-v)}. \tag{14}$$

The formula (14) is proven in [8].

There is a map

$$\mu : K_t(T^2) \to \mathbb{C}\emptyset \oplus \mathbb{C}H_1(T^2; Z_2)$$

introduced in [16]. Let

$$\mu\left(\sum_{(p,q)} a_{(p,q)} s_{(p,q)}\right) = a_{(0,0)}\emptyset + \sum_{(p,q)\neq(0,0)} a_{(p,q)}[(p,q)],$$

where $[(p,q)]$ is the Z_2–homology class in $H_1(T^2; Z_2)$ corresponding to $d = \gcd\{p,q\}$ copies of a $(p/d, q/d)$ curve on the torus. The map μ has as its kernel the submodule of all commutators. Hence any linear functional on the five dimensional space that is the image of μ is a trace. It is easy to check that there is a three dimensional family of traces that are invariant under diffeomorphism. In this set up

$$\mathcal{YM}\left(\sum_{(p,q)} a_{(p,q)} s_{(p,q)}\right) = a_{(0,0)}.$$

This is the same trace as the one induced from the inclusion of $K_t(T^2)$ into the non-commutative torus [8].

Towards uniqueness of the Yang-Mills measure, it should be normalized, just as the symplectic measure on moduli space needs to be normalized. It should also be invariant under diffeomorphism, and be local. Locality is made up by two rules. One for cutting a surface along an arc and one for removing a point from a closed surface. If we formalize our rules correctly, we get the following:

Theorem 3. *The Yang-Mills measure is the unique, local, diffeomorphism invariant trace on $K_t(\Sigma_g)$ up to normalization.* □

5. Roots of Unity

Fusion no longer holds in $K_t(M)$ when t is a root of unity. However, when $t = e^{\frac{\pi i}{2r}}$ then one can take a quotient, where an appropriate form of the fusion identity is true. This can be done by setting any skein containing the $(r-1)$-st Jones-Wenzl idempotent equal to zero. The quotient is denoted $K_{r,f}(M)$. The *reduced skein* is a central object in the construction of quantum invariants of 3-manifolds [9, 21, 22].

The Yang-Mills measure on a surface with boundary is obtained the same way as for other values of t. Since $[r] = 0$, the iterative procedure for finding a skein in the annulus that annihilates handle-slides terminates, to yield

$$\sum_{i=0}^{r-2}(-1)^i[i+1] \bigcirc^i .$$

There is an induced trace,

$$\mathcal{YM} : K_{r,f}(\Sigma_g) \to \mathbb{C},$$

constructed the same way as for other t except that there is no need to take a limit because the formula is a finite sum.

Notice that Σ_g is the boundary of a handlebody H_g (it doesn't make any difference which one). There is an action of $K_{r,f}(\Sigma_g)$ on $K_{r,f}(H_g)$ given by gluing skeins in $\Sigma_g \times I$ into a collar of the boundary of H_g. The action gives a map

$$\phi : K_{r,f}(\Sigma_g) \to \mathrm{End}(K_{r,f}(H_g)).$$

As we are working at a root of unity, $K_{r,f}(H_g)$ is a finite dimensional vector space. Denote its dimension by d, and let $\omega = \mathcal{YM}(\emptyset) = \sum_{i=0}^{r-2} \frac{1}{[i+1]^{2g-2}}$. The Yang-Mills measure is:

$$\mathcal{YM}(\alpha) = \frac{\omega}{d}\mathrm{tr}(\phi(\alpha)).$$

From [23] the map ϕ is injective and onto. Hence we can identify $K_{r,f}(\Sigma_g)$ with $\mathrm{End}(K_{r,f}(H_g))$. The Yang-Mills measure is zero on commutators. Thus it factors through

$$\mathrm{End}(K_{r,f}(H_g))/[\mathrm{End}(K_{r,f}(H_g)), \mathrm{End}(K_{r,f}(H_g))].$$

This quotient is a 1-dimensional vector space. Hence any two linear functionals that factor through this quotient are equal if they agree on the identity matrix. The trace also vanishes on commutators, thus it factors through the commutator quotient. The normalization in the formula causes the two induced linear functionals to be the same.

Next we address the cases of $t = \pm 1$. Since the formula for the measure of a spine is an even function of t, we only need to consider one value. The value $t = -1$ is more convenient as the correspondence between $K_{-1}(F)$ and the $SU(2)$-characters of $\pi_1(F)$ is simpler. The skein of the disjoint union of curves c_i corresponds to the function that sends the representation ρ to

$$\prod_i -\mathrm{tr}(\rho(c_i)).$$

Theorem 4. *The Yang-Mills measure is well defined on $K_{\pm 1}(\Sigma_g)$ for $g > 1$. Let s_c be an admissibly colored trivalent spine of Σ_g. If t_n, with $|t_n| \neq 1$, is a sequence of complex numbers converging to ± 1 then*

$$\lim_{n\to\infty} \mathcal{YM}_{t_n}(s_c) = \mathcal{YM}_{\pm 1}(s_c).$$

Proof. The formulas for working with skeins in $K_{-1}(F)$ are the same as the ones for $|t| \neq 1$ except that quantized integers are replaced by ordinary integers. These formulas are the limits as $t \to -1$ of the values we have been using. Revisiting the fundamental estimate (8), we see that,

$$\frac{|\mathrm{Tet}\begin{pmatrix} i & i & i \\ k_1 & k_2 & k_3 \end{pmatrix}|}{\sqrt{|\theta(i,i,k_1)\theta(i,i,k_2)\theta(i,i,k_3)|}} \leq \sqrt{\frac{\theta(k_1,k_2,k_3)}{(-1)^{i+k_3}(k_3+1)(i+1)}} \tag{15}$$

from which we conclude that the right hand side is less than or equal to

$$\frac{C(k_1,k_2,k_3)}{\sqrt{i+1}}.$$

Considering the series for the Yang-Mills measure of a spine, comparison to the p-series implies that it converges as long as the surface has genus greater than 1. Similarly, the Yang-Mills measure is invariant under handle-slides.

The convergence statement follows from the fact that the series that define the Yang-Mills measure at t_n converge absolutely, and the terms of the series converge to the terms of the series for the Yang-Mills measure at -1. □

For a surface of genus 1 we divide the partial sums, as before, by the number of terms in the sum, and the series then converges.

Theorem 5. *The Yang-Mills measure at $t = -1$ is the symplectic measure on $\mathcal{M}(\Sigma_g)$.*

Proof. Using Weyl orthogonality to compute Witten's Yang-Mills measure for a surface of area ρ yields that its value on the spine s_c is given by the series

$$\sum_{i=0}^{\infty}(-1)^i(i+1)e^{-\rho c_2(i)}\prod_{j=1}^{6g-3}\frac{1}{\theta(i,i,k_j)}\prod_v \mathrm{Tet}\begin{pmatrix} i & i & i \\ k_{v1} & k_{v2} & k_{v3} \end{pmatrix},$$

where the edges of s_c carry colors k_i, and k_{v_i} are the colors of the edges ending at the vertex v, and $c_2(i)$ is the value of the quadratic Casimir operator on the $(i+1)$-dimensional irreducible representation of $SU(2)$. As both Witten's series and our series converge absolutely, and Witten's formula converges term by term to our formula as $\rho \to 0$, the limit of Witten's Yang-Mills measure is equal to our Yang-Mills measure at $t=-1$. Finally, Forman [7] showed that the limit as $\rho \to 0$ of Witten's measure is the symplectic measure on $\mathcal{M}(\Sigma_g)$, normalized as in [7]. $\square$

Suppose now that $|t| = 1$ and t is not a root of unity. Evaluation of the Yang-Mills measure on the empty skein on a surface of genus g yields $\sum_{i=o}^{\infty} \frac{1}{[i+1]^{2g-2}}$. As t is not a root of unity the number $[i+1]^{2g-2}$ gets arbitrarily close to 1 infinitely often, which means that the series does not converge. Therefore the Yang-Mills measure does not exist away from roots of unity on the unit circle.

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The Kauffman bracket skein module of a twist knot exterior

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Abstract We compute the Kauffman bracket skein module of the complement of a twist knot, finding that it is free and infinite dimensional. The basis consists of cables of a two-component link, one component of which is a meridian of the knot. The cabling of the meridian can be arbitrarily large while the cabling of the other component is limited to the number of twists.

AMS Classification 57M27; 57M99

Keywords Knot, link, skein module, Kauffman bracket

1 Introduction

At first glance, and in original intent [13], the Kauffman bracket skein module is a formal extension of the Kauffman bracket polynomial to an arbitrary 3-manifold. As Kauffman's polynomial (for framed links in S^3) is equivalent to the Jones polynomial (for oriented links in S^3), one may think of the skein module as a generalization of the Jones polynomial. More recently the module has taken on a different significance: it is now seen as a deformation of the $SL_2(\mathbb{C})$-characters of the fundamental group [4, 5, 14]. Using this interpretation of the skein module of a knot exterior, Frohman, Gelca and the second author here constructed a quantum version the A-polynomial [6]. This is related back to the Jones polynomial [15, 16] (not simply by generality) and has implications for the hyperbolic volume conjecture [9, 12]. Despite all this, there have as yet been no computations of Kauffman bracket skein modules for hyperbolic manifolds.

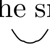
Early computations [13] depended on an I-bundle structure for the manifold, since projection along the I factor gave a natural mechanism for controlling complexity. The only other successful method [1, 7, 8] has been to consider the effect of adding a single 2-handle to a handlebody. This creates a presentation with fairly simple generators (any basis for the module of the handlebody), but having an unwieldy set of relations. Eliminating redundant relations is the most difficult part of the task. What is needed is an effective method of creating relations among relations, or *syzygies*, and then keeping track of which relations can be removed.

This has been managed for all genus one manifolds [7, 8], and for $(2, q)$-torus knot exteriors [1]. In principle, the combinatorics ought to be accessible for genus two manifolds with toral boundary (one added handle), but the computations are quite daunting in practice. Even for $(2, q)$-torus knots, the trick was managed only with help from a particularly nice basis.

The innovation in this paper is a simpler method of keeping track of the relations. We use the established handle addition technique, but we twist the handlebody instead of the handle, which simplifies many bracket computations. Our viewpoint also leads to a comfortable and practical method for producing syzygies that reduce the initial presentation to a simple basis.

2 The theorem

Let M be an orientable 3-manifold. A framed link in M is an embedding of a disjoint collection of annuli into M. Framed links are depicted by link diagrams showing the cores of an annuli lying flat in the projection plane (i.e. with blackboard framing).

Two framed links in M are equivalent if there is an isotopy of M taking one to the other. Let $\mathcal{L}_M$ denote the set of equivalence classes of framed links in M, including the empty link. With $R = \mathbb{Z}[t^{\pm 1}]$, form a free module $R\mathcal{L}_M$ with basis $\mathcal{L}_M$. Define $S(M)$ to be the smallest submodule of $R\mathcal{L}_M$ containing all expressions of the form  $- t$  $- t^{-1}$)(and $\bigcirc + t^2 + t^{-2}$, where the framed links in each expression are identical outside balls pictured in the diagrams. The Kauffman bracket skein module $K(M)$ is the quotient

$$R\mathcal{L}_M / S(M).$$

A q-twist knot (right-handed, if not amphichiral) is the alternating knot formed by inserting a left-to-right string of q half-twists into the coupon in Figure 1. The 2-twist knot in Figure 2, for example, is the familiar figure-8 knot. Let M_q be the twist knot exterior, and denote by xy the 0-framed, two component link also pictured in Figure 1. The meridian is x and the other component is

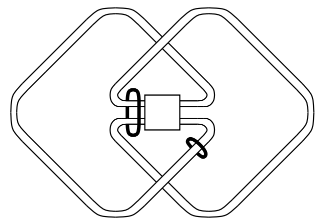


Figure 1: Exterior of a q-twist knot and the link xy

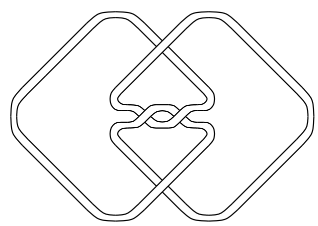


Figure 2: Figure-8 knot as a 2-twist knot

y. In general, $x^l y^m$ denotes the cable of xy consisting of l parallel copies of x and m parallel copies of y. The exponents run over non-negative integers and 1 denotes the empty link.

Theorem 2.1 *$K(M_q)$ is free with basis $\{x^l y^m \mid m \leq q\}$*

3 Initial presentation

The knot exterior M_q decomposes into a pair of genus two handlebodies glued along the 4-punctured sphere S shown in Figure 3. Let H be the closure of the component of $M_q - S$ containing the coupon. Figure 4(a) depicts H, slightly deformed so that the upper left and lower right punctures are in the foreground.

The portions of the knot outside H are parallel to a pair of arcs in S that cut it into an annulus. Therefore, M_q is homeomorphic to H with a 2-handle attached along this annulus. Its core is shown in Figure 4(b).

There is a standard argument [1, 2, 7, 8, 10] that says $K(M_q)$ is $K(H)$ modulo skeins differing by slides across the 2-handle. We find this language to be a little imprecise, so we will rephrase it in terms of relative skeins. Suppose that the core of the attaching annulus is given the blackboard framing in S. We cut out a very small bit of this curve, as indicated in Figure 4(c), leaving a framed

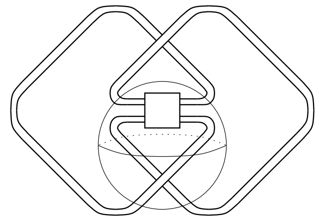
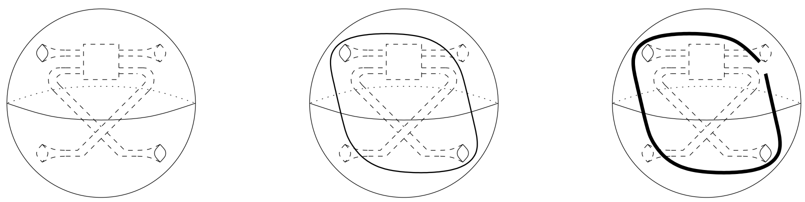


Figure 3: Decomposing sphere S in M_q



(a) (b) (c)

Figure 4: (a) H (b) Core of the attaching annulus (c) The core as a relative link

arc whose ends are a pair of framed points in ∂H. Following [11, 13], let $K_1(H)$ be the skein module of H relative to those two framed points.

Let L be a relative link in H. Since the ends of L are very close together we can unambiguously define the *completion* of L to be the result of gluing its ends together. The *slide* of L is formed by gluing its ends to the cut open core of the attaching annulus. Completion and slide are denoted by $c(L)$ and $s(L)$. Let $r(L) = c(L) - s(L)$ and extend linearly to $r\colon\ K_1(H) \to K(H)$. The image of r is the set of all possible relations in $K(H)$ induced by handle slides. Therefore,

$$K(M_q) = K(H)/r(K_1(H)).$$

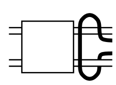
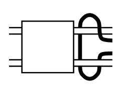
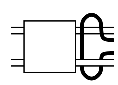
Since $K(H)$ is free, the quotient provides a presentation of $K(M_q)$. Any basis for $K(H)$ serves as a generating set. For relations, choose generators for $K_1(H)$, apply r, and express everything in terms of the basis of $K(H)$. The more efficient your generating set for $K_1(H)$, the more efficient your presentation. However, even a basis for $K_1(H)$ yields unnecessary relations. Computing $K(M_q)$ thus becomes a search for all relations among the relations in this presentation.

We need to fix a basis for $K(H)$. Let x and y be the knots in Figure 1, but only up to isotopy in H. Let z be a meridian that is not isotopic to x in H. As usual, x, y and z are 0-framed. The set of cables, $\mathcal{B} = \{x^l y^m z^n\}$, is a basis for $K(H)$ [13].

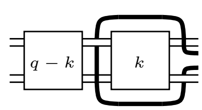
We also need to fix generators of $K_1(H)$, but first some notes on multiplicative notation for (possibly relative) links in H.

- The notation is commutative and associative.
- x^l, y^m and z^n denote cables.
- If L is a (possibly relative) link then $x^l L$ means the union of L, pushed away from the knot boundary, with a cable of x very near the knot boundary.
- Similarly for $z^n L$.
- If σ is a (possibly relative) skein then $x^l z^n \sigma$ is defined by distributing $x^l z^n$ across any linear combination of links representing σ. This is well defined because representatives of σ differ by skein relations that take place away from the knot boundary.
- If σ is written in terms of the basis $\{x^i y^j z^k\}$ then $x^l z^n \sigma$ is just polynomial multiplication.
- In general, $y^m L$ is not well defined, but there are some specific embeddings of L for which we want $y^m L$ to make sense. These are explained below.

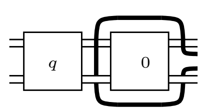
If L is one of the relative knots

$X =$  , $Z =$  or $U =$ 

then $y^m L$ denotes a copy of y^m inserted into the twist coupon. If $0 \leq k \leq q$, let

$Y_k =$ 

where the coupons contain $q-k$ and k twists. By $y^m Y_k$ we mean a cable of y inserted into the coupon containing k twists, even if $k=0$. Lastly,

$y^m Y_{-1} =$ 

with y^m inserted in the coupon that contains no twists.

Lemma 3.1 *$K_1(H)$ is generated by $\{x^l y^m z^n L \mid L = X,\ Y_0,\ Z, \text{ or } U\}$.*[1]

[1]It's actually a basis, but the proof is annoying and the result is unnecessary.

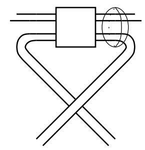
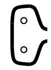
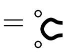


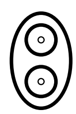
Figure 5

Proof Given any relative link in H, it can be isotoped into the oval neighborhood shown in Figure 5. Once there, grab the top and bottom of the oval and twist in opposite directions a quarter turn each. This should make the tubes perpendicular to the page so that

$X =$  , $Y_0 =$  , $Z =$  and $U =$  .

If not, twist the opposite way and it will.

Now resolve according to a relative version of the argument in [3, Lemmas 1–3]. Each term of the resolution will be a cable of



together with one of X, Y_0, Z or U. The modification introduced in [14, Theorem 6.2] lets us force an X to end up above the cabled link and a Z to end up below it. Neither Y_0 nor U can become entangled. Now we untwist the oval neighborhood, returning X, Y_0, Z and U to their initial embeddings. This will twist the cabled link , but it can be further resolved into a polynomial in x, y and z. □

4 Sufficient relations

In this section we locate in $r(K_1(H))$ sufficient relations to eliminate all but $\{x^l y^m \mid m \leq q\}$ from $\mathcal{B}$. It turns out that powers of z are easy to eliminate and that powers of y index the complexity of other computations. For this reason, we introduce the notation $\sigma \sim y^m$, meaning $\sigma = t^{\pm p} y^m$ modulo the span of $\{x^i y^j z^k \mid j < m\}$.

Where x and z are concerned, the relation submodule behaves like an ideal.

Lemma 4.1 *If $\sigma \in r(K_1(H))$, then $x^i z^k \sigma \in r(K_1(H))$.*

Proof Suppose L is a relative link in H. Since x and z are nowhere near the attaching annulus on ∂H, it's easy to see that $x^i z^k r(L) = r(x^i z^k L)$. This extends to all of $K_1(H)$. □

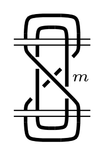
In practice, you compute a relation by grinding some $r(\sigma)$ down to a polynomial in x, y and z. Lemma 4.1 then says that any formal multiple of that relation by $x^i z^k$ is another relation. For example, from the relation $r(y^m Z)$ we obtain a class of relations:

$$\begin{aligned} x^{l-1} z^n r(y^m Z) &= x^{l-1} z^n (y^m z - y^m x) \\ &= x^{l-1} y^m z^{n+1} - x^l y^m z^n \end{aligned} \tag{1}$$

Relations (1) can be used to eliminate $\{x^l y^m z^n \mid n > 0\}$ from the presentation of $K(M_q)$. Powers of y are more troublesome. To eliminate $\{x^l y^m \mid y > q\}$, we need some new relations.

Lemma 4.2 $c(y^m Y_k) \sim y^{m+k+1}$

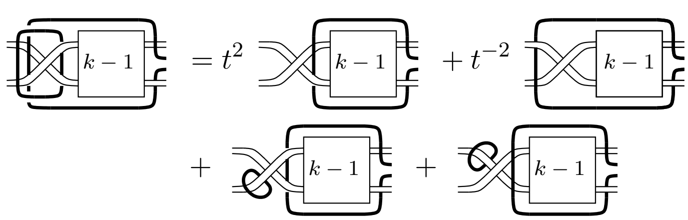
Proof Induct on k. If $k = 0$, we have

$c(y^m Y_0) = -t^{-3}$ 

By counting wrapping numbers in each term of the resolution one can see that $c(y^m Y_0) \sim y^{m+1}$.

Another wrapping number argument shows that no term of $c(y^m Y_{-1})$ has a power of y larger than m.

For $k \geq 1$ consider the relation



(2)

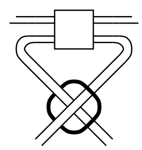
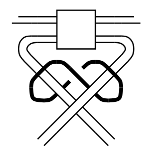
Insert y^m into the coupon and take the closure of every term to get

$$c(y^{m+1} Y_{k-1}) = t^2 c(y^m Y_{k-2}) + t^{-2} c(y^m Y_k) + y^m(\text{meridians}),$$

which can be solved for $y^m Y_k$. □

Lemma 4.3 $s(y^m Y_q) \sim y^{q+m}$.

Proof Note first that

$s(Y_q) =$  $= -t^3$  (3)

$$= -t^4 c(Y_{q-1}) - t^2(\text{meridians})$$

Then insert y^m into the coupon and apply Lemma 4.2. □

Lemmas 4.2 and 4.3 imply $r(y^m Y_q) \sim y^{q+m+1}$. Extended to include powers of x, there relations serve to eliminate any remaining terms of $\mathcal{B}$ with y-degree greater than q. Hence, with

$$\mathcal{R} = \{x^i z^k r(y^m Z)\} \cup \{x^i r(y^m Y_q)\}$$

Proposition 4.4 *The presentation $\mathcal{B}$ modulo $\mathcal{R}$ reduces to the free presentation of Theorem 1.*

To finish the proof of Theorem 1 we must find relations among the relations $r(K_1(H))$ sufficient to write them all in terms of $\mathcal{R}$. Such a relation among relations is called a *syzygy*.

5 Syzygies

Here we show that $\mathcal{R}$ generates $r(K_1(H))$. Recall that $\mathcal{R}$ contains relations of the form

$$r(x^l y^m z^n Z), \text{ and}$$
$$r(x^l y^m Y_q)$$

We need to show that the span of $\mathcal{R}$, denoted $\langle \mathcal{R} \rangle$, contains all relations of the form

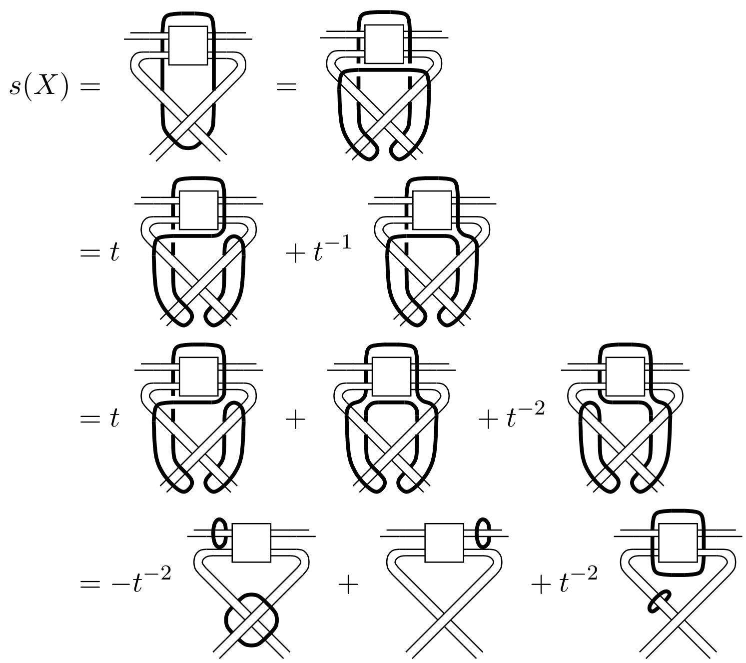
$$r(x^l y^m z^n X),$$
$$r(x^l y^m z^n Y_0), \text{ and}$$
$$r(x^l y^m z^n U)$$

Lemma 5.1 *If L is any link in H (or any skein in $K(H)$), then $xL - zL \in \langle \mathcal{R} \rangle$.*

Proof Express L in terms of the basis $\mathcal{B}$ and then apply (1). □

Lemma 5.2 $r(x^l y^m z^n X) \in \langle \mathcal{R} \rangle$

Proof Slide X and resolve as



Modulo terms of the form $xL - zL$, this is

$$s(X) = -t^{-2} x s(Y_q) + c(X) + t^{-2} x c(Y_q)$$

which is the syzygy $r(X) = -t^{-2} r(xY_q)$. Inserting y^m into the coupon and multiplying by $x^l z^n$ gives the syzygy

$$r(x^l y^m z^n X) = -t^{-2} r(x^{l+1} y^m z^n Y_q)$$

Finally, on the right hand side, convert z's to x's by repeated applications of

$$\begin{aligned} r(x^i y^m z^k Y_q) &= r(x^i y^m z^k Y_q) - r(x^{i+1} y^m z^{k-1} Y_q) + r(x^{i+1} y^m z^{k-1} Y_q) \\ &= z r(x^i y^m z^{k-1} Y_q) - x r(x^i y^m z^{k-1} Y_q) + r(x^{i+1} y^m z^{k-1} Y_q) \end{aligned}$$

This will express $r(x^l y^m z^n Y_q)$ as $r(x^{l+n} y^m Y_q)$ plus terms of the form $xL - zL$. Lemma 5.1 insures that all terms are in $\langle \mathcal{R} \rangle$. □

Lemma 5.3 *Modulo relations of the form $xL - zL$, we have the syzygy*

$$r(y^m Y_q) = t^4 r(y^m Y_{q-1})$$

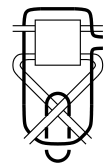
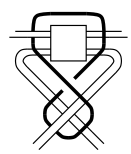


Figure 6

Proof Leaving obvious isotopies to the reader,

$$t^4 s(Y_{q-1}) = -t$$



$$= -t^2 x^2 - c(Y_q)$$

Subtract this equation from Equation (3) and insert y^m as usual. □

Lemma 5.4 *For* $0 \leq k \leq q$, $r(x^l y^m Y_k) \in \langle \mathcal{R} \rangle$.

Proof Induct downward on k. If $k = q$ we are looking at $r(x^l y^m Y_q)$. If $k = q-1$, apply the syzygy from Lemma 5.3 multiplied by x^l.

If $k \leq q-2$, apply r to Equation (2) $k+1$ twists in the coupon. Modulo terms of the from $xL - zL$, this becomes the syzygy

$$r(yY_{k+1}) = t^2 r(Y_k) + t^{-2} r(Y_{k+2}) + r(xX) + r(xZ)$$

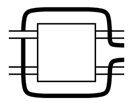
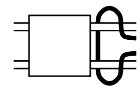
Solve for $r(Y_k)$, multiply by x^l, and insert y^m in the usual place. □

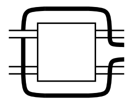
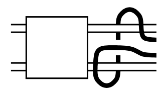
Lemma 5.5 $r(x^l y^m z^n Y_0) \in \langle \mathcal{R} \rangle$.

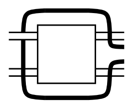
Proof Convert $r(x^l y^m z^l Y_0)$ to $r(x^{l+n} y^m Y_0)$ as in the proof of Lemma 5.2. Then apply Lemmas 5.1 and 5.4. □

Lemma 5.6 $r(x^l y^m z^n U) \in \langle \mathcal{R} \rangle$.

Proof Consider the link in Figure 6. The relative component is Y_q, and the closed component isotops into the coupon where it resolves into some polynomial $p(x, y, z)$. On the other hand, by resolving the crossings as shown we obtain (modulo terms of the form $xL - zL$)

$pY_q = t^2U + x$  $+ xX + t^{-2}$ 

$= t^2U + x$  $+ xX - t$ 

$= t^2U + x$  $+ xX - t^2Y_0 - xX$

Now apply r to this equation to obtain the syzygy

$$r(pY_q) = t^2r(U) \pm r(xZ) - t^2r(Y_0)$$

(The sign of $r(xZ)$ depends on the number of twists in the coupon.) Insert y^m, multiply by x^lz^n, and solve for $r(U)$. Except for the term $r(x^ly^mz^nU)$, convert all z's to x's as usual. The resulting linear combination will lie in $\langle \mathcal{R} \rangle$ □

We have shown that $\langle \mathcal{R} \rangle = r(K_1(H))$, so $K(M_q)$ must be presented as in Proposition 1.

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AC 2008-1703: ENHANCING PRECALCULUS CURRICULA WITH E-LEARNING: IMPLEMENTATION AND ASSESSMENT

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Enhancing Precalculus Curricula with E-Learning: Implementation and Assessment

Abstract

During Fall semester of 2007, a semester-long, quasi-experimental study was conducted at Boise State University to investigate the effectiveness of a systematically sequenced and managed, self-paced e-learning activity on improving students' academic performance and motivation. A total of 125 students enrolled in 3 different sections of a Precalculus class participated in the study. The e-learning activity was implemented in 2 of the 3 sections as a required homework assignment. Students enrolled in one of the 2 selected sections were all engineering majors. The 3rd section was a control group that did not use the e-learning activity. A pre-test, measuring students' entry-knowledge levels, was administered at the beginning of the semester, and a post-test was administered at the end of the semester. Students' learning styles were measured with the Gregorc Style Delineator™. Then, the relationships among the students' learning styles, their academic performance, and self-regulated studying behaviors such as the number of hours they spent on weekly e-learning homework assignments were investigated. This study revealed that using an e-learning activity as a homework assignment improved students' knowledge in Precalculus about the same as did traditional homework that was collected, graded and returned daily. Moreover, we found that different types of learning styles were associated with different degrees of knowledge improvement in Precalculus. Several recommendations on instructional strategies related to students' learning styles are discussed.

Introduction

To facilitate learning processes and to help students produce successful learning, especially during the early years of their study, educators often seek innovative instructional technology. One such technology is e-learning. Presently, e-learning is already deeply integrated into school curricula to motivate students and facilitate learning. Numerous studies have revealed the benefits of implementing self-paced e-learning strategies in traditional curricula for improving critical learning variables such as motivation, self-efficacy, goal-orientation, satisfaction, and persistence.[1] Especially, there has been a fair amount of acceptance and practice among the community of science and engineering education community that traditional teaching can be greatly benefited by incorporating e-learning strategies.[2-6] Leading academic organizations such as the Sloan Consortium also advocate that incorporating online learning strategies into the engineering curricula can augment some of the ABET engineering competencies.[2]

E-learning is also ideal for individualized learning. In contrast to lecture-based classroom learning, computer-based learning programs allow students to adjust the pace, sequence and method of learning to better fit their learning behavior and needs. A study by Yoshioka, Nishizawa, and Tsukamoto[7] revealed that individualized exercises improved calculating skills of engineering students in a fundamental mathematics class. A significant advantage associated with e-learning is that students can learn at their own convenience and are less dependent on the instruction given in class, making it advantageous for nontraditional students that may find it difficult to attend class on a daily basis.

For example, ALEKS (Assessment and LEarning in Knowledge Spaces) is a web-based e-learning program.[8] It provides a systematically sequenced and managed, self-paced e-learning activity, designed to help improve math skills. ALEKS breaks down the Precalculus curriculum into topics, or problem types, that students must work through in order to master the material and complete the course. It is possible to customize a course to include only desired topics; this course was customized and consisted of 178 topics from a list of about 250 total Precalculus topics.

Each student takes an initial assessment in ALEKS to determine which topics he or she has already mastered and which topics he or she is ready to learn. Following this initial assessment, the students begin working in “Learning Mode”. Here the students are presented with a list of topics selected by the web based engine that, based on their assessment, they have the prerequisite knowledge to learn. A student then picks a topic to work on and is given several problems from that topic to practice. When the student types in an answer (very few problems are multiple choice), ALEKS provides immediate feedback concerning the correctness of the given response. If the student has trouble with a certain topic, there is always a complete explanation available for any problem. When the student has answered a sufficient number of problems from the chosen topic correctly, that topic is added to the student’s Knowledge State and the student can move on to a new topic. As the student masters the topics in this manner, more complex topics become available for him or her to work through, with the end goal being complete mastery of the Precalculus curriculum.

In addition to allowing students to work problems in Learning Mode, ALEKS periodically reassesses the students. These 20-30 question assessments occur after a student has completed 20 new topics or spent 10 hours logged into ALEKS. If a student answers a question incorrectly during an assessment, that topic is removed from the student’s Knowledge State and the student must re-demonstrate mastery of that topic in Learning Mode. This provides an excellent way for the students to review and to reinforce topics from throughout the semester, as well as to ensure that the students retain the topics they have learned.

ALEKS provides a personalized, time-efficient environment in which each student is able to work through the Precalculus curriculum at his or her own pace. If a student begins the course already having mastered certain topics, and demonstrates this mastery on an assessment, ALEKS does not require the student to work through problems from that type. Rather, the student is free to move on and spend time working on topics that they have not yet mastered. Many students informally commented throughout the semester that they appreciated this feature of ALEKS.

Working problems using ALEKS also has significant advantages over doing traditional “pencil and paper” homework. First, the student receives immediate feedback as to whether he or she is doing the problem correctly. While this is true for almost any e-learning strategy, ALEKS has the additional advantage that the student is required to work several problems from each topic correctly before that topic is considered mastered and the student is able to move on. Therefore, if a student works a problem incorrectly, that student must go back through his or her work and not only find the mistake, but correct the mistake and answer the problem correctly. This is not only a very useful process for students to practice, but a process that is very hard to require of students in a more traditional classroom setting with handwritten and hand-graded homework. Also, a significant advantage to using progress in ALEKS as homework in lieu of written homework assignments, is that it significantly reduces the load on the instructor while still providing critically needed feedback and student accountability.

When incorporating e-learning into their curricula, another important element that educators should take into account is learners' characteristics such as pre-knowledge levels, personalities, or learning styles. There are various instruments that measure people's different cognitive tendencies or learning styles, including the Gregorc Style Delineator™. The Style Delineator measures four qualities of concreteness, abstraction, sequence, and randomness in people's perception toward, and ordering of, their world.[9] As shown in Table 1, dominant learning styles are identified with one of four style types: concrete-sequential (CS), abstract-sequential (AS), concrete-random (CR), and abstract-random (AR). Every individual has the ability to orient himself or herself toward all four styles. However, people tend to have strong orientation toward one or two, or sometimes even three, dominant style(s). The Style Delineator reveals a score for each style type, identifying the dominant learning style(s) among the 4 types. For example, a person might score 39, 19, 26, and 16 for CS, AS, CR, and AR, respectively, resulting in a dominant learning style of CS.

Table 1. Four Learning Style Types Identified by Gregorc Style Delineator.

	Concrete	**Abstract**
Sequential	CS	AS
Random	CR	AR

Gregorc explains that people with different dominant styles tend to have different views of their world and exhibit different characteristics. People with dominant CS styles view and approach their experiences in an ordered, sequential, and one-dimensional manner. They tend to follow a 'train of thought' with a clear beginning and a clear end, and they excel in making, gathering, and controlling objects. People with dominant AS styles also approach their experiences in an ordered and sequential manner, but their approach is two-dimensional, which is analogous to a tree with multiple branches. They value knowledge, and they are willing to gain knowledge for the sake of knowledge. People with dominant CR styles use intuition and instinct and are concerned more with ideals than with materials, and more with attitudes than with facts. They pay attention to applications, methods and processes of knowledge. People with dominant AR styles behave in a non-linear and multi-dimensional manner, their thinking processes are anchored in feelings, and they concentrate their energies on social relationships.

However, no one possesses one 'pure' style; every individual is capable of orienting himself or herself toward all four styles. Because learners tend to prefer learning environments that support and stimulate their dominant style, understanding learning styles helps educators evaluate and modify their instructional methods and strategies.

We conducted a semester-long study in fall of 2007 to investigate the effectiveness of using the e-learning program, ALEKS, on improving academic performance and motivation of students in Precalculus classes. We also investigated the relationships between the students' learning styles, their degree of improved knowledge in Precalculus, and their self-regulated studying behaviors while using ALEKS.

Method

Research Questions

This study aims to answer the following questions:

1. Does the use of an e-learning activity (ALEKS) have a significant effect on improving students' knowledge in Precalculus?
2. Are there strong relationships between students' learning styles and the degree of improved knowledge in Precalculus?
3. Are there strong relationships between students' self-regulative behaviors (the total time spent and the level of Math skills mastered while using ALEKS) and the degree of improved knowledge in Precalculus?
4. How do engineering students perceive the use of ALEKS in their Precalculus class as a supplementary learning activity?

The first three research questions were answered by testing the following null hypotheses, and the last research question was investigated by using descriptive statistics and qualitative data:

1. The use of an e-learning activity (ALEKS) has no significant effect on improving students' knowledge in Precalculus.
2. There are no strong relationships between students' learning styles and the degree of improved knowledge in Precalculus.
3. There are no strong relationships between students' self-regulative behaviors (the total time spent and the level of Math skills mastered while using ALEKS) and the degree of improved knowledge in Precalculus.

Research Design and Participants

A nonequivalent control group design was used in this quasi-experimental study. A total of 129 students enrolled in 3 sections of MATH 147 Precalculus class in the fall semester of 2007, but 4 students withdrew during the semester. Therefore, a total of 125 students participated in this study. Among them, 88 students (70.40%) were male and 38 students (29.60%) were female. The students in the 1st section of the class ($N = 48$) were all engineering majors and were taught by a male instructor. The students in the 2nd section ($N = 40$) and the 3rd section ($N = 37$) were a mixture of various majors across the disciplines (with 6 and 9 of them being engineering majors in sections 2 and 3, respectively). Both the 2nd and 3rd sections were taught by the same female instructor. All 3 sections of the class were held for 50 minutes, 5 times a week, Monday through Friday.

Section 1 and section 2 were the experimental group which participated in an e-learning activity (the use of ALEKS) as a weekly homework assignment. We verified that section 1 and section 2 were not significantly different in terms of their pre-test scores. Section 3 was a control group in which an e-learning activity was not used. Table 2 describes the different conditions of the groups.

All three sections moved through the material according to the same schedule. The schedule was devised in a way that allotted approximately 10 classes for the first 79 ALEKS topics (chiefly review from intermediate algebra topics), and then moved through the remaining material (99 topics in ALEKS) at an average rate of about 1.6 topics per class (8 topics per week). Class grades were comprised for all three sections as follows: homework was 30% of the grade; each of five exams was worth 11%, and the final comprehensive exam was 15% of the final grade. The homework grade in the e-learning groups (sections 1 and 2) was set according to the percentage of the assigned material that was completed, with 8 deadlines at approximately 2 week intervals throughout the semester. These dates corresponded to the completion of the appropriate chapter in the assigned textbook. Meanwhile, in section 3, homework was assigned, collected and graded by the instructor on a daily basis.

Table 2. Experimental and Control Groups.

		Student					**E-**
Group	**Section**	**Major**	**Male**	**Female**	**Total**	**Instructor**	**learning**
Experimental	1	Engineering	44	4	48	Instructor 1	ALEKS
	2	Various	23	17	40	Instructor 2	ALEKS
	Subtotal		67	21	88		
Control	3	Various	21	16	37	Instructor 2	None
	Total		88 (70.40%)	37 (29.60%)	125 (100%)		

Instruments and Procedures

<u>Pre- and Post-Knowledge Tests</u>: A pre-test was administered at the beginning of the semester to measure students' entry-knowledge levels in Precalculus, and a post-test at the end of the semester. Eleven identical questions were included in both tests, and 105 students completed both tests (20 missing data when excluding missing cases).

<u>Gregorc Style Delineator</u>: To assess students' learning styles, the Gregorc Style Delineator was administered during the semester, and 117 students completed the instrument (8 missing data).

E-Learning Activity (ALEKS): Students in the experimental group (section 1 and section 2) were asked to use ALEKS as a homework assignment. The system kept track of the total time individual students spent with ALEKS and the level of Math skills they mastered in ALEKS, and 81 sets of data were retrieved from the system after the semester was over (7 missing data when excluding missing cases).

Exit Survey: At the end of the semester, the engineering majors (section 1) submitted an exit survey with 21 questions. The exit survey measured students' perceptions toward the use of ALEKS and their motivation and confidence levels in Math skills for continuing their study in engineering.

Data Analysis: The data were analyzed using SPSS 15.0 for Windows (2006) [10]. Statistical procedures used for inferential statistics include a Wilcoxon signed ranks test, a Mann-Whitney *U* test, and Pearson correlation coefficients.[11, 12]

Results

Students' Overall Learning of Precalculus

The possible range of the pre-test and post-test scores was zero to 100. The pre-test scores of all entire participants ranged from 0 to 47 ($M = 9.86$, $SD = 8.25$), and the post-test scores ranged from 14 to 100 ($M = 70.27$, $SD = 18.25$) (see Table 3). The pre-test scores and post-test scores were fairly skewed (Skewness = 1.49 and -1.06, respectively). The difference between individual students' pre-test scores and their post-test scores is the degree of improved knowledge (i.e., learning) ($M = 60.40$, $SD = 17.09$). The normality test on the knowledge improvement scores showed that its normality assumption was not met (Shapiro-Wilk = .95, $p < .00$). Therefore, a nonparametric Wilcoxon signed ranks test was conducted to reveal whether or not the difference between the pre-test scores and the post-test scores was significant.[11] The test revealed the difference was significant at a .01 level, $Z(104) = -8.89$, $p < .00$, indicating that overall, students significantly improved their knowledge in Precalculu*s* during the course of a semester.

Group Differences in Knowledge Improvement in Precalculus

The mean values of the pre-test scores and post-test scores for the experimental group (sections 1 and 2 combined) were 8.58 ($SD = 7.35$) and 67.82 ($SD = 19.72$), respectively; therefore, the average degree of improved knowledge was 59.23 ($SD = 18.04$). The mean values of the pre-test scores and post-test scores for the control group were 12.78 ($SD = 9.48$) and 75.87 ($SD = 12.95$), respectively; therefore, the average degree of improved knowledge was 63.09 ($SD = 14.62$).

Table 3. Descriptive Statistics for Pre-Test and Post-Test Scores Between Groups.

		Pre-Test	**Post-Test**	**Difference**
Experimental	*M*	8.58	67.82	59.23
(*N* = 73)	*SD*	7.35	19.72	18.04
Control	*M*	12.78	75.87	63.09
(*N* = 32)	*SD*	9.48	12.95	14.62
Total	*M*	9.86	70.27	60.40
(*N* = 105)	*SD*	8.25	18.25	17.09

Effects of ALEKS on Knowledge Improvement in Precalculus

The first null hypothesis was: The use of an e-learning activity (ALEKS) has no significant effect on improving students' knowledge in Precalculus. As shown in Table 3, the control group produced a higher average post-test score than the experimental group did. However, the control group's pre-test scores were also higher than the experimental group's pre-test scores. Because the assumptions of normality were not met for the pre-test, post-test, and degree of improved knowledge variables, we conducted multiple Mann-Whitney *U* tests to examine the differences in pre-test scores, post-test scores, and knowledge improvement between the two nonparametric independent samples.

The *U* tests revealed significant differences in pre-test scores and post-test scores between the experimental and control groups, $Z = -2.36$, $p < .05$, and $Z = -2.00$, $p < .05$, respectively. However, the degree of knowledge improvement between the two groups was not significantly different, $Z = -.58$, $p > .05$ (see Table 4). Therefore, the first hypothesis was not rejected. There was no significant difference in the degree of knowledge improvement between section 1 and section 2 (i.e., engineering majors vs. non-engineering majors) of the experimental group either, $Z = -1.32$, $p > .05$.

Table 4. Results of Independent-Samples Mann-Whitney *U* Tests.

Observation	***Mann-Whitney U***	**Z**	**Sig. (2-tailed)**
Pre-test	829.00	-2.36	.01
Post-test	880.00	-2.00	.04
Knowledge improvement	1083.50	-.58	.55

Learning Styles and Knowledge Improvement in Precalculus

The most frequently identified dominant learning style among the students was concrete-sequential (CS); 60 students (51.79%) scored CS as their dominant style. Abstract-random (AR) was the most frequently identified weakest learning style among the students; 46 students (39.31%) scored AR as their weakest style.

Although the normality assumption for the degree of knowledge improvement variable was not met, the normality assumptions for the four sets of learning style scores were not violated. Therefore, Pearson correlation coefficients were computed. To minimize the chances of making a Type I error across the 10 correlations, the Bonferroni approach was used and a p value of less than .005 (.05/10 = .005) was considered for significance.[12] An interesting finding from the correlational analyses was that the scores of the two sequential types (CS and AS) and the scores of the two random types (CR and AR) among students were negatively correlated at the .005 significant level (see Table 5). This implies that when students have a strong *sequential* tendency or preference in a concrete or abstract manner (CS or AS), they tend to exhibit a weak *random* tendency or preference in those manners (CR or AR).

The second null hypothesis was: There are no strong relationships between students' learning styles and the degree of improved knowledge in Precalculus. This null hypothesis was rejected as we found that the more AS tendency or preference students had, the more they increased their knowledge of Precalculus (Pearson's $r = .28$, $p < .005$). On the other hand, when using a p value of .05 as the significant level by taking a risk of making a Type I error, it was found out that the more CR tendency or preference students had, the less they increased their knowledge of Precalculus during the course (Pearson's $r = -.24$, $p < .05$). However, as explained above, the possible Type I error when using a p value of .05 across 10 correlations should be noted, and this result should be interpreted with caution. Also, it is important that these results indicate correlation, not causation; therefore, it should not be interpreted as if the characteristics of AS and CR *caused* the observed results.

Table 5. Correlations Matrix among Learning Styles and Degree of Knowledge Improvement.

		CS	**AS**	**CR**	**AR**	**Knowledge Improvement**
Pearson Correlation	CS	-	.11	-.59**	-.51**	.13
	AS	-	-	-.36**	-.59**	.28**
	CR	-	-	-	.02	-.24*
	AR	-	-	-	-	-.15

** Correlation is significant at the 0.005 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Self-Regulative Behaviors While Using ALEKS and Knowledge Improvement in Precalculus

The third null hypothesis was: There are no strong relationships between students' self-regulative behaviors (the total time spent and the level of Math skills mastered while using ALEKS) and the degree of improved knowledge in Precalculus. To test the hypothesis, we analyzed the total time (measured in hours) students spent with ALEKS and the level of Math skills they mastered in ALEKS obtained from the experimental group (section 1 and section 2). See Table 6.

Table 6. Descriptive Statistics for Total Time Spent and Mastery Level Achieved in ALEKS.

		Total Time Spent [b]	**Math Skills Mastered**
Section 1	*M*	115.69	88.07
(*N* = 41)[a]	*SD*	39.68	11.71
Section 2	*M*	67.59	85.03
(*N* = 40)	*SD*	36.00	17.94
Total	*M*	91.64	86.55
(*N* = 81)	*SD*	44.77	15.09

[a] 7 missing cases when excluding missing cases listwise
[b] measured in hours

The normality assumptions for all three variables (total time spent, mastery level, and degree of knowledge improvement) were not met; therefore, Spearman's rho, a nonparametric equivalent of the Pearson correlation coefficient, was calculated. The results showed that the level of Math skills mastered in ALEKS and the degree of improved knowledge were significantly correlated at a .01 level, but the total time spent with ALEKS and the degree of improved knowledge were not (see Table 7).

Table 7. Correlations Matrix between Learning with ALEKS and Degree of Knowledge Improvement.

		Total Time Spent	**Mastery Level**	**Knowledge Improvement**
Spearman's rho	Total Time Spent	-	.03	-.09
	Mastery Level	-	-	.62**
	Learning	-	-	-

** Correlation is significant at the 0.01 level (2-tailed).
(Listwise *N* = 73)

Engineering Students' Perceptions toward the Use of ALEKS

The fourth research question was: How do engineering students perceive the use of ALEKS in their Precalculus class as a supplementary learning activity? The exit survey revealed that students thought that using ALEKS as a supplementary learning activity helped them to learn Math (*M* = 5.5 on a scale of 1 to 7 when 7 is the highest score). Figure 1 presents the frequency of students' responses to the statement "ALEKS helped me learn Math" on a 7-point scale. The students who rated the usefulness of ALEKS as low mentioned that they did not like its highly structured and controlled format. On the other hand, the students who rated the usefulness of ALEKS as high commented that they liked the nature of self-paced learning and the feedback on their learning progress provided by the system.

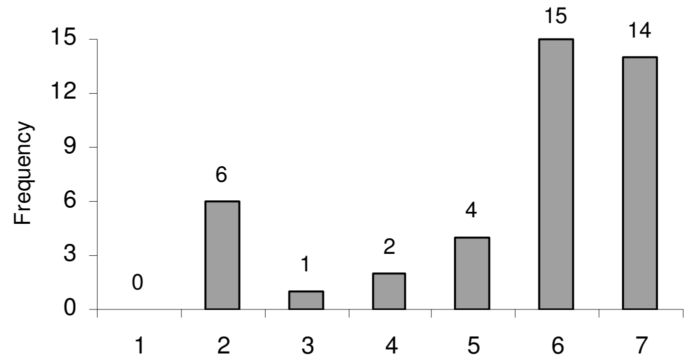

Figure 1. Frequency of Responses to Statement "ALEKS helped me learn Math."

Another question in the survey measured the engineering students' confidence levels about their Math preparation for calculus; the average score was 5.36 on a scale of 1 to 7, where 7 is the highest score. The data were negatively skewed (Skewness = -1.08, see Figure 2). No significant correlations were found between the engineering students' learning styles and their perceptions on the usefulness of ALEKS or their confidence levels in Math preparation for Calculus.

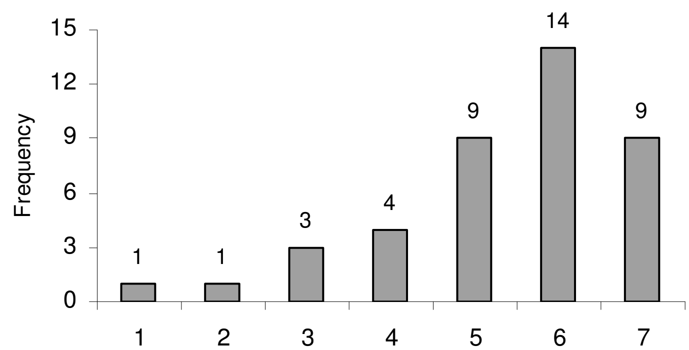

Figure 2. Frequency of Responses to Statement "I am confident about my math preparation for calculus."

Some other results are qualitative. The instructor for sections 2 and 3 indicated that it felt as though she was teaching two different classes. She remarked that the ALEKS material seemed easier, and that there seemed to be less material to cover. Also, she noted less attendance in the ALEKS section, with ½ to ⅔ of the class attending section 2, and with ⅔ to ¾ of the class attending section 3 (no ALEKS). She also noted that the ALEKS section required about five hours less grading than the non-ALEKS section. The instructor for section 1 also reported low attendance on a daily basis. This raises an interesting question -- to what degree can the use of

ALEKS can compensate for the absence of classroom learning? It should be noted that section 3 had written homework collected daily, which promoted attendance. Sections 1 and 2 did not have anything collected daily. Thus, the control group in this study (section 3) consisted of "best practices" in terms of mathematics instruction. A more closely matched control group would have only collected and graded homework about once every two weeks to coincide with the deadlines for student achievement in ALEKS. One would predict such a control group to be less successful in terms of overall mathematics learning than this "best practices" control group was. To investigate these new research questions, future research might be conducted to correlate students' attendance rates, their use of ALEKS, homework due dates and their academic performance.

Conclusions

This study found that using an e-learning strategy (ALEKS) as a homework assignment improved students' knowledge in Precalculus about the same as traditional homework that was collected, graded and returned daily. Based on the positive results that instructors at the university had had with ALEKS in the past,[13,14] it was somewhat surprising that the experimental (ALEKS) group did not outperform the control group. As the study was quasi-experimental, though, some threats to internal and external validity could not be effectively controlled, and conclusions from the study are necessarily guarded -- with the use of a convenience sample instead of random selection and random assignment, other factors in addition to the use of the e-learning activity could have influenced the results.

Findings of this study support the notion that a self-paced e-learning system can be effectively used as a supplementary learning activity. For example, a closer look at the students' self-regulative behaviors while using ALEKS revealed that the level of Math knowledge mastered in ALEKS was significantly correlated with the level of improved knowledge in Precalculus measured by the gap between a pre-test and a post-test. This finding is somewhat expected, as both results indicate students' improved knowledge in Math (therefore, the results are correlated). However, helpful implications can be drawn from this finding: First, instructors can rely on ALEKS as a homework engine that provides students with timely, reliable feedback while maintaining student accountability to accomplish the homework goals. For instructors that grade daily homework assignments, this has a profound impact on instructor time, freeing time that would have been otherwise allocated to grading of homework. Second, instructors can monitor students' use of ALEKS to detect low-level performers, and provide personal feedback and additional guidance. In other words, the use of a self-paced e-learning activity provides instructors with data and opportunities that enable them to direct their time and attention to individual students who need individualized feedback. It also makes effective use of in-class instruction while reducing the grading burden.

Another interesting finding was that the more AS type of learning style students had, the more amount of knowledge in Precalculus they gained. However, this is not too surprising, since the characteristics of AS include a high level of aptitude in abstract thinking and problem solving such as Math or Music. Instead, more attention should be paid to the group(s) of students whose learning styles are negatively correlated with their performance in Math. For example, this study indicates the possibility that the more CR tendency or preference students have, the less amount of knowledge in Precalculus they gained during the course, compared to their AS counterparts. Although this study is unable to support generalizablity of this finding, a reasonable recommendation is to provide students in Math classes, especially those with a strong CR tendency, with more 'concrete examples' of abstract Math problems and learning guidance for following step-by-step, sequential learning processes when solving problems (e.g., a job aid, a checklist, or a workbook).

Future Work:

The e-learning strategy, ALEKS, has now been used at Boise State University for three years.[13,14] As a result of this, it has been observed by several mathematics instructors at Boise State University that students that have used ALEKS in Precalculus, do very well in subsequent mathematics courses. One instructor that volunteered to participate in an ongoing study observed, "I had a student [in fall, 2006] who was struggling with Precalculus due to deficits in prerequisite material. The student started working with ALEKS as part of course work for a different class, [an engineering class] and I noticed within a few weeks that the number of errors the student made was decreasing. Having drilled on basic skills, the student was able to focus more on the Precalculus material, instead of being lost in the "basic algebra" steps. This student was ultimately successful in Precalculus, and I attributed that success, at least in part, to work on ALEKS." We postulate that an important outcome of using ALEKS that occurs during Precalculus throughout the semester, is the repair of prerequisite skills. To date, we have not quantified this by measuring prererequisite knowledge for student participants. Future work will measure the extent of prerequisite knowledge at the beginning and end of the course, as the hierarchical nature of the content in ALEKS forces students to exhibit mastery of prerequisite knowledge before learning new material. Longitudinal studies are also underway to quantify the long term effects of this e-learning strategy on student success as they progress through the calculus sequences.

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AC 2009-1783: THE IMPLEMENTATION OF AN ONLINE MATHEMATICS PLACEMENT EXAM AND ITS EFFECTS ON STUDENT SUCCESS IN PRECALCULUS AND CALCULUS

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The Implementation of an Online Mathematics Placement Exam and its Effects on Student Success in Precalculus and Calculus

Introduction

Engineering education research on the impact of freshman engineering courses reveals a close connection between graduation rate and first semester GPA.[1] The same research also explains the importance of first-semester math placement, so as to provide students with the necessary background for success. For example, students at Purdue University that earned a grade of A in a pre-calculus course in the first semester had the same engineering retention rate as students who earned a B in the first semester calculus class.[1] Yet, if those same students are placed based on their SAT math scores, such students would probably fail calculus if taken in their first semester.[1] A recent study on parameters that affect student success indicated that the grade earned in a student's first college level mathematics class was significantly correlated to whether or not those students persisted in engineering, while the level at which they began mathematics study at the university was not.[2] French, et al. conclude in their study of indicators of engineering students' success and persistence, that achievement of good grades at the student's university is an indicator of persistence, and suggests that retention programs focus on academic achievement.[3] These studies highlight the importance of timely and accurate student placement in mathematics in terms of success in engineering programs.

A number of different math assessment tools are widely used by universities for student placement in mathematics courses. These tools include the mathematics portions of the ACT [4] and SAT,[5] the mathematics AP exams,[4] COMPASS[4] examinations and CLEP[5]exams. Many universities and mathematics departments also have internal exams used for math placement that they have developed over the years and routinely administer. Student scores on the ACT and SAT exams are also used by most universities as part of their admissions criteria, and it is common practice to record and use for both admissions and placement the highest score achieved by students on these examinations. Thus, information about what students know, or presumably knew at some point in their history, is available in the form of ACT or SAT or both to mathematics departments. These scores are frequently used for first semester mathematics placement at the precalculus and calculus levels. However, the ACT/SAT information does not provide a *current* measure of a student's knowledge in mathematics. Thus, for example, if a student last took either of these examinations in the middle of their junior year of high school, and then did not take mathematics during their senior year, a significant change in current math knowledge would be expected to occur. Also, students who continued in their mathematics instruction in their senior year of high school but did not retake the SAT or ACT examination would be placed too low. At Boise State University, which is an accessible metropolitan university, it is not uncommon to encounter students that took the ACT or SAT one time only. For example, in fall 2008, among first-time first semester freshmen, 34% of engineering students at Boise State University took one of the exams (ACT or SAT) one time only, most likely during their junior year of high school.

This paper reports on a novel online math assessment strategy originally developed and deployed in fall 2007 at the University of Illinois, where it was administered to approximately 3500 students, and which now requires it as a math placement exam for all incoming first-year students. The methodology by which the assessment method was rapidly implemented at both the University of Illinois and by Boise State University is presented, together with some faculty perceptions associated with the implementation.

Online Mathematics Assessment: ALEKS

ALEKS (Assessment and Learning in Knowledge Spaces)[6] is a web-based, artificially intelligent assessment and learning system that uses adaptive questioning to determine what a student knows and what they do not yet know in a course. ALEKS was developed from an assessment and teaching system for arithmetic that was based on Knowledge Space Theory.[7] The early development was funded by the National Science Foundation in 1992. It is now a commercial system that is used by individuals and institutions to learn many levels of Mathematics. ALEKS is accessible from any computer with web access and a java-enabled web browser. Students are required to work problems and enter the solution; there are very few multiple choice answers associated with the system. In 2006, Carpenter, et al.[8] showed that student preparedness in Calculus could be predicted with ALEKS, a study that prompted several other universities to deploy ALEKS as an instructional tool to assist with Precalculus and Calculus learning.[9-11]

This study reports on the use of ALEKS as an assessment tool only – that is, the assessment aspect of ALEKS is separated from the teaching system aspect of ALEKS in this study. The application of this unproctored, internet-based system as an assessment tool is novel. Being internet-based, the system provides significant benefits to students as a result of the easily accessed remediation aspect, which is optional for students. It also provides unlimited opportunity for re-assessment. The course product that student knowledge is assessed within for this study is termed "Preparation for Calculus," which if accessed in learning mode, contains 251 topics divided as shown in Table 1. A typical assessment asks between 29 and 32 questions.

Table 1: Preparation for Calculus Curriculum (ALEKS)

Curriculum Area in ALEKS	Number of Topics
Real Numbers	30
Equations & Inequalities	30
Linear & Quadratic Functions	41
Exponents & Polynomials	30
Rational Expressions	27
Radical Expressions	21
Exponents & Polynomials	21
Geometry & Trigonometry	51

Prior Assessment Strategies

Boise State University uses a variety of indicators of student knowledge in mathematics in order to place students in Precalculus and Calculus courses. These indicators include the Math ACT and SAT, the AP Calculus AB exam and the COMPASS examination, see Table 2. The minimum scores necessary to place into Precalculus and into Calculus are shown, together with the ALEKS assessment benchmarks used in this study.

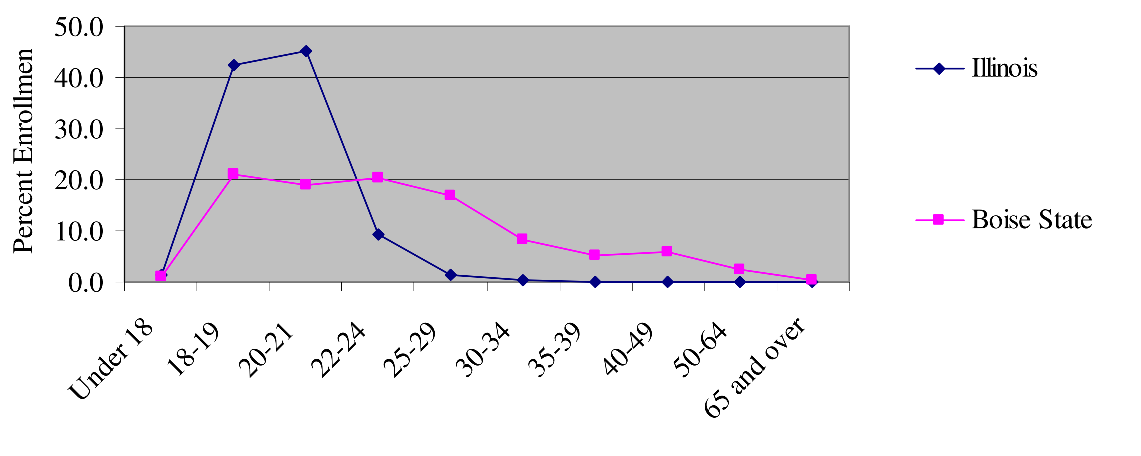
Table 2: Math Placement at Boise State University

Math Level	ACT score	SAT score	COMPASS	AP Exam (AB)	ALEKS
Precalculus	23	540	61 (ALGP)	N/A	40%
Calculus	29	650	51 (TRIG)	3	70%

Institutional Information:

University of Illinois is a very large urban campus that awarded more than 1200 bachelor's degrees in engineering and computer science in 2007. The total undergraduate enrollment in fall 2007 was 30,895, including 6,940 first-time freshmen. By contrast, Boise State University has approximately half the total enrollment of University of Illinois, with a total of 17,574 undergraduate students in fall 2008, and a full-time equivalent enrollment of 14,608. Its engineering college is young, having been formed in 1997, and approximately 130 engineering bachelor's degrees were awarded in 2007. There were 1900 first-time full time freshmen in fall of 2007. The enrollment distribution, by age, for both universities that deployed the online ALEKS assessment is shown in Figure 1, which illustrates the need for a *current* measure of mathematics knowledge for Boise State University students, many of whom are years beyond high school.

Figure 1 Age Distribution of Students



Both universities that deployed the ALEKS assessment strategy focused on two math levels, Precalculus and Calculus I. At the University of Illinois enrollment in these two courses in fall 2007 was approximately 3500. At Boise State University enrollment in these two courses in fall 2008 was approximately 750. This study presents the implementation strategy used at both universities that enabled the system to be rapidly deployed and institutionalized, together with first semester results from Boise State University.

Implementation Strategy:

The University of Illinois developed the following implementation strategy during their first deployment of the ALEKS assessment system in fall 2007. Boise State University adopted a nearly identical strategy for their deployment in fall 2008. The strategy required a benchmark score within the ALEKS Preparation for Calculus curriculum, prior to the end of the open enrollment period at the beginning of the semester (add/drop). This benchmark score was set at 40% of the curriculum for the Precalculus course and 70% for the Calculus course. The personal motivation for students to take the assessment and to achieve the benchmark was based on the fact that achievement of the benchmark score would consist of 10% of their grade in their upcoming course. In other words, each student's first assignment for their course, due by end of add/drop, was to achieve either 40% or 70%, depending on which course they were enrolled in. If this benchmark was not achieved, the premise was that students would self-select down one math level rather than get a zero on such a large portion of their grade. This premise proved true, and there was approximately 99% compliance with student self-selection of courses. At University of Illinois, less than 1% of students altogether elected to remain in the course without the benchmark score in fall 2007. At Boise State University only 1.5% of students (out of 733) elected to remain in the course without the benchmark score in fall 2008.

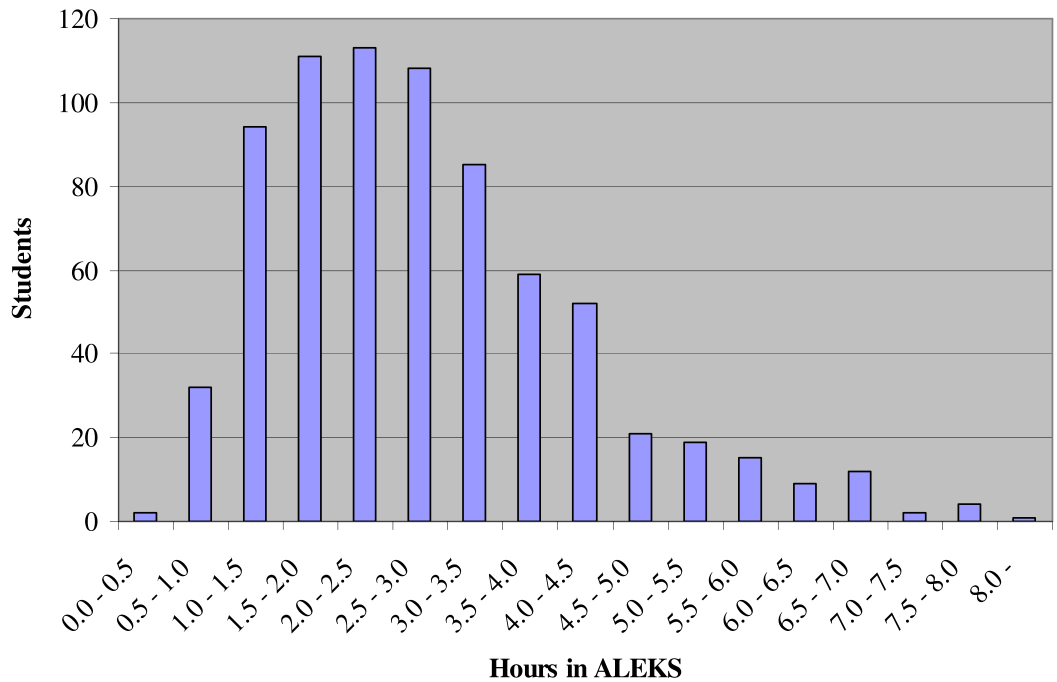
At both universities all students that registered for the course, whether during the summer or prior to that if returning students, were notified electronically of this important first assignment, and sent a hyperlink to the assignment at least five times prior to the start of classes. Each university maintained their own website containing up-to-date information including FAQs and detailed instructions. Students first went to the university website to obtain the instructions, and then began the assessment through the ALEKS website. Students entered a particular "course number" that identified their university to the ALEKS system, and also entered their university student identification number. This enabled the results to be archived by ALEKS and available in various downloadable formats to each university. Assessment licenses, enough for one per student taking the assessment, were purchased by each institution by their respective Provosts' offices. This allowed each student to be assessed once at no cost to them and was essential to the rapid implementation of the new assessment strategy at each institution. Students could elect, at their own expense, to take additional assessments for $3.60 (fall 2008 pricing information). Alternatively, students could elect to purchase an assessment and learning module ($36.90 fall 2008 pricing) which provided automatic and nearly unlimited reassessment to students for six weeks. As a further incentive, at Boise State University the Provost provided funds to reimburse 50% of the purchase price for any student who used the Learning Module to successfully meet the benchmark. Approximately 5% of students took advantage of this offer.

Results at Boise State University

Assessment Data

A total of approximately 750 students took the ALEKS assessment. An average time of 160 ± 90 minutes was spent; approximately 15 minutes of this time would have involved learning to use the methods of entering answers within the ALEKS tutorial which is launched prior to any assessment questions. Figure 2 shows a histogram of time spent doing the assessment for all students that took the assessment at Boise State University in fall 2008.

Figure 2: ALEKS Assessment Duration



Effect of Assessment on Student Success

We seek to answer the question "What effect did the ALEKS requirement have on student success rates." No answer is possible without a definition of *success.* Both Calculus and Precalculus at Boise State University are taught in individual sections with individual instructors solely responsible for all exams, assessments, grading rubrics, and final letter grades. There are no pre-determined learning outcomes, and even if there were, there are no standardized or even commonly agreed upon assessments that could be used to indicate *success.* The only measurement we have available is the Pass Rate, defined as follows:

- ABC = number of A's, B's and C's, including plus/minus grades

- DWF = number of D’s, F’s and W’s, including plus/minus and CW grades.
- Pass Rate = ABC / (ABC + DWF)

Note that the denominator may not always match enrollment, since there are a small number of audits and unresolved incompletes. These grades are appropriately not part of pass rate computation.

Enrollment trends

There are two ways that the ALEKS assessment could reasonably influence success rates.

1. Primarily, we expect students who do not meet the minimum assessment to drop the class before the 10th day of enrollment (Sept 8, 2008).
2. Less significantly, some students may discover their lack of preparation and self-remediate through ALEKS.

This first of these should be visible in enrollment data, and indeed this was the case for fall 2008. Historically we see nearly full enrollment at the beginning of the term followed by a decline of approximately 5% across the first 10 days. Expected versus actual enrollments are shown in Table 3.

Table 3: Enrollment, Fall 2008 at Boise State University

	Precalculus		Calculus	
Peak enrollment	434	100%	309	100%
Predicted 10th day	412	95%	294	95%
Actual 10th day	367	85%	278	90%
Forced out by ALEKS?	45	10%	16	5%

The last row of the table is a rough calculation. However, the ALEKS assessment requirement had a clear impact on enrollments. The day-to-day changes in enrollment are shown below. The converging graphs are total enrollment and successful ALEKS assessment. The sharp change on Aug 29 corresponds to the official deadline for completing the ALEKS assessment. Late assessments were allowed for a few students who added after the first day of class.

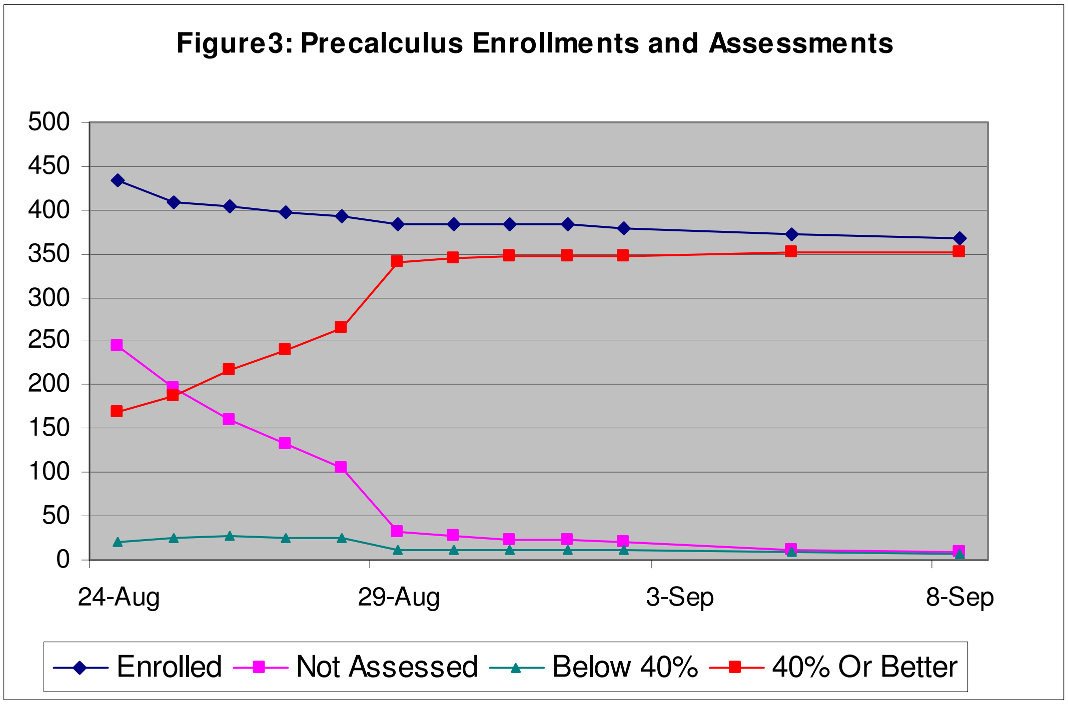


Figure3: Precalculus Enrollments and Assessments
500
450
400
350
300
250
200
150
100
50
0
24-Aug
29-Aug
3-Sep
8-Sep
Enrolled
Not Assessed
Below 40%
40% Or Better

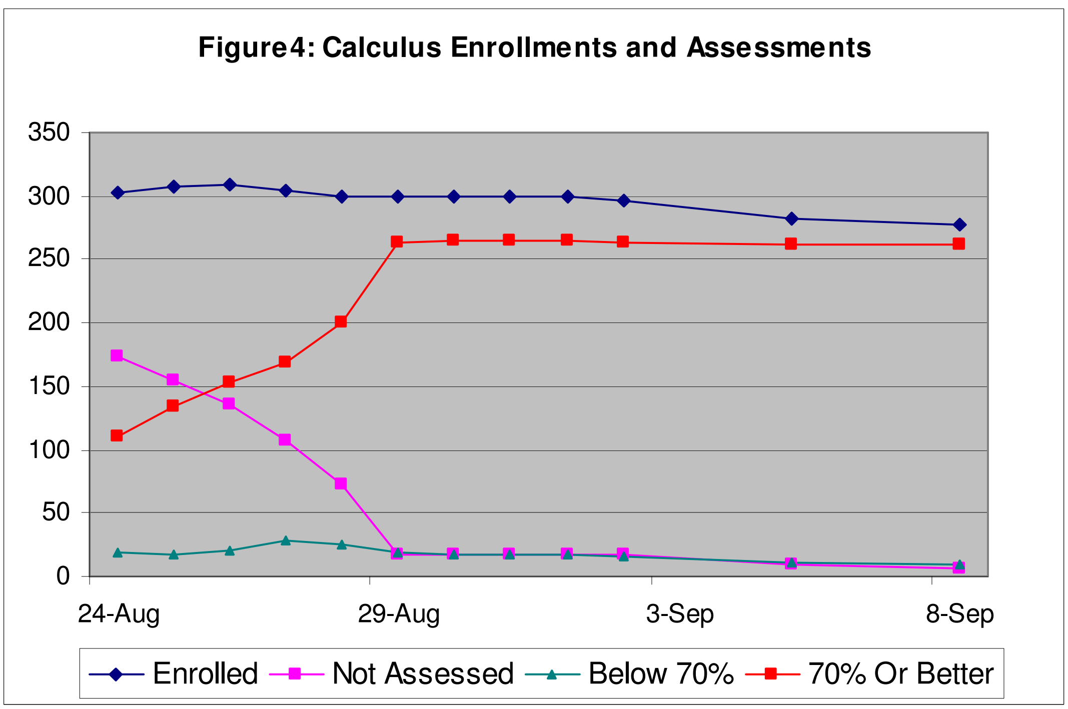
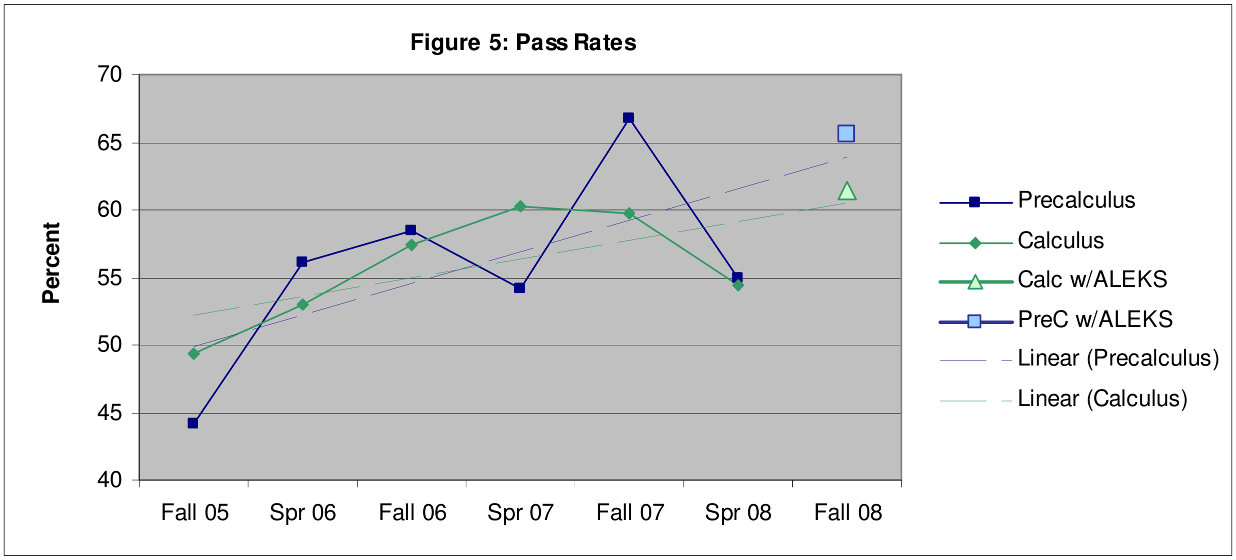


Figure4: Calculus Enrollments and Assessments
350
300
250
200
150
100
50
0
24-Aug
29-Aug
3-Sep
8-Sep
Enrolled
Not Assessed
Below 70%
70% Or Better

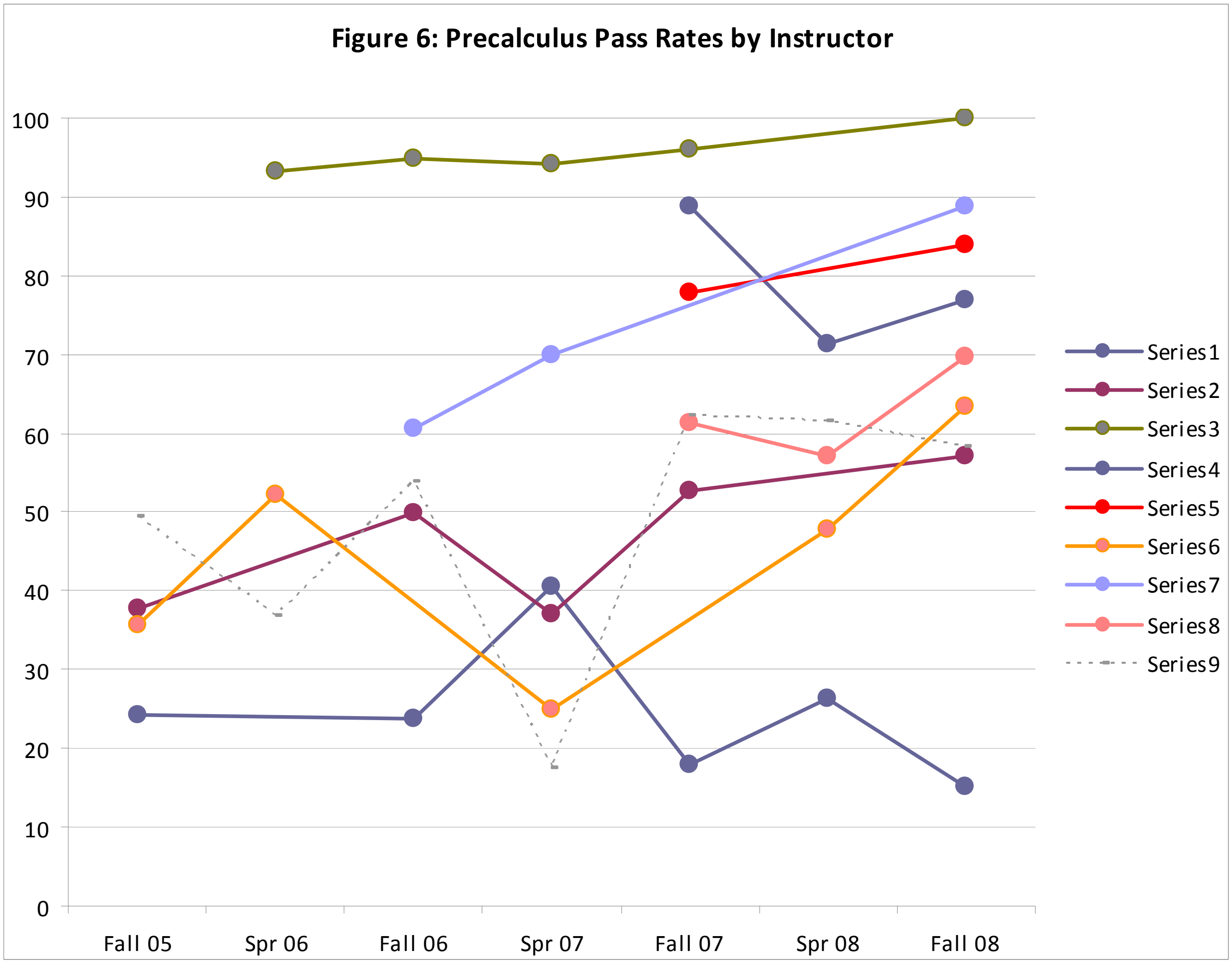
Pass Rates

The simplest measurement of the impact of the ALEKS assessment is to compare historical pass rates to pass rates in fall 2008. The chart below shows pass rates across the university from fall 2005 through fall 2008. From fall 2005 through spring 2008 there is a slight positive trend, indicated with a dashed trendline. The effect of ALEKS in fall 2008 is a barely perceptible bump above the projected trend. This fails to control for any factors except the historical trend. In particular, it does not attempt to control for influence of individual instructors.



Instructor Influence on Precalculus Pass Rates

To examine the effect of ALEKS on individual instructors we first discard any instructors who did not teach Precalculus in both fall 2008 and at least one prior semester. This removes 13 instructors and 40% of the student data (860 of 2106 records). Pass rate data for the remaining eight instructors are shown below. (The dashed line represents the aggregate pass rate seen in the discarded data. We include this to point out that its removal is not unduly sanitizing.)



Clearly there is enormous and persistent variability from instructor to instructor. Rather than analyze the effect of ALEKS across all sections, we compute the effect on each instructor using three different methodologies.

1. **Year-over-Year**: defined as fall 08 pass rate minus fall 07 pass rate. This seeks to control for the effects of changes in the student body, both over time and from fall to spring.
2. **Before-After**: defined as fall 08 pass rate minus aggregate pass rate from prior semesters (fall 2005 to spring 2008). This does not control for possible trends in the student body, but does give a much larger data set. We consider this the weakest measure.
3. **Trend**: defined as fall 08 pass rate minus the pass rate predicted by a best fit regression over prior semesters. This would theoretically give the best control over both changes in student body and changes in the instructor's pedagogy or methods. However the data are extremely sparse and short time series will often give hugely misleading predictions.

For some instructors it is not possible to do all three of these – two did not teach in fall 07, and four have very short trends (either one or two prior semesters). Results are shown in Table 4.

Table 4: Pass Rate Gains for Individual Precalculus Instructors

Instructor	Before-After Gain	Year-over-Year Gain	Gain against Trend
A	-11.1	-2.7	-10.9
B	-7.1	-12.0	
C	5.3	3.8	3.0
D	6.2	6.2	
E	10.6	8.4	
F	12.7	4.5	6.6
G	20.8		21.2
H	23.4		
Average	7.6	1.4	5.0

Significance

It appears that regardless of methodology, most instructors saw a positive change in their fall 2008 pass rates. This suggests testing the null hypothesis, “Requiring the ALEKS assessment does not affect an individual instructor’s pass rate.” If true, then the number of instructors who show positive gain follows a binomial distribution. The 5% rejection region would be positive gain by 7 or more instructors. Under normal hypothesis testing parameters of 95% confidence we would have to accept the null hypothesis. The precise probabilities for $n = 8$ are $P (>= 7) = 0.035$ and $P (>= 6) = 0.145$, so we could reject with 85% confidence.

Analysis of Calculus Pass Rates

One might apply the same analysis to Calculus sections. Unlike Precalculus, which is mostly taught by full time lecturers with heavy and repetitive teaching loads, Calculus is taught by a large rotation of research faculty with lighter and more varied loads. Since fall 2005 there have been 27 instructors of Calc I. Of these, only eight taught Calculus in fall 2008 and two of those had no prior experience. Restricting to the remaining six removes 70% of the data (1263 of 1793 records), and leaves few year-over-year or trend comparisons.

Table 5: Pass Rate Gains for Individual Calculus Instructors

Instructor	Before-After Gain	Year-over-Year Gain	Gain against Trend
U	-10.6		
V	-1.9	-1.9	
W	-0.2		-7.7
X	15.2	15.2	
Y	19.7		
Z	29.4		
Average	8.6	6.6	-7.7

These are inconclusive results and only apply to 30% of Calculus students. Even the weak null hypothesis of "ALEKS does not affect instructors' results" is clearly impossible to reject.

An Alternative Analysis

Since instructor level analysis for Calculus gave nearly random results and drastically restricted the data set we propose an alternate analysis, aggregating all pass fail data for various groups of instructors. For each of Precalculus and Calculus there are four reasonable groups:

- Group I: All instructors.
- Group II: Discard from Group I the instructors with the highest and lowest pass rate trends. (Note the two clear outliers in Figure 6.)
- Group III: All instructors who taught in Fall 2008 and at least one applicable prior term.
- Group IV: Group III with outliers removed (as identified in Group II).

For each group one may compare fall 2008 aggregate pass rate against the rate for fall 2007 (Year-over-Year) or against all prior terms (Before-After). "All prior terms" refers to six semesters of data, between fall 2005 and spring 2008, inclusive. The advantage is reasonably large data sets. The disadvantage is failure to control for instructor influence beyond hoping that it averages out. Precalculus results are presented in Table 6. Note the heavy influence of outliers.

Table 6: Precalculus Aggregate Pass Rates						**Percentage Reduction in DWF Rate**
Before-After						
	All prior terms		Fall 2008		Raw Increase	
	Sample Size	Pass Rate	Sample Size	Pass Rate		
Group I	1795	56.2%	353	65.6%	9.4%	**21%**
Group II	1318	51.4%	282	67.0%	15.6%	**32%**
Group III	926	61.2%	274	67.8%	6.8%	**17%**
Group IV	449	52.3%	203	70.4%	18.1%	**38%**
Year-over-Year						
	Fall 2007		Fall 2008		Raw Increase	
	Sample Size	Pass Rate	Sample Size	Pass Rate		
Group I	403	66.7%	353	65.6%	(-1.1%)	**(-3%)**
Group II	297	63.5%	282	67.0%	3.5%	**10%**
Group III	211	70.6%	202	68.2%	(-2.4%)	**(-8%)**
Group IV	105	65.7%	131	72.5%	6.8%	**20%**

Fortunately, individual Calculus instructors do not display such extremes in pass rates. This makes outliers difficult to detect but also lowers their impact. We therefore conclude with results for Calculus with just Group I and Group III.

Table 7: Calculus Aggregate Pass Rates						Percentage Reduction in DWF Rate
Before-After						
	All prior terms		Fall 2008		Raw Increase	
	Sample Size	Pass Rate	Sample Size	Pass Rate		
Group I	1519	55.8%	274	61.4%	5.6%	**13%**
Group III	294	47.6%	236	60.4%	12.8%	**24%**
Year-over-Year						
	Fall 2007		Fall 2008		Raw Increase	
	Sample Size	Pass Rate	Sample Size	Pass Rate		
Group I	302	59.7%	274	61.4%	1.7%	**4%**
Group III	65	57.1%	80	64.6%	7.5%	**17%**

In summary, it appears that the addition of the ALEKS assessment as a course requirement has a positive impact on the pass rates of students in both PreCalculus and Calculus. Year-over-year impacts are less pronounced than historical averages compared to the ALEKS semester. This may reflect other ongoing efforts to improve performance in these two courses at Boise State University, as suggested by the positive trends in overall pass rates prior to fall 2008.

Math Instructor Survey

A survey of mathematics instructors at the Precalculus and Calculus levels was conducted at the end of the fall 2008 at Boise State University, in an effort to capture instructor perceptions of various questions that related to the assessment. Using a scale of 1 to 7, where 1 is "Disagree strongly," 4 is "Neither agree nor disagree," and 7 is "Agree strongly," mathematics instructors at the Precalculus and Calculus levels were surveyed at the close of the fall 2008 semester. Highly experienced instructors (those who had taught the courses seven or more times) agreed strongly (7.0 out of 7.0) that student placement is critical in terms of student success in math courses. They agreed slightly/moderately (5.4) that this semester's students had adequate preparation for the course. They agreed slightly (5.0) that the last semester they taught the course, that students also had adequate preparation for the course. They agreed moderately (5.9) that there was a difference in students' mathematics preparation this semester as opposed to previous semesters, but they agreed only slightly (4.6) that students were significantly more prepared in fall 2008 as compared with previous semesters. They agreed slightly/moderately (5.4) that a greater percentage of students who started the course persevered as opposed to in prior semesters. They neither agreed nor disagreed (3.9) that student performance on quizzes, tests, and other assessments indicated greater mastery of course material this semester as opposed to previous semesters. A total of seven instructors fell into the category of "highly experienced instructors." When the responses of all surveyed instructors were included (15 responses), the same trends were observed but to a slightly lower degree. One unsolicited remark from an instructor indicated, "The main difference I noted was that I was missing the students who made 10's, 20's or 30's (percents) on the first test. After that first test, I did not really see much difference in the students' work. Many of my students scored 50 or above on ALEKS and did poorly in the course. I see no relationship between their ALEKS score and their performance in Precalculus." This remark highlights the fact that these instructors were aware of their students' ALEKS scores, which may have influenced their survey responses.

Discussion

The ALEKS Assessment: Accessible and Unproctored

The ALEKS assessment strategy is online, enabling the assessment to be widely accessible – anywhere, anytime. Both universities elected to conduct the assessment in an unproctored environment. The rationale for this included first and foremost, the fact that placement into a particular Precalculus or Calculus course did not eliminate any university requirements. That is, students did not receive "credit" for the prerequisite course by receiving a certain score on the ALEKS assessment. Rather, they simply placed into the appropriate level that they showed themselves to be ready for. Second, it was not deemed inappropriate for students to use the assessment process as part of a personal review of mathematics. That is, if students took a long time to answer questions (while they looked up or remembered how to solve various problems), it was considered time well spent in review. In fact, following their initial assessment, approximately 5% of students at Boise State University went on to purchase the online assessment and learning module, on which they spent an average of 18 hours. Finally, if students received personal assistance during the assessment, it clearly was a self-limiting behavior that would result in subsequent poor academic performance in the student's mathematics course.

Upward Mobility

An interesting outcome of using the ALEKS assessment strategy was that it gave students whose SAT or ACT scores placed them in Precalculus an opportunity to enroll in Calculus, and those that placed at College Algebra or lower, an opportunity to enroll in Precalculus. A remark frequently heard from engineering students during summer orientation at Boise State University, was, "I took Calculus in high school, why do I need to enroll in Precalculus again?" The ALEKS assessment strategy gave those students a chance to demonstrate, to themselves as well as to the mathematics department, that they were indeed ready for Calculus. The Chair of Mathematics at Boise State University personally interviewed each student that did not have the required ACT/SAT/COMPASS/AP scores (according to Table 2), but that did realize sufficient ALEKS scores. A total of 37 students (about 5%) fell into this category; 7 of them placed into Precalculus, 24 enrolled in Calculus, and three others enrolled in other mathematics courses. Although the sample sizes are too small for meaningful analysis, the pass rate for these Precalculus students was about the same as for the Precalculus students with the same instructor group, and the pass rate for these Calculus students was slightly lower (8%) than Calculus students with the same instructor group.

Summary

A novel online assessment strategy for assessing current student knowledge in the Preparation for Calculus curriculum was deployed in an unproctored environment at two universities. This strategy consisted of requiring benchmark assessment scores in the curriculum that is offered in an online environment through the ALEKS Corporation. Students were highly motivated to comply with the assessment requirement because 10% of their grade was based on their achievement of the benchmark assessment level set for their course. These levels were 40% for Precalculus and 70% for Calculus. Each university paid for one assessment for the students, and students were allowed to re-assess as many times as they wished. Analysis of the data from Boise State University yields the conclusion that the addition of ALEKS assessment as a course requirement has a positive impact on student pass rates. Depending on the analysis selected, whether before-after, or year-over-year, these improvements demonstrated a raw increase in pass rate for Calculus of (5.6, 12.8, 1.7 and 7.5%), which corresponded to percentage reductions in DWF rates of (13, 24, 4 and 17%). For Precalculus, if all instructors are included, raw increases in pass rate of (9.4, 6.8, -1.1 and -2.4%) are seen, corresponding to percentage reductions in DWF rates of (21, 17, -3 and -8%). If the Precalculus instructors with the highest and lowest pass rates are not included in the analysis, raw increases in pass rate of (15.6, 18.1, 3.5 and 6.8%) are seen, corresponding to percentage reductions in DWF rates of (32, 38, 10 and 20%). All in all, the ALEKS online assessment strategy is an excellent tool for assessing <u>current student knowledge</u> so as to assure proper placement in Precalculus and Calculus.

Future Work:

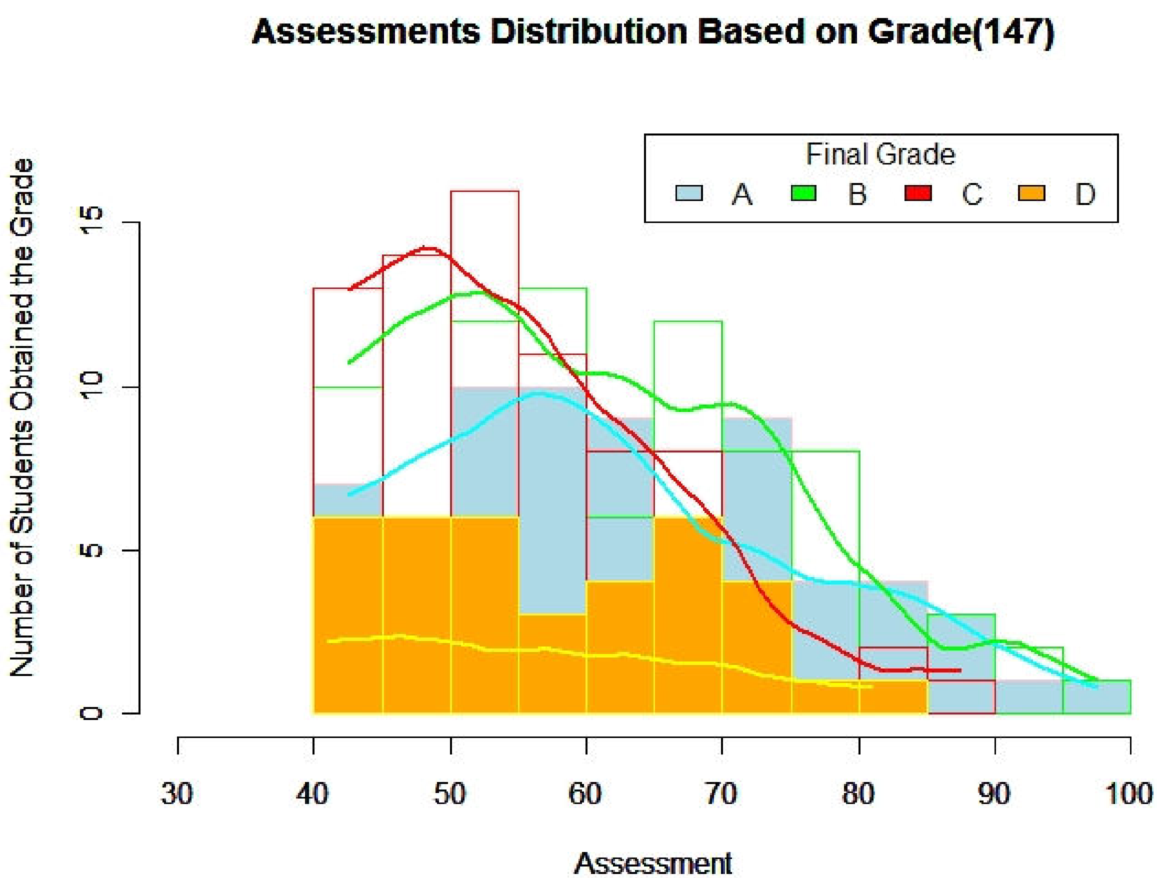
The absence of any definition of success other than a pass rate that is heavily dependent on individual instructors makes analysis of any other variable difficult. Fortunately, longitudinal analysis allows other measures. Success in subsequent courses (although equally skewed by instructor variation) is a possible measure. Another valuable measure will be the ALEKS assessments taken by students entering Calculus in spring 2009 and future semesters. This will function as a post test for students who completed Precalculus in the prior term. Longitudinal analysis can allow for recalibration of data discussed in this paper. It will also provide an additional comparison of various cohorts of students after they complete the full sequence of Precalculus, Calculus I and Calculus II.

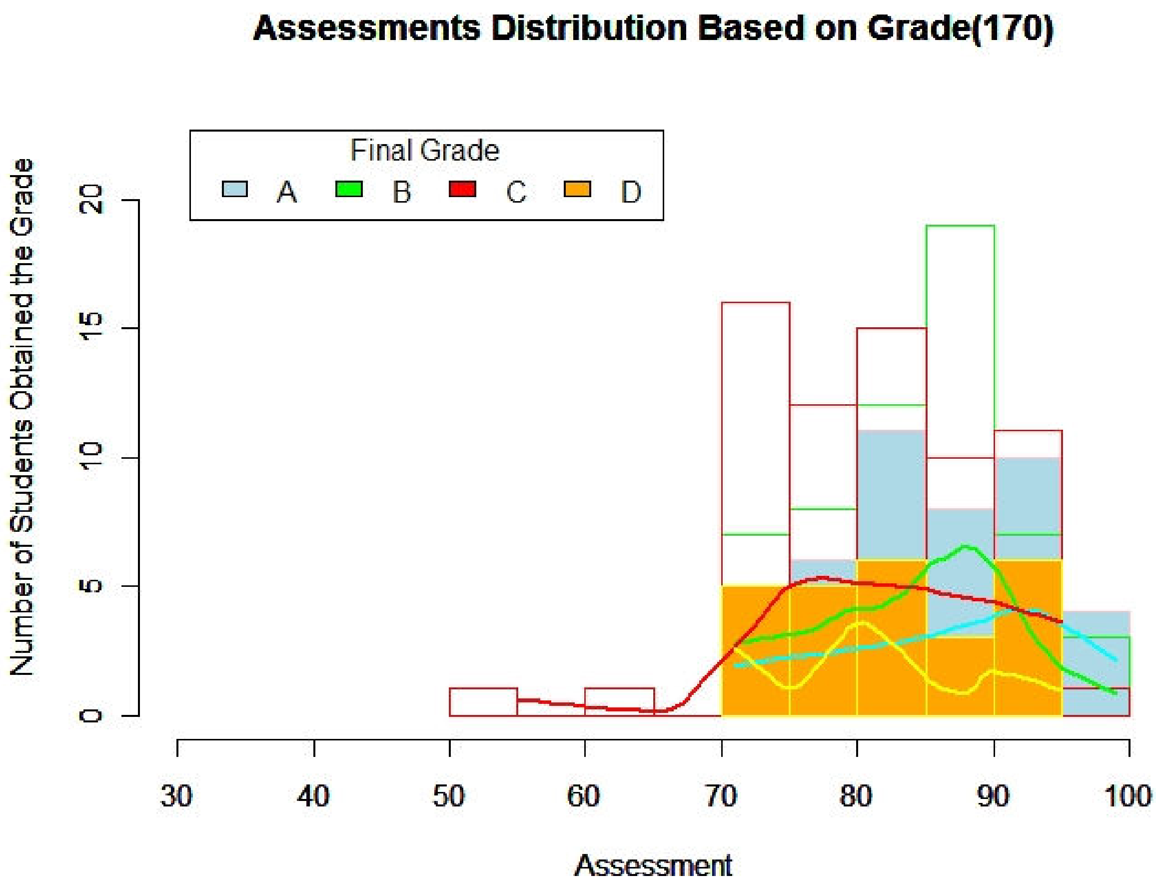
Acknowledgments

The authors gratefully acknowledge the support of the William and Flora Hewlett Foundation's Engineering Schools of the West Initiative, which helped jumpstart a long lasting focus on student success at Boise State University.

Added in Proof, ALEKS and course grades.

During the review period after this paper was submitted, we completed additional analysis of the correlation between ALEKS score and final grades at Boise State University. For each course, Calculus and Precalulus, the data set was scrubbed of all grades except A, B, C or D. These were assigned weights of 4, 3, 2 and 1 respectively, and the least squares regression was computed with independent variable ALEKS score and dependent variable final grade. The regression slope was positive but quite small. However, in both classes we were able to reject the null hypothesis, "Slope is 0" with p-value < 0.1. In other words, there is 90% confidence that letter grades are positively correlated with ALEKS score. The data may be visualized in the following histograms. Math 147 is Precalculus. Math 170 is Calculus.




Assessments Distribution Based on Grade(170)
Final Grade
A
B
C
D
Number of Students Obtained the Grade
0
5
10
15
20
30
40
50
60
70
80
90
100
Assessment

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AC 2009-1873: IMPROVING STUDENTS' LEARNING IN PRECALCULUS WITH E-LEARNING ACTIVITIES AND THROUGH ANALYSES OF STUDENTS' LEARNING STYLES AND MOTIVATIONAL CHARACTERISTICS

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Improving Students' Learning in Precalculus with E-Learning Activities and through Analyses of Student Learning Styles and Motivational Characteristics

Abstract

During the spring semester of 2008, a quasi-experimental study with 138 students who were enrolled in 4 sections of an undergraduate Precalculus class was conducted. The study investigated (1) the effectiveness of using a systematically sequenced and managed, self-paced e-learning program, ALEKS, on academic performance of students with different learning styles, and (2) the relationship among the students' dominant learning styles, motivational characteristics, and overall performance in the Precalculus class. Students in the experimental group, consisting of 2 of the 4 sections of the course, were assigned to complete ALEKS as homework assignments throughout the semester. Students in the control group, consisting of the other 2 sections of the course, completed a series of traditional paper-and-pencil homework assignments instead. Students' dominant learning styles were measured by Gregorc Style Delineator™. Their motivational orientations and learning strategies were measured with the Motivated Strategies for Learning Questionnaire. A pre-test and a post-test, measuring students' entry- and exit-knowledge levels in Precalculus, were administered in both experimental and control groups at the beginning and at the end of the semester. This study revealed that *sequential-type* students who used ALEKS outperformed *sequential-type* students who completed handout homework assignments and *random-type* students who used ALEKS or handout homework assignments by one letter grade, although this difference was not statistically significant. Several instructional implications related to students' learning styles, motivational characteristics, and academic performance are discussed. Especially, students with a dominant abstract-random style may need more tailored learning support to be more successful in a Precalculus class.

Theoretical Frameworks

Effective Delivery Media and Methods: E-Learning vs. Traditional

Computer technology has been a paradigm-shifting agent in education since the first computer generation of mainframes during the 1960s and 1970s, and throughout the second generation of desktop computers and the third generation of the Internet and the World Wide Web during the 1980s and 1990s.[1] E-learning is especially ideal for individualized instruction. In contrast to one-to-many classroom learning, web technologies can help adjust the pace, sequence, and method of instruction to better fit each individual student's learning behavior and needs. Presently, e-learning is deeply integrated into school curricula to facilitate learning,[2] and a fair amount of literature discusses that traditional science, technology, engineering and math (STEM) education can be greatly benefited by incorporating e-learning strategies.[3, 4, 5, 6, 7]

One such e-learning program available in STEM education is ALEKS (Assessment and LEarning in Knowledge Spaces).[8] This web-based program provides a systematically sequenced and managed, self-paced environment, designed to help students improve Math skills. In

ALEKS, a variety of different mathematics levels, or courses can be selected, and within each course, the curriculum can be customized through selecting/deselecting certain topics. This research is focused on the Precalculus curriculum, and consisted of 181 topics in all. Students must successfully work through the topics in order to master the content. At any given time, a variety of topics may be selected to be learned by the student; however, each topic has a set of prerequisite topics that must be mastered before it may be worked on. Thus, for example, a student may not proceed to learn a rather complicated trigonometry topic until various prerequisite algebra topics within the Precalculus course are mastered. ALEKS provides immediate feedback concerning the correctness of the student's response (see Figure 1). It also provides elaborated explanations for any problem. As the student masters the topics, the data are added to the ALEKS MyPie, which presents a summary of the student's current performance level and offers more complex topics available for him or her to work through, with the end goal being mastery learning of Precalculus (see Figure 2 and Figure 3).

Using an e-learning program such as ALEKS for practicing Math skills implies potentially significant advantages over using traditional "pencil and paper" homework assignments. First, the student immediately receives diagnosed feedback as to whether he or she is doing the problem correctly. Although delayed feedback may be appropriate in certain context because it allows students to have sufficient time to solve problems on their own which may in turn increase retention of the information,[9] it is often important to provide immediate feedback to students who are working on a series of drill-and-practice type Math problems so that they are able to master each topic before they move on.

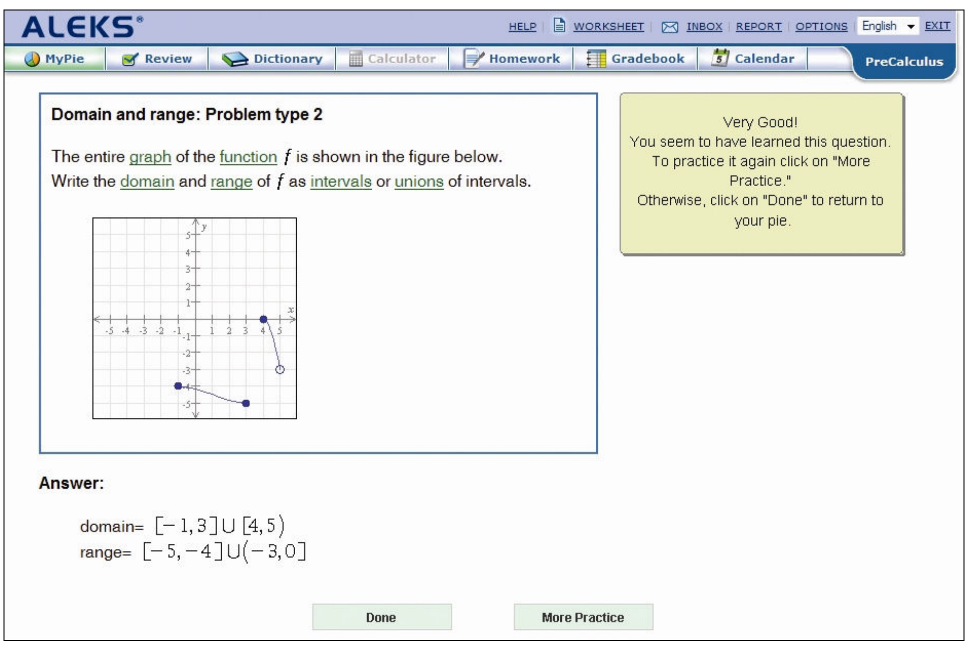

Figure 1. A screen shot of the Learning Mode in ALEKS. [Note: ALEKS product screen shot reprinted with permission from ALEKS Corporation.]

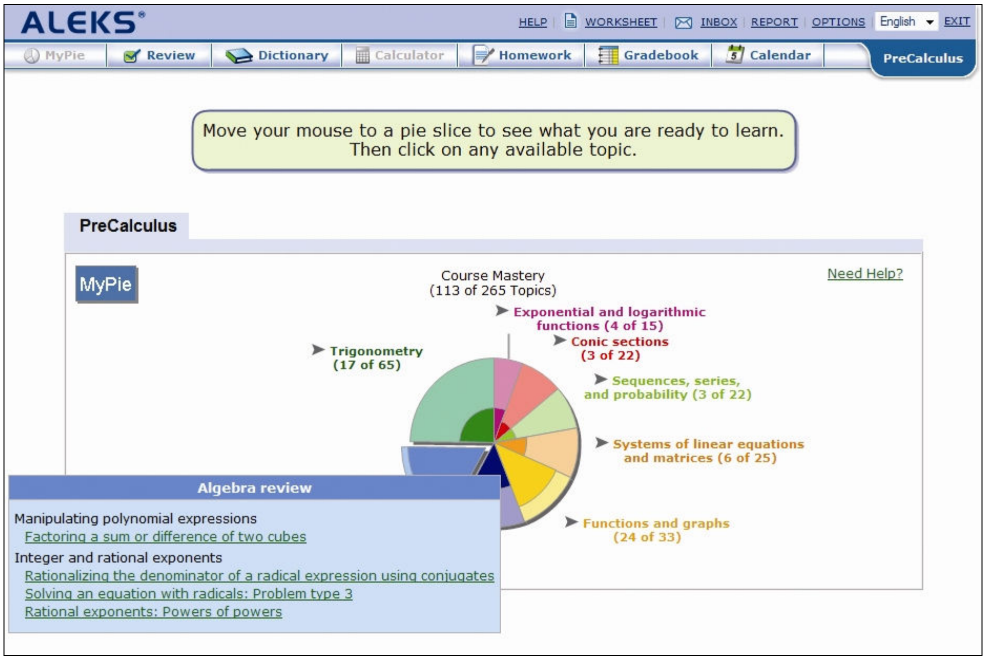

Figure 2. A screen shot of MyPie in ALEKS: The darkened portion of each pie slice represents the topics that the student has mastered and the lighter portion represents what the student has yet to learn. [Note: ALEKS product screen shot reprinted with permission from ALEKS Corporation.]

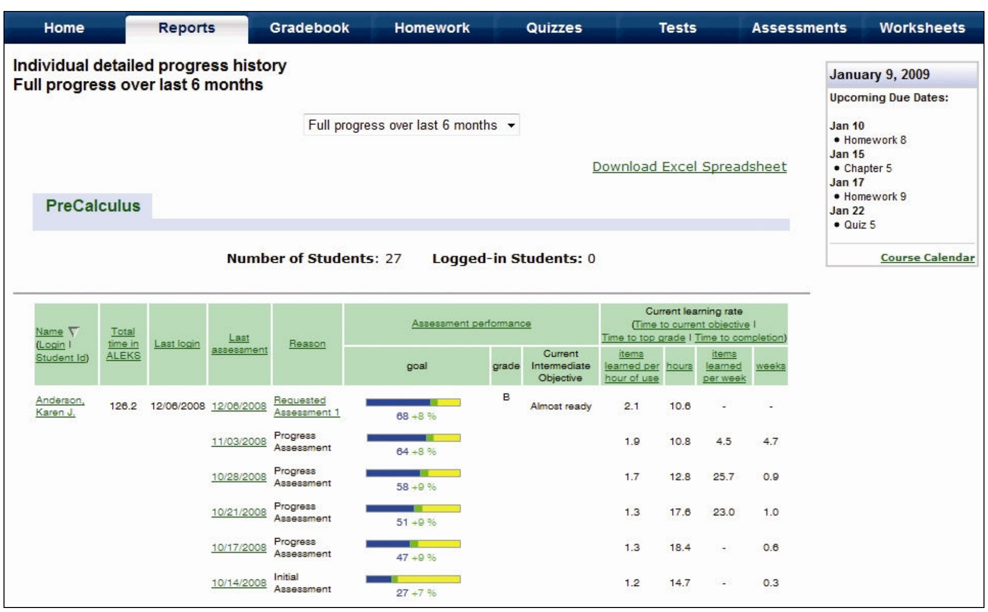

Figure 3. A screen shot of Learning Progress in ALEKS. [Note: ALEKS product screen shot reprinted with permission from ALEKS Corporation.]

Using a program such as ALEKS as homework in lieu of a series of written homework assignments can also reduce the load on the instructors, allowing them to spend more time on other responsibilities such as curriculum improvement, student advising, and professional development. Therefore, it is important to investigate whether an e-learning program such as ALEKS is more effective than, or at least as effective as, traditional pencil and paper homework assignment on helping students improve Math skills.

Relationship between Learning Styles and Academic Performance

Individual students in the same classroom may have different learning experiences due to their characteristics such as learning styles. For example, some students may learn information in a sequential manner more effectively while others tend to approach new information in a more random fashion. Therefore, educators should take into account such characteristics of learners in order to deliver instruction with more effective media and methods for individual students.

A well-known instrument for measuring learning styles is Gregorc Style Delineator™. The Style Delineator measures four qualities of concreteness, abstraction, sequence, and randomness in people's perception toward, and ordering of, their world.[10] Perceptual abilities are the ways through which people obtain information – in a concrete or abstract way. Ordering abilities are ways in which people organize information – in a sequential or random way. The instrument identifies degrees of abilities with four style types: concrete sequential (CS), abstract sequential (AS), concrete random (CR), and abstract random (AR) (see Table 1). Every individual is believed to be capable of orienting himself or herself toward all four styles; however, people tend to have strong orientation toward one or two, which are viewed as their dominant styles.

Although no one particular learning style is better than another, research has shown strong correlation between dominant learning styles and academic performance in certain learning subjects. Especially, the sequential-random dimension is shown to be a stronger predictor than the concrete-abstract dimension in many areas including STEM.[11] For example, research has revealed that sequential learners perform significantly better than random learners in computer application courses[12] and other Science and Math-related courses, while random learners excel in Fine Arts courses.[13]

Table 1. Four Learning Style Types Identified by Gregorc Style Delineator.

Sequential (S)		**Random (R)**	
Concrete (C)	**Abstract (A)**	**Concrete (C)**	**Abstract (A)**
Concrete-Sequential (CS)	Abstract-Sequential (AS)	Concrete-Random (CR)	Abstract-Random (AR)

Motivational Orientations and Learning Strategies

In addition to learning styles, students' motivational orientations and learning strategies that they use also likely influence their learning processes. These characteristics can be measured with the Motivated Strategies for Learning Questionnaire (MSLQ), which was developed by a group of researchers in the University of Michigan in the early 1990s.[14] The instrument is intended to measure motivational orientations college students exhibit and learning strategies they use in a college course. The complete MSLQ contains 15 sub-scales, including 6 sub-scales for motivational orientations and 9 sub-scales for learning strategies (see Table 2).

The MSLQ has been used in research to help understand the nature of learner motivation and use of learning strategies in various subject areas such as statistics, chemistry, technology, social studies, and physical education.[15] Research has shown that learner characteristics measured by the MSLQ have strong associations with their self-regulative learning processes and academic performance. Based on research conducted by Pintrich and his colleagues at the University of Michigan, the MSLQ has become a standard instrument for conducting research on self-regulation and motivation. The generally accepted conclusion is that positive motivational orientations (e.g., intrinsic goal, high task value, high self-efficacy, and low test anxiety) are related to higher levels of self-regulated learning strategies, which in turn are related to better academic performance.[16] Research conducted with the MSLQ can enable instructors to diagnose student characteristics and to develop appropriate instructional strategies to help students improve learning.[17]

Table 2. Sub-Scales of the Motivated Strategies for Learning Questionnaire.

Category	Sub-Category	Sub-Scale	Explanation
Motivational orientations	Value components	1. Intrinsic goal orientation	Perceiving themselves to participate in a task for reasons such as challenge, curiosity, and mastery.
		2. Extrinsic goal orientation	Perceiving themselves to participate in a task for reasons such as grades, rewards, performance, evaluation by others, and competition.
		3. Task value	Learners' evaluation of how interesting, how important, and how useful the task is.
	Expectancy components	4. Control belief	Learners' beliefs that their efforts to learn will result in positive outcomes.
		5. Self-efficacy for learning and performance	A self-appraisal of one's ability to accomplish a task as well as one's confidence in having skills to perform that task.
	Affective components	6. Test anxiety	Cognitive thoughts and emotional feelings toward taking tests.
Learning strategies	Cognitive and metacognitive strategies	7. Rehearsal	Reciting or naming items from a list to be learned.
		8. Elaboration	Building internal connections between items to be learned by paraphrasing, summarizing, creating analogies, and generative

			note-taking.
		9. Organization	Clustering, outlining, and selecting the main idea in reading passages.
		10. Critical thinking	Applying previous knowledge to new situations in order to solve problems, reach decisions, or make critical evaluations with respect to standards of excellence.
		11. Metacognitive self-regulation	The awareness, knowledge, and control of cognition.
	Resource management strategies	12. Time and study environment	Scheduling, planning, and managing one's study time, and setting places to do class work.
		13. Effort regulation	Students' ability to control their effort and attention in the face of distractions and uninteresting tasks.
		14. Peer learning	Dialoguing and collaborating with peers.
		15. Help seeking	Recognizing needs for help, identifying others who can provide help, and asking for help.

Research to Improve Students' Learning of Precalculus

Based on the literature review presented above, it was questioned if using an e-learning program such as ALEKS, compared to using traditional handout-type homework assignments, could be an effective method for helping students learn Precalculus, and whether or not the highly structured e-learning environment in ALEKS would benefit students with different learning styles and motivational characteristics differently. Therefore, a semester-long study was conducted to investigate (1) the effectiveness of using a systematically sequenced and managed, self-paced e-learning program, ALEKS, on academic performance of students with different learning styles (sequential and random), and (2) the relationship among the students' dominant learning styles, their motivational orientations and learning strategies, and their overall academic performance in Precalculus. The research findings would help Precalculus instructors select effective media and methods for handling homework assignments, address individual students' needs based on their learning styles and motivational characteristics, and improve their learning. The research method used in this study is described in the following section.

Method

Research Questions

The study aimed to answer the following two research questions:

1. Is the homework activity administered via ALEKS more effective in helping students with different learning styles (sequential vs. random) learn Precalculus than is the traditional paper-and-pencil handout type homework activity?
2. If any, what relationship exists among Precalculus students' dominant learning styles, their motivational orientations and learning strategies, and overall academic performance in Precalculus?

The first research question was answered by testing the following null hypothesis:

Ho_1: There is no significant difference in students' learning of Precalculus due to the use of different types of homework activity (an e-learning program ALEKS vs. traditional handout assignments) and the students' learning styles (sequential vs. random).

The second research question was answered by examining correlation among multiple variables measured with Gregorc Style Delineator, the MSLQ, and final points students earned from the course.

Subjects

Subjects participated in this study were 138 students enrolled in 4 sections of a Precalculus class offered at a medium-size university in the northwestern region of the U.S. during the spring semester of 2008. It was a 5-credit course; all classes were held for 50 minutes daily, Monday through Friday. The same textbook and course topics were used in all sections. Eighty-three students (61%) were male, and 55 students (39%) were female. The average age of the students was 22 (SD = 5.32, *Min.* = 18, and *Max.* = 55).

Research Design

A quasi-experimental factorial research design was used in this study. The two independent variables used for answering the first research question were (1) the type of homework assignments administered in the Precalculus class (e-learning vs. handout), and (2) students' dominant learning styles (sequential vs. random). Two female instructors were assigned to teach the four sections of the class (each instructor taught two sections). To reduce potential instructor bias, one of the two sections taught by the same instructor was randomly assigned to an experimental group and the other section was assigned to a control group (see Table 3). Students in the experimental group used the systematically sequenced and managed, self-paced e-learning program, ALEKS, while students in the control group completed a series of traditional paper-and-pencil, handout-type homework assignments instead. The dependent variable was students' learning of Precalculus.

Table 3. Experimental and Control Groups Taught by Two Instructors.

	Experimental Group using ALEKS homework (N =72)	**Control Group** using handout homework (N = 66)
Instructor A	Section 004 (N = 36)	Section 003 (N = 29)

Instructor B	Section 005 ($N = 36$)	Section 006 ($N = 37$)

Research Instruments and Procedure

Students' Pre- and Post-Knowledge in Precalculus: A pre-test was administered at the beginning of the course, and a post-test at the end of the course, and 111 students (80.43%) completed both tests. The pre-test contained 11 questions, and the post-test contained 16 questions, 11 of which were identical to the ones included in the pre-test and the remaining 5 questions of which were also directly related to the topics measured in the pre-test. The scores were recorded in percentage of accuracy.

Students' Learning Styles: To assess students' dominant learning styles, Gregorc Style Delineator was administered during the course, and 104 students (71.33%) completed the instrument.

Students' Motivational Characteristics: Students' motivational orientations and learning strategies were measured with the 15 sub-scales of MSLQ, and 112 students (78.33%) completed the instrument.

Homework Assignments via ALEKS vs. Handouts: Students in the experimental group were given access to ALEKS to complete their homework assignments. Students were assigned to complete 9 intermediate objectives in ALEKS by established deadlines across the semester, and the system kept track of the progress. These were selected to align with the 9 chapter completion deadlines in the accompanying textbook. At the end of the semester, students in the experimental group completed, or mastered 85.0% of the total topics assigned in ALEKS. Students in the control group were provided with handout type homework assignments almost daily. Students were asked to turn in their homework assignments by the first or second following class meeting. The instructors returned the assignments with scores within 2-3 days. Discounting any students that received less than 5% on their total homework grade, students in the control group received an average score of 79.0% on the handout homework assignments. Although the practice questions provided in ALEKS and the questions included in the instructor-developed handout homework assignments were not identical, they were directly related to the topics that students were learning in the course. In both groups, students were aware that the homework assignments were worth 30% of the final grade.

Data Analysis: The data were analyzed with descriptive and inferential statistics using SPSS 17.0 for Windows.[18, 19]

The overall research procedure is illustrated in Figure 4.

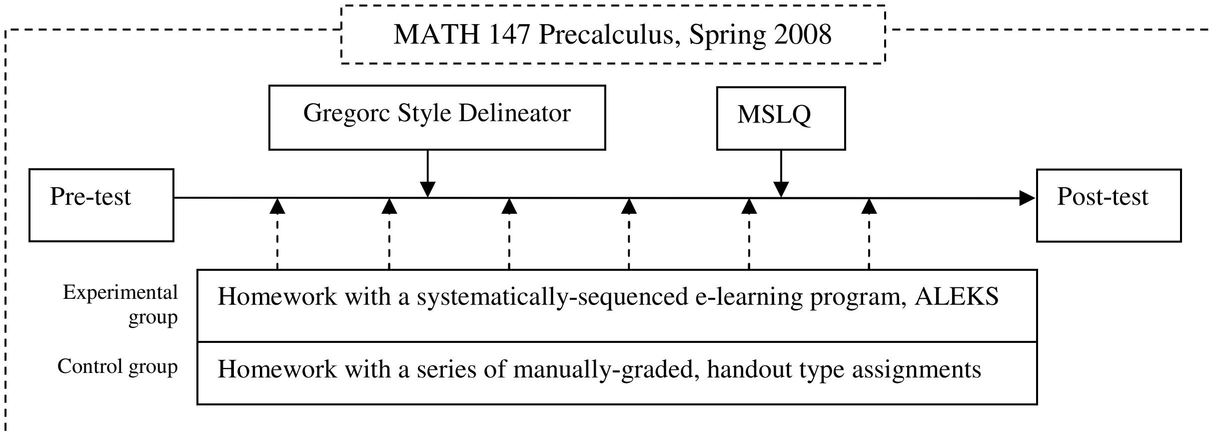

Figure 4. Research procedure.

Results

Effects of Using E-Learning vs. Handouts on Students' Learning with Different Learning Styles

A complete set of pre-test and post-test scores and learning style data were obtained from 98 of the 138 participants (71.0%), and cases with any missing values were excluded in the following data analysis. Most students (93%) demonstrated one of the four styles as their dominant style, and 7 students (7%) showed two or three styles as equally dominant. In those tied cases, computer-generated random numbers were used to select a dominant style.[13] The most frequently identified dominant learning styles among the students were in order, concrete-sequential (N = 42), abstract-random (N = 24), concrete-random (N = 20), and abstract-sequential (N = 12). Among 98 students, 54 of them were sequential-type (CS and AS), and 44 of them were random-type (CR and AR).

The average pre-test scores among the four groups (the experimental-control groups by 2 learning style groups) were not significantly different (see Table 4). Therefore, the post-test scores were compared to test the group differences due to the type of homework activity.

Table 4. Descriptive Statistics of Pre-Test Scores Between Groups.

Homework	Learning Style	*N*	*Mean*	*SD*
ALEKS	Sequential	25	9.36	9.738
	Random	19	12.00	8.888
	Total	44	10.50	9.367
Handout	Sequential	29	9.69	8.824
	Random	25	11.32	8.240
	Total	54	10.44	8.518
Total	Sequential	54	9.54	9.171

	Random	44	11.61	8.431
	Total	98	10.47	8.862

The null hypothesis set to answer the first research question was: There is no significant difference in students' learning of Precalculus due to the use of different types of homework activity (an e-learning program ALEKS vs. traditional handout assignments) and the students' learning styles (sequential vs. random).

A 2 x 2 ANOVA was conducted to test the null hypothesis. The average post-test scores of the experimental and control groups were 68.56 (*SD* = 19.87), and 62.00 (*SD* = 20.36), respectively. The average post-test scores of the sequential and random learner groups were 66.45 (*SD* = 20.71) and 63.09 (*SD* = 19.87), respectively. The two-way ANOVA indicated no significant effects due to the types of homework assignments (ALEKS vs. handout homework), $F(1, 94) = 2.05$, $p > .05$, and learning styles (sequential vs. random), $F(1, 94) = .81$, $p > .05$, on students' learning of Precalculus; therefore, the first null hypothesis was retained. The interaction effect on students' learning of Precalculus was not significant either, $F(1, 94) = 1.55$, $p > .05$. However, it is noteworthy that sequential learners who used systematically sequenced and managed ALEKS performed a letter grade higher (*M* = 72.38, *SD* = 18.40) than sequential learners who used handout homework assignments (*M* = 61.34, *SD* = 21.53) and random students who used ALEKS or handout homework assignments (*M* = 63.53, *SD* = 21.10, and *M* = 62.76, *SD* = 19.32, respectively). The group mean differences are presented in Table 5 and illustrated in Figure 5.

Table 5. Descriptive Statistics of Post-Test Scores Between Groups.

Homework	**Learning Style**	***N***	***Mean***	***SD***
ALEKS	Sequential	25	72.38	18.401
	Random	19	63.53	21.107
	Total	44	68.56	19.879
Handout	Sequential	29	61.34	21.534
	Random	25	62.76	19.327
	Total	54	62.00	20.362
Total	Sequential	54	66.45	20.716
	Random	44	63.09	19.878
	Total	98	64.94	20.309

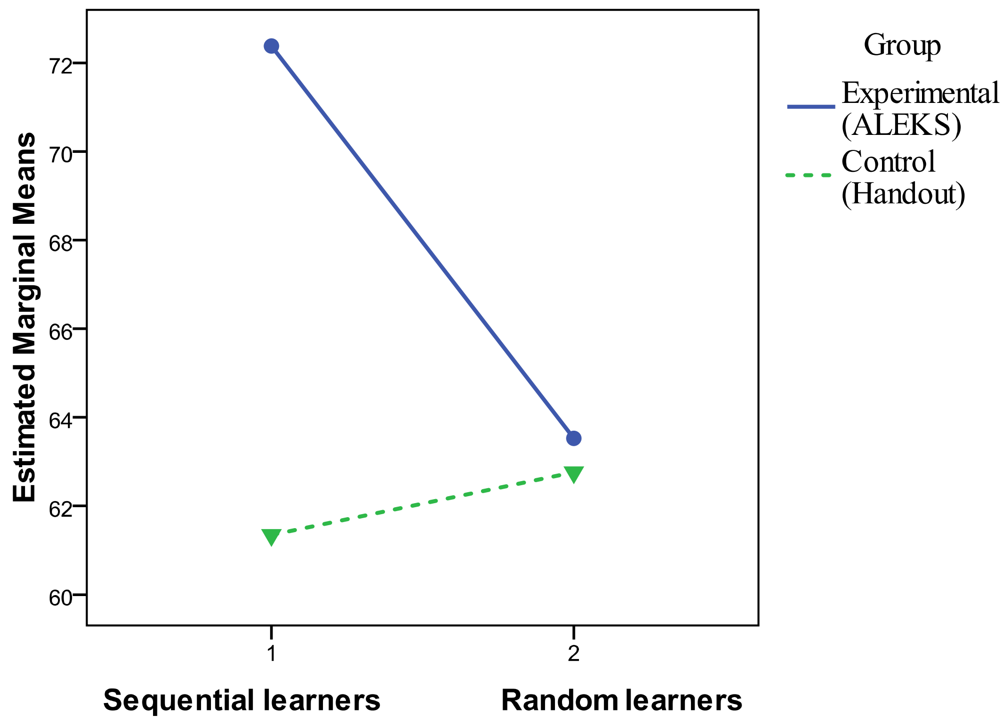

Figure 5. A line graph illustrating the group means of post-test.

Learning Styles, Motivational Characteristics, and Academic Performance in Precalculus

The second research question was: If any, what relationship among Precalculus students' dominant learning styles, their motivational orientations and learning strategies, and overall academic performance in Precalculus exists?

The measure of overall academic performance was the final points earned from the homework activity (30%), 5 quizzes (50%), and the post-test (20%), which determined the final grade of the course. As shown in Table 6, overall, the final points were positively associated with students' intrinsic goal orientation (*rho* = .208), task value (*rho* = .230), control belief (*rho* = .323), self-efficacy levels (*rho* = .655), management of time and study environment (*rho* = .261), and their ability to control effort and attention from distraction (*rho* = .348), and were negatively associated with students' test anxiety (*rho* = -.326) and seeking peer learning (*rho* = -.282).

Table 6. Correlations Among Learning Styles, Motivation, and Academic Performance.

Category	**Sub-Scale**	**CS**	**AS**	**CR**	**AR**	**Final Points**
Motivational	Intrinsic Goal Orientation	.025	.016	.162	-.183	.208*
Orientations	Extrinsic Goal Orientation	.113	.059	-.175	-.038	.135
	Task Value	.148	.232*	.077	-.385**	.230*
	Control Belief	.084	.102	.084	-.187	.323**

	Self-Efficacy	.107	.160	.055	-.253*	.655**
	Test Anxiety	-.104	.034	-.145	.172	-.326**
Learning Strategies	Rehearsal	-.045	-.105	.097	-.008	.096
	Elaboration	.002	-.009	.036	-.032	-.016
	Organization	-.015	.024	-.004	-.023	.064
	Critical Thinking	-.133	.027	.133	-.003	-.021
	Metacongnitive Self-Regulation	.051	-.011	-.043	-.025	.151
	Time and Study Environment	.184	-.021	-.075	-.132	.261*
	Effort Regulation	.200	.090	-.107	-.172	.348**
	Peer Learning	-.008	-.101	.010	.072	-.282**
	Help Seeking	.003	-.075	-.130	.121	-.198
Academic Performance	Final Points	.178	.142	-.062	-.247*	1.000

* Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).
Listwise *N* = 96

Only a few small degrees of correlations were found among students' dominant learning styles, their motivational orientations and learning strategies, and final points earned from the course. Task value is about the person's evaluation about how important, how useful, and how interesting the task is. A positive correlation of the task value scores to the abstract-sequential scores (*rho* = .232) confirms that AS-strong students tend to think that it is important and interesting to learn Precalculus that requires abstract-sequential thinking. On the other hand, a notable observation was that students' abstract-random scores were negatively correlated with task value (*rho* = -.385), self-efficacy (*rho* = -.253), and the final points they earned from the course (*rho* = -.247). It can be interpreted in two ways: 1. AR-*strong* students tend to think that learning Precalculus is not interesting or they are not good at learning Precalculus, and they tend to produce lower final points, or 2. AR-*weak* students tend to think that learning Precalculus is interesting or they are good at learning Precalculus, and they tend to produce higher final points. The first interpretation seems to be more plausible with the sample used in this study, because as shown in Figure 6, AR-dominant students performed a letter grade lower than other learning style groups. CS-dominant students scored highest on the final points (*M* = 78.68, *SD* = 12.75) while AR-dominant students scored lowest (*M* = 68.65, *SD* = 17.18). However, the mean differences in the final points shown by their dominant learning styles were not significant at the .05 level. Nonetheless, attention should be paid to this trend associated between AR-dominant students and their tendency toward learning Precalculus.

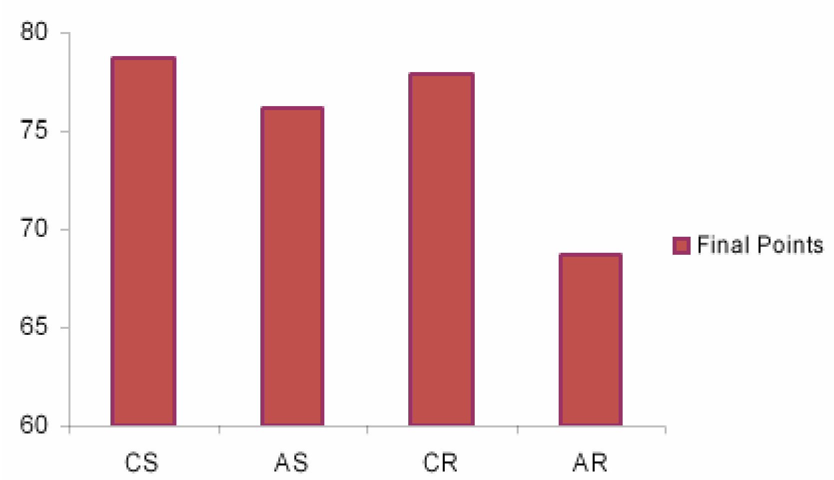

Figure 6. A comparison of average final points by students' dominant learning styles.

Conclusions

Discussion

The purpose of this study was to investigate (1) the effectiveness of using a systematically sequenced and managed, self-paced e-learning program, ALEKS, on academic performance of students with different learning styles in a undergraduate Precalculus class, and (2) the relationship among the students' dominant learning styles, their motivational orientations and learning strategies, and overall academic performance in the Precalculus class. The study revealed that the use of ALEKS and the use of handout homework assignments did not contribute to making statistically significant differences in students' learning of Precalculus. However, a notable trend was observed that sequential students who used ALEKS performed a letter grade higher than sequential learners who used handout homework assignments or random students who used either ALEKS or handout homework assignments.

Interpretations of the above findings are as follows. Learning styles indicate people's abilities in perceiving information and their preferences as to how the information should be arranged.[10] When the learning environment is designed to support their dominant abilities and preferences, learners tend to find it more enjoyable and perhaps perform better as a result. Therefore, it is plausible that ALEKS which is a systematically sequenced and managed learning environment could be more appealing to sequential learners than it was to random learners; and as a consequence, sequential learners who used ALEKS outperformed other groups of learners by one letter grade. However, this study also revealed that random students, especially AR-dominant students, tend not to value the task of learning Precalculus, tend to have less self-efficacy in succeeding in the Precalculus class, and in fact, did not perform as well as other groups of learners. Therefore, another possible interpretation is that it is not just because the ALEKS learning environment supported sequential-type students more and random-type students less, but it could be also because the learning subject matter was more appealing to sequential students and less to random students. This implies that learning is a product of triadic interactions among learners' characteristics, the learning environments, and the learning subjects (see Figure 7).

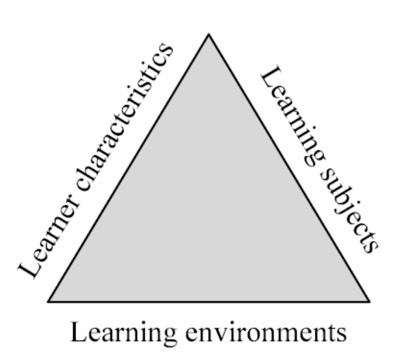

Figure 7. Triadic interactions among learners, learning environment, and learning subject.

This study revealed that the use of ALEKS was not significantly better or worse than the use of traditional handout homework assignments. However, as expected, cost-effectiveness was a benefit of using ALEKS for handling homework assignments. The two instructors indicated that it took about 5 hours per week to manually administer the handout homework assignments, which means that the use of ALEKS freed up 5 hours of their time per week. It implies that an e-learning program such as ALEKS could be substituted for a traditional, time-consuming method for handling homework assignments, allowing instructors to engage in other responsibilities such as curriculum improvement, student advising, and research. However, it should not be over-generalized that e-learning is more effective than instructor-facilitated learning, leading to a conclusion that the entire Precalculus course be taught via self-paced e-learning.

Another interesting observation by the instructors was low class attendance in the two sections of the experimental group that used ALEKS (about 50% attendance), while the students in the control group maintained high class attendance throughout the course (80-90% attendance) although it varied day by day in both cases. It is unknown whether the research results would have been different if the students in the experimental group had also kept as high class attendance levels as the students in the control group.

Limitations of the study

Several limitations exist in this quasi-experimental study. First, it was not feasible to randomly select a sample from the population; therefore, a convenience sample was used. Although two sections taught by the same instructor were randomly assigned to either an experimental group or a control group, potential instructor bias is still a threat to external validity of the study. Also, as a common condition in most educational research settings, the students were asked to participate in the study on a voluntary basis, and complete data obtained from only 71% of the sample were used for data analysis; therefore, the findings of this study should be generalized with some caution. A few threats to internal validity existed, as there were some factors related to the use of ALEKS and handout homework assignments that could not be controlled. For example, by nature, homework assignments, whether they were administered via ALEKS or handouts, were completed in uncontrolled environments; therefore, other confounding factors could interact with the treatment. Although students who used ALEKS might have enjoyed more flexible deadlines to meet, it is also possible that they had less access to the treatment (the use of ALEKS) as it required a computer connected to the Internet, compared to students who used simple handouts.

Recommendations

Based on the findings of this study, the following recommendations are provided to educators who teach Precalculus or related topics:

1. Measure students' learning styles in the beginning of the course. The information would enable the instructors to be aware of their students' potential strengths and weaknesses in performing in classroom and to tailor their instructional strategies toward the individual students with different needs. For example, random students who are studying a subject that demands sequential thinking such as Math may need more attention from the instructors.
2. Have students be aware of their dominant learning styles and motivational characteristics. It can help them self-monitor their learning behaviors and give them opportunities to self-correct ineffective study habits, and develop more effective learning behaviors.
3. Have instructors be aware of their own dominant learning styles and reflect on their preferred approaches for teaching their subjects. Instructors also have their dominant learning styles which are often their preferred teaching styles.[20] For example, CS-dominant instructors may use CS-friendly strategies in their courses. Some common behaviors of CS-dominant people include being adept at following precise step-by-step directions for completing assignments and being good at meeting deadlines. A learning environment with such expectations from the instructor may not appeal to AR students, who like group discussions and collaborative work and tend not to pay attention to meeting deadlines as much.[21]
4. Conduct a more rigorous experimental study in which a sample is randomly drawn from the population. Use a large sample. Measure students' learning styles first to group them into the four learning style groups, and then randomly assign members of each group into an experimental or control group. That way, the findings of the study would provide more statistical power and generalizability.

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AC 2011-2500: THE IDAHO SCIENCE TALENT EXPANSION PROGRAM: FRESHMAN ORIENTATION FOR STEM MAJORS

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The Idaho Science Talent Expansion Program: Improving Freshmen Retention for STEM Majors

Abstract

During summer of 2010, we conducted a series of freshman orientation programs that were held for new science, technology, engineering and mathematics (STEM) majors at Boise State University. Approximately 320 students were advised in this manner, during seven summer orientation sessions. This was a significant change from previous years, which used a college-specific approach to advising, thereby limiting various retention programs and opportunities designed and promoted by the College of Engineering to engineering majors. The motivation for these changes was a Science Talent Expansion Program award from the National Science Foundation, and the fact that the retention rates for freshmen engineering majors is approximately 10% higher as compared with science and mathematics majors. The grant proposed to (1) integrate the science and mathematics majors with the engineering majors during summer orientation, (2) expand student learning community offerings to STEM majors, (3) create a General Sciences course for STEM students who are underprepared in mathematics, and (4) offer an elective, non-credit bearing mathematics online review course, free of charge, to students entering the university in STEM majors. An underlying and important rationale for widening the advisement base to include all STEM majors in an inclusive manner is the fact that many freshmen are unsure of their major. Therefore orientation materials were prepared that emphasized the commonalities between majors and the underpinning courses and their prerequisites. The results of these four activities, to date, will be presented together with strategy revisions planned for summer 2011.

Introduction

Boise State University, with the largest enrollment and highest academic admission standards among Idaho's public universities, is the state's comprehensive metropolitan research university. The university has been experiencing, year after year, exceptional growth to meet the needs of the area's emerging technology economy. The Boise metropolitan area has recently earned national Top 10 rankings for overall patents, high-tech output, business and career climate, livability, and engineers per capita.[1-5]

The College of Engineering was formed in 1997 as a result of the university's steady growth and diversification coupled with the state's technology boom. Since its inception, the college has grown explosively with more than 60 new faculty, and a 16th place U.S. News & World Report 2010 ranking among public masters level engineering colleges. Since 2000, the college has added one doctorate, six masters and one new undergraduate program, Materials Science and Engineering (2005). In the College of Arts and Sciences, a new masters program in Mathematics was added in 2005, a doctoral programs in Geosciences was added in 2006, and a masters program in Chemistry began in 2010. Two additional doctoral programs are in progress at the university, in Biomolecular Sciences and in Materials Science and Engineering.

Table 1: B.S. STEM degree offerings

College of Engineering	College of Arts & Sciences
Civil Engineering	Biology
Computer Science	Chemistry
Electrical Engineering	Geology, Geophysics
Materials Science and Engineering	Mathematics
Mechanical Engineering	Physics

The undergraduate degree offerings in STEM majors are listed in Table 1. The undergraduate enrollment trend in STEM majors at Boise Stateuniversity since 2004 is shown in Figure 1. Geosciences consists of two majors, Geology and Geophysics, which are grouped together. Mathematics comprises Applied Mathematics (a B.S. degree), the Bachelors of Arts in Mathematics and also the B.S. in Mathematics. There has been a positive trend in enrollment for a number of years.

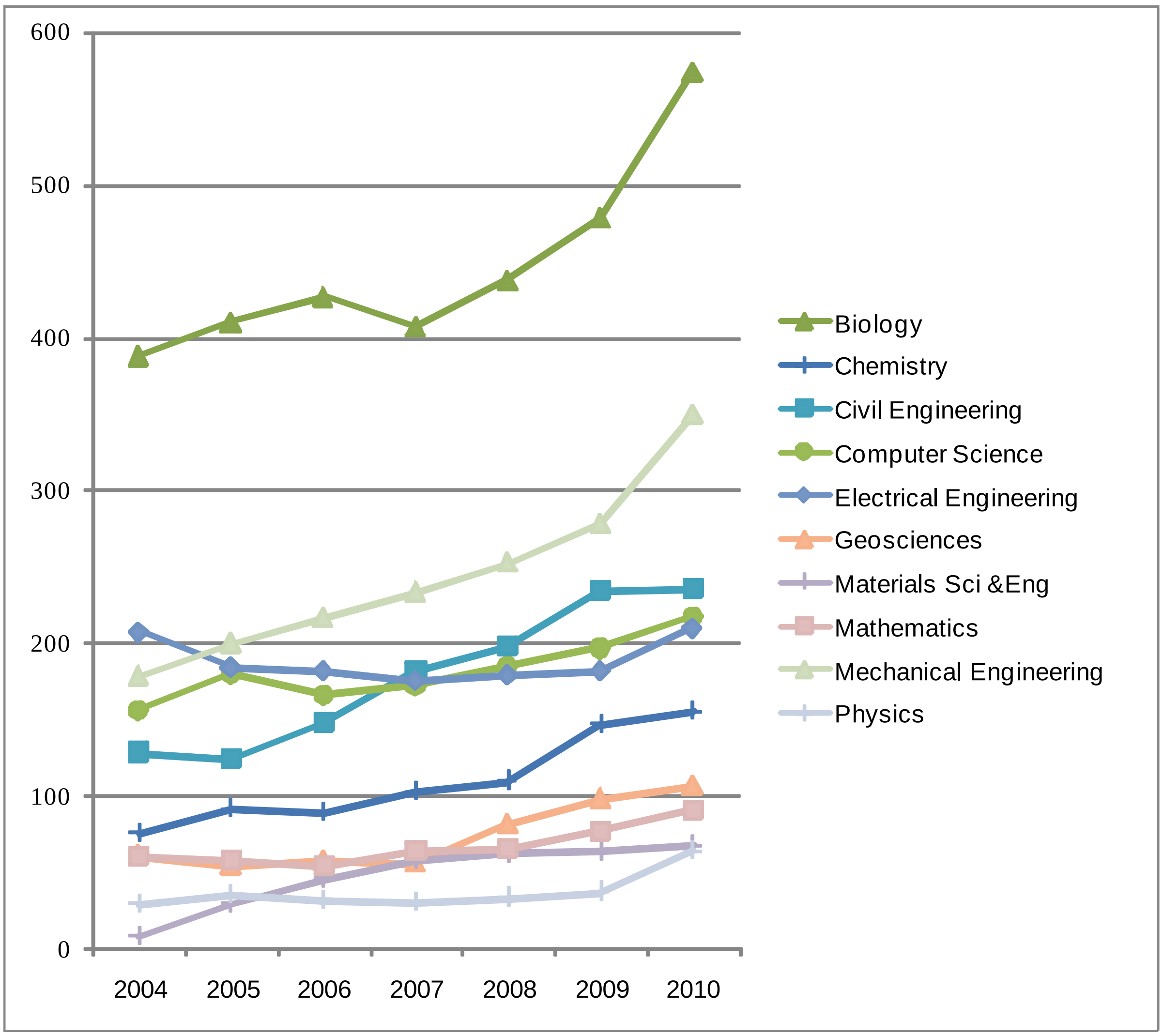

Figure 1: FTE Fall Enrollment

An analysis of STEM degrees awarded shows a total number of Boise State University graduates that has varied between a low of 161 in 2004-2005, and a high of just over 200, see Table 2.

Table 2: STEM graduates, six-year trend

Academic Year	2004	2005	2006	2007	2008	2009
Engineering and Computer Science	96	97	112	91	111	106
Science and Mathematics	65	106	89	73	78	95
Total STEM Graduates	161	203	201	164	189	201

One of the program outcomes of the Boise State University National Science Foundation Idaho Science Talent Expansion Program (STEP) is to increase the first year retention level of first-time STEM freshmen from a weighted average of 57% to a target level of 70%. This level was thought to be a good balance between what had been already achieved by COEN (63.3% in 2007-8) and what might be realistically achieved in a five-year time frame, recognizing that not all students will be retained in any major. This outcome was selected as a step along the path toward increasing the number of STEM undergraduate degrees conferred, which is the program's overarching objective. The focused attention on first year retention is based on the fact that the first-time full time freshman to sophomore retention for majors within the College of Engineering (COEN) is approximately 10% higher than the retention for STEM majors in the College of Arts & Sciences (COAS), see Table 3.

Table 3: First-time full time freshman to sophomore retention

Year	Group	Number in group	% retained at university	% retained in any STEM major	% retained in original STEM group (COEN or COAS)
2005-6	All majors	1755	63.4	NA	NA
	COEN	163	66.9	58.3	57.1
	COAS	77	68.8	51.9	46.8
2006-7	All majors	1867	63.8	NA	NA
	COEN	174	71.8	58.0	55.7
	COAS	97	67.0	47.4	46.4
2007-8	All majors	1900	66.4	NA	NA
	COEN	215	75.8	63.3	60.0
	COAS	90	71.1	52.2	51.1
Average over three years	All majors	1841	64.6	NA	NA
	COEN	184	71.9	**60.1**	57.8
	COAS	88	68.9	**50.4**	48.1

Although a first-time full time freshman to sophomore retention level of 60% is not tremendous, this higher level achieved by the engineering majors at Boise State University is likely a result of a set of initiatives that were put in place between 2002 and 2007 as part of $1M in funding from the William and Flora Hewlett Foundation's Engineering Schools of the West Initiative. This funding resulted in institutionalized practices within the College of Engineering, that comprise part of the basis for the STEP program's target outcome of increasing the first-time freshman to sophomore retention.[7-11] These practices include conducting or creating for all STEM majors: (1) summer orientation with a high level of engagement by faculty, (2) freshman learning communities, (3) coursework for STEM majors that are not yet calculus ready (and therefore not qualified for chemistry, or physics or engineering classes required by their majors) and (4) mathematics preparation and placement.

This paper reports on these program initiatives and discusses how they were expanded to include all STEM majors as well as on strategy revisions planned for summer 2011.

Results and Discussion

1. Summer orientation

For the first time, all incoming first year Boise State students majoring in the STEM disciplines were oriented to their curriculum requirements as a group during summer orientation sessions. This was a substantial change over prior orientation sessions which segregated students by college. This provided a framework which emphasized the commonality of the STEM curriculum between engineering, mathematics and science majors in lower division courses and the relative ease of moving between these majors in the freshman/sophomore years. This is important because many freshmen are unsure of their major – in engineering alone, fall 2010, there are 125 “undecided engineering” undergraduate students, comprising 11.3% of the engineering and computer science undergraduate students. All students were advised of the STEM core courses they must take in an overview presentation. Next, students were categorized by specific discipline for individualized course advising using peer advisors and STEM faculty (32 advisors over the summer). The objective was to help the students identify as a STEM major, begin connecting them with an advisor, and to identify the STEP project coordinator and other resources available to them.

The advisors assisted students in selecting the appropriate math course, promoted the use of ALEKS, a mathematics online learning module (described in part 4), and encouraged enrollment in Student Learning Communities (SLCs). To continue support and assistance, advisors accompanied the students to a computer lab to complete online course registration. Over 323 STEM students were advised and registered in seven sessions in the summer of 2010.
To emphasize cross disciplinary cooperation and build support, we presented a salary chart for STEM majors that grouped engineering salaries instead of categorizing them by specific discipline (chemical, civil, electrical, etc.). This communicates that other STEM fields are attractive options to students and faculty rather than the stark message: “engineering has seven of the ten top salaries.” This is important in building university, cross-college and discipline cooperation because too often engineering is presented as the only high income career choice and may alienate faculty from math and the sciences. Table 4, below, depicts how the salary information was presented in the large group student briefing.

Table 4: Ten Highest Paying College Majors[12]

Major	Average first year salary	Average mid-career salary
Engineering	$59,000	$101,000
Economics	$50,200	$101,000
Physics	$51,100	$98,800
Computer Science	$56,400	$97,400
Statistics	$48,600	$94,500
Biochemistry	$41,700	$94,200
Mathematics	$47,000	$93,600
Construction Management	$53,400	$89,600
Information Systems	$51,400	$87,000
Geology	$45,000	$84,200

Lessons learned: The initial overview presentation contained more information than the students could absorb on day two of orientation. The time allotted for advising and registration of the freshmen did not provide sufficient time for the advisors to emphasize our new academic support tools including Student Learning Communities, the GenSci (General Science) class for pre-calculus ready STEM majors, and ALEKS; nor did the STEM students remain in a group when they moved to computer labs which made it difficult for the advisors to respond to their questions in a timely fashion.

New actions: We have requested time on day one in summer 2011 to do the majority of advising followed by a time on day two to address problems such as missing course placement data (SAT scores, AP credits) or other student records. We are also developing a training tool to educate our advisors about our STEM academic support tools, tips for positioning the tools and better articulating the benefits so they are able to succinctly advise the students. Advisors will attend a short training session on the registration system from the student perspective to enhance their skills on the system. We have requested and confirmed a dedicated computer lab for all STEM majors during registration to allow advisors to address registration issues in real time. Finally, increased communication between summer orientation and the Registrar's office is planned, so as to ensure students' ACT, SAT and AP scores are in the system prior to course registration. This will ensure that the system recognizes that the student has achieved appropriate prerequisites for the critical math and science courses in which they need to enroll.

2. Freshman learning communities

STEM major focused Student Learning Communities (SLCs) based on math learning readiness for first term freshman are designed to encourage a diverse group of learners to become engaged with STEM faculty and other SLC members. This practice has worked well for the engineering majors and has now been expanded to all STEM majors. Additional benefits to SLCs are 1) experienced faculty who are committed to student success are assigned to these classes, and 2) SLCs provide guaranteed enrollment in high demand core science courses (which often become oversubscribed) and provide a pool of potential study group members. Eleven sections of academic STEM and two engineering SLCs were formed for Fall 2010 entering freshman and sophomore continuing students to foster connections, form study groups, and learn core knowledge. Over 204 students representing all 11 STEM disciplines were enrolled.

We distributed the listings of these SLCs to our STEM advisors prior to beginning summer orientation for incoming new and transfer STEM students. At summer orientation, we used time during our STEM orientation presentations to explain and emphasize the advantages to students of enrolling in SLCs. Following the presentations, students were advised in groups related to their disciplines; in these discussions the benefits of enrolling in SLCs were again emphasized. In addition, we pointed out that in many cases, the only path to registering for STEM service classes such as mathematics, chemistry, and physics was to select a SLC. Finally, during the actual registration portion of orientation, when students were confronted with the reality of selecting the classes, we once more emphasized that perhaps the best (and sometimes only option) to register for their required classes was to select an SLC which contained the classes they needed.

The structure of the first year SLCs is listed in Table 5. For Spring 2011, we designed seven sections of SLCs and are updating advisors of the availability. We have been particularly aware of the rapid enrollment in the STEM calculus sequence, and calculus based chemistry and physics which nine of the 11 STEM majors must take.

Table 5: First Year Fall Student Learning Communities (SLC)

Level	Math Course Name	Math Credits	GenSci Credits	Suggested General Education Course	Total LC Credits	LCs (section size 20-25)
C2	Calc II	4	5	--	9	1
C1	Calc I	4	4	Comm	11	2
PC	PreCalc	5	2	Comm	10	2
PC	PreCalc	5	--	Comm	8	2
A-II	Alg. II	4	2	Comm	9	2
A-II	Alg. II	4	--	Comm	7	2

Lessons learned: Although engineering advisors understood and supported SLCs, other STEM advisors were unfamiliar with them and did not fully realize the advantages to the students. We included a communications (Comm) course, which is required for engineering majors but a social science elective for other STEM majors, so students did not perceive that course to be required. Demand for SLCs containing calculus, chemistry and physics exceeds course availability. Science courses with labs do not have wait lists; this is due to limitations in the software system used by the university which makes it difficult to quantify the additional slots needed. Course additions are tightly controlled due to resource limitations that include both funding and difficulty in securing qualified instructors.

New actions: We are developing a SLC tutorial with tips for advisors to ensure they understand the benefits for the students. We will structure the SLCs with greater care to ensure we provide the most benefit from the offerings. We are developing guidelines for instructors on SLCs, which contains tips such as encouraging them to stagger the course test dates between classes in the SLC. The need for more seats and the issues associated with not really knowing what the unmet need for math and science course seats has been brought to the attention of the appropriate people at the university. Measures are being taken to step ahead of the need to avoid shortfalls in math and science course offerings.

3. Coursework for STEM students who are not ready for calculus.

To improve STEM freshman retention by increasing engagement, as well as improving scientific reasoning skills, *GenSci 197 (Scientific Thought and Reasoning)* was developed for the students who are not ready for calculus. From an effort to understand the STEM population at Boise State University, and where to focus our efforts, an analysis was done of first time STEM freshmen who first enrolled in fall 2007, 2008 or 2009. The student math level enrollment shows that the majority of STEM students, 65%, enroll in math courses at levels below calculus. These 567 students show a STEM major retention level of 72%, one year later. By contrast, 35% of students enroll in math courses at calculus level or higher, and are retained in STEM majors one year later at a level of 84%. This indicates that the students not ready for calculus have a greater chance of leaving their programs. Thus, attention and grant resources were directed toward this majority of students who have selected a STEM major but who are not mathematically prepared for calculus. In an effort to improve student engagement by giving them a science class to take and at the same time improve their reasoning skills, *GenSci 197* was developed and offered in fall, 2010.

The goal of the *Scientific Thought and Reasoning* class is to improve students' scientific reasoning skills through explicit instruction in the areas of the nature of science, observations vs. inference, measurements including precision and accuracy, unit conversion and analysis, identification and control of variables, hypothetico-deductive reasoning, probability, interpreting graphical and tabular data, and lab report writing. This course is based on a course at Wright State University (SM101), developed and taught by Dr. Kathy Koenig, which has succeeded in increasing reasoning skills of students.[13] We are using the same workbook.

The goals of many STEM courses include problem solving procedures. It has been suggested that Piagetian Formal reasoning skills are required for solving problems.[14] Piagetian reasoning is a developmental theory which describes that students start as children with less sophisticated reasoning skills and develop toward Formal reasoning. The aspects of Formal Operational reasoning as suggested by Piaget are combinatorial reasoning, separation and control of variables, proportional reasoning, probabilistic reasoning, correlational reasoning, and hypothetico-deductive reasoning. When some of the formal schemes are absent or not fully developed, they may only be applied to familiar situations and not systematically.

"*One can be said to be reasoning at the Formal operational level when Formal operational schemes have become explicit and useful as general problem solving procedures.*"[14]

On the basis of this developmental reasoning theory, it has been suggested that classroom activities may play a significant role in the development of student reasoning.[15] From the results of their study to enhance the development of student reasoning, Karplus and his team devised a strategy referred to as a *Learning Cycle*.[15] This learning cycle consists of an exploratory activity upon which later conceptual understanding can be built, referred to as the *Exploration* phase, which confronts the student with questions that cannot be answered with their familiar pattern of reasoning. The *Invention* phase starts with the students speculating about possible explanations for the questions raised by the exploration in order to develop a new reasoning pattern. During the last phase, *Application*, the students apply the new reasoning pattern to additional examples. All three of these phases contribute to the development of formal reasoning skills. The text used for GenSci 197 utilizes this Learning Cycle, which is evident in the sequence of the workbook activities and the experiments performed.[13] Students working through the course activities are placed in situations where formal reasoning is purposefully being encouraged.

The research method used and preliminary results obtained from the first semester's offering are presented below.

Method

Research Question: The study aimed to begin to answer the following research question: What is the effect of explicit reasoning instruction upon student scientific reasoning? This research question was answered by testing the following null hypothesis:

> H01: There is no statistically significant change in student reasoning after engaging in explicit reasoning instruction.

Population: Subjects participating in this study were 55 first semester freshman STEM students enrolled in two sections of *Scientific Thought and Reasoning* during the fall semester of 2010. It was a two credit class which met twice a week for 50 minute classes. The same textbook and course topics were used for both sections. 43 students (78%) were male, and 12 students were female. This group of students were not calculus-ready, with 23 students entering intermediate algebra and 32 students entering pre-calculus.

Research Design: A quasi-experimental pre-test post-test experiment design was planned for this experiment. The independent variable is the direct instruction in reasoning, and the control variable is the student scores on the Lawson's Test of Scientific Reasoning. The change in students reasoning may thus be measured and tested for statistical significance. Also the gain of the treatment students will be compared to published reasoning gains.

Instrument and procedures: The Lawson's test of Scientific Reasoning[16] is a well accepted and reliable assessment tool for measuring student reasoning.[17] While the Lawson's test covers all aspects of Piagetian reasoning, since our course only covered Control of Variables, Probabilistic Thinking, and Hypothetico-deductive reasoning, only these aspects were included in the scores for our students. A pretest was administered at the beginning of the course and a post-test at the end of the course. Of the 55 students in the course, 43 students (78%) completed both tests. The data was analyzed using SPSS 17.0 for Windows.

Results: A paired samples t test was conducted to evaluate to see if there is a relationship between the amounts of explicit reasoning instruction that STEM students receive and the students' scientific reasoning. For this test the student scores on aspects of interest were used. The test results indicated that the mean post-test score (M=9.95, SD=2.72) was statistically significantly greater that the pre-test score (M=8.86, SD=3.04), with t(42)=-2.85, p=0.007. To estimate the practical significance of this increase Cohen's *d* was calculated, *d*=0.38. This result supports the research hypothesis and the prediction. In a post-hoc analysis the items were analyzed by reasoning aspect to see if the students did gain reasoning ability in any one aspect. The students showed a statistically significant gain in Control of Variables, t(42)=-2.85, p<0.001. Cohen's d was calculated where d=0.58. The students also showed a statistically significant gain in Probabilistic Thinking, t(42)=-2.86, p,0.001. Cohen's d was calculated where d=0.37. There was no statistically significant difference for the aspect of hypothetico- deductive reasoning. Figure 2 and Table 6 show the student results for the total score on aspects of interest and the student scores for Control of variables.

Table 6: Descriptive statistics of pre- and post-test scores

	mean	SD
Pre-test	8.86	3.04
Post-test	9.95	2.72

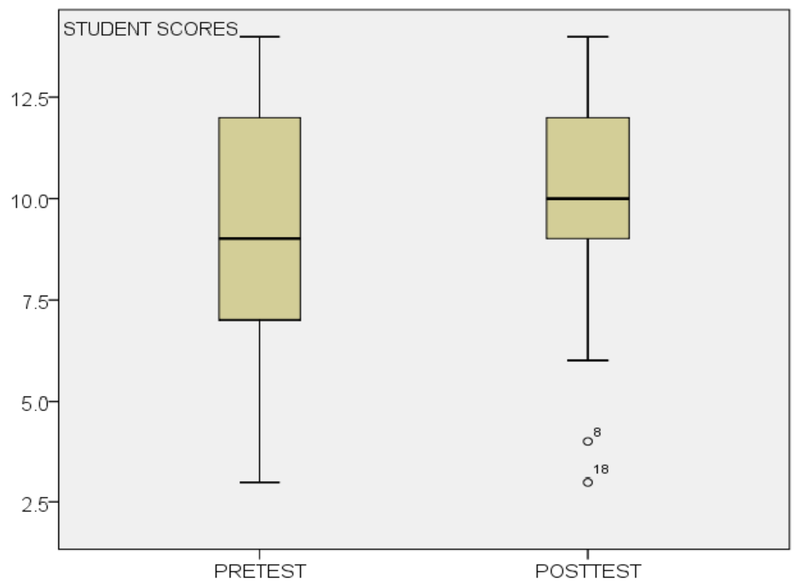

Figure 2: Student total scores, aspects of interest

Finally, it is noted that studies indicate a small change in student reasoning scores for students taking one semester of typical science classes. Maloney showed that students in a physics class can show an average gain as small as 0.31 on the Lawson's test.[17] In comparison the students in Gensci 197 showed an average gain of 1.1.

Discussion: The analysis of the data supports the hypothesis that there is a relationship between the amounts of explicit reasoning instruction that STEM students receive and the students' scientific reasoning. When students were given a semester's worth of explicit instruction in scientific reasoning their scores on a reasoning test increased. The effect size for this overall increase was small (d=.38), indicating a small gain in practical terms. This is a disappointing result. All aspects of the course will be reviewed in an effort to increase the gain in future sections of this course.

The item analysis indicated a larger practical gain (d=.58) for the aspect of control of variables. This is not surprising as a significant portion of the course dealt with this topic, and the students had multiple tests covering this topic. It is noted that the hypothetico-deductive reasoning did not increase, even though this was covered significantly during the course. Looking back at the theory behind hypothetico-deductive reasoning it is noted that Fuller *et. al.*[14] indicate that hypothetico-deductive reasoning uses all aspects of scientific reasoning in concert. Since in our course we did not have time to address proportional thinking, combinatorial or correlational thinking, it is possible that the lack of these reasoning skills prevented the students from excelling at hypothetico-deductive reasoning. In the future we plan to increase the contact time of this course from 2 hours/week to 4 hours/week, so that we may include proportional, combinatorial, and correlational thinking.

The preliminary results of this study show a practical gain in the aspects of student reasoning that were taught. The trial run of Gensci 197 focused primarily on control of variables, and yielded a student gain in this aspect of reasoning. This suggests that the class has the potential to improve student reasoning. We are suggesting that future offerings on Gensci 197 include more time to allow other aspects of reasoning to be addressed. The correlation between improved student reasoning and retention will be explored in a future longitudinal study.

Lessons learned; limitations: Several limitations exist in this study. There is a need for a control group, in order to conduct a complete experiment where the control variable of reasoning instruction can be explored. Also, as is a common practice in educational research settings, students were asked to participate in the study on a voluntary basis. Some students did not complete both the pre-test and the post-test. Of the 55 students in the course, complete data was obtained from 43 (78%) of the students and used for data analysis. Therefore the findings should be generalized with some caution.

4. Mathematics preparation/placement

It is well understood how critical the role of mathematics is to the success of STEM majors. A number of efforts have been undertaken at our institution to investigate ways to use mathematics placement and instruction to help STEM students succeed.[8,10,11] Some of these efforts have included using the online mathematics learning system, ALEKS, marketed through www.aleks.com. These efforts over the past several years have included using the system as a math placement tool (since abandoned), using ALEKS as part of an introductory engineering course, using ALEKS as part of or even all the homework for Precalculus classes, and requiring that students who enroll in either Precalculus or Calculus I, demonstrate a minimum competency defined by mastery of a percentage of "knowledge space" in the ALEKS course termed: "Preparation for Calculus." This percentage is presently enforced at 40% for Precalculus and at 70% for Calculus I. The method of enforcement is that it is treated as a take home exam (unproctored, repeatable, and due during the first week of class), worth approximately 10% of a student's total course grade.

This paper reports on a new initiative that commenced in Fall, 2010 as a result of the NSF STEP grant. The grant budgeted for the purchase of online learning licenses from ALEKS, Inc.,[18] as well as for bookstore awards. The licenses were for 77 days of learning, a $47 value. The intent was to use summer orientation as an opportunity to inform students of these mathematics learning subscriptions, and to incentivize student participation in embarking upon this personal review/learning effort by advertising that students were eligible to be considered for $100 bookstore awards if they achieved 15 hours or more of active online learning.

It was hoped that students would use the free access to begin work on ALEKS shortly after their orientation sessions and continue to work across the summer. The expectation was that students who put in significant time would be more likely to succeed in fall semester math courses.

Results: Over 75 students took advantage of the free subscriptions. In some cases students used both the "Preparation for Precalculus" and "Preparation for Calculus" ALEKS courses to build their skills for entering fall math courses. Table 7 differentiates the students who were enrolled only in Preparation for Precalculus with a cell color that is blue. Because a specific assessment score was a partial requirement for taking MATH 147 and 170, many students only chose to use the learning tool until they obtained the target score. Table 7 presents summary data showing the time spent by each student who used a free subscription and spent two or more hours learning mathematics online. The grade each student earned in their subsequent university math course is shown in conjunction with the university math course in which they enrolled. MATH 25 is Elementary Algebra; MATH 108 is Intermediate Algebra, MATH 147 is Precalculus, MATH 170 is Calculus I and MATH 175 is Calculus II.

The publicized bookstore award eligibility criterion was spending 15 or more hours online learning ALEKS. Of particular note are the 12 students who did so, indicated in bold in Table 7. <u>All these students passed their mathematics class</u>; 3 earned grades of A, 7 earned grades of B, and 2 earned grades of C. The average amount of time these students spent was 30.4 h, with a standard deviation of 8.0 h. Of these students, 7 of them completed 100% of the knowledge space for the course they enrolled in (6 were in Prep for Calc, and 1 in Prep for PreCalc, which is the suitable course for those who enroll in Intermediate Algebra, Math 108).

Table 7: Hours spent in math preparation in online learning (ALEKS) with grade earned in subsequent math class

Combined ALEKS preps courses	Math course: 25	108			147					170						175	
A	4.6	7.7	**26.1**	**18.2**	11.6	7.7				11.3	11.8	13.4	**35.4**				
A-					10.6	10.7											
B+		4.2			**38.3**					3.2	10.4						
B		10.6	**18**							2.3	4.5	6.1	8.6	**30.2**	**32.1**		
B-										**26.4**	**26.8**	**43.1**					
C					3.5	6.9	7.3			2.3	2.7	3.6	**29.3**				
C-										5.5	12.4	**40.5**					
D					2.7	3.9											
F					2	2.2	2.5	3.3	11.9	2	8.6					9.2	9.9

Lessons Learned: Summer orientation sessions are crowded with information and hectic. In the scramble to register for classes, advising on how to use ALEKS was not always clearly communicated. Data on the start date and total hours worked indicate that a significant number of students did not use the software in the manner that we had intended. This is also reflected in the numerous queries about how to use ALEKS that the Math Department received in the last few weeks of the summer. It appears that both better advising techniques and stronger incentives will be needed.

New Actions: (1) Restructure orientation sessions as described in Section 1. The two day format should reduce the concentration of information flowing at students in the short advising window currently available. It is also possible that students could use computer facilities on campus to begin their ALEKS work under our supervision. (2) Track our success rate for inducing appropriate use of ALEKS. Collect data on how many students are advised about ALEKS in summer orientation and compare that number to the number that end up using the software for a suitable amount of learning over the summer. (3) Add incentives; showcase student success. Grant funds do not allow for more than the free license and the bookstore award. However we can provide data on the performance of students who used ALEKS appropriately in Summer 2010. We also plan to feature profiles, with permission, of certain students on the Idaho STEP website with their photos and 'words of wisdom' they would like to pass along to future students. (4) Begin a longitudinal study of the performance of the students who used ALEKS appropriately in Summer 2010. As longitudinal data builds this may provide us with stronger evidence that can be used to incentivize students in Summer 2011 and future years. (5) Add a third group of students for comparison. The current protocol is to offer the ALEKS software to all declared STEM majors passing though orientation. The result is that all STEM majors self select into those who use the software appropriately and those who do not. The protocol can be modified to collect performance data on students who are not STEM majors.

Summary

Retention of qualified students at all universities is a common goal, one of particular importance as universities strive to maximize efficiency in face of reduced resources. This study reported on four program activities that were targeted toward first-year success for undergraduate STEM majors. These pilot studies have resulted in numerous lessons learned that are informing this program's second year activities. Creating a STEM freshman learning cohort instead of one that is discipline-based has provided a stronger sense of belonging to the STEM community as a whole. This approach has engaged not only the students, but also has helped the university as a whole to appreciate the commonalities among STEM majors as well as the challenges that STEM majors face.

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AC 2012-5051: BOTH SIDES OF THE EQUATION: LEARNER AND TEACHER

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Both Sides of the Equation:
Learner and Teacher

Abstract

An engineering professor decided to retake a first-semester calculus course under the tutelage of the chair of mathematics at Boise State University. While completing the course with 37 other students, she had in-depth experiences as a student of a calculus class as well as an experienced educator with a strong background on STEM retention. During the course, she recorded her observations and experiences in the classroom. The math professor also shared reflections on his teaching, observations of his students, and perspectives on the influence of her presence in his class.

The two professors' reflections enabled us to identify a set of student assumptions and learning behaviors that would likely influence their learning outcomes in both positive and negative ways. We developed a survey questionnaire based on the identified student assumptions and learning behaviors. At the end of the course, we administered the survey with the calculus students in order to obtain the students' perspectives. By triangulating the three sources of information and through our self-reflections on the results, we have generated recommendations on teaching strategies to which math and engineering instructors might need to pay attention, in order to better understand students and to provide them with more meaningful learning experiences.

Introduction

Of the many factors affecting student success in engineering, competency in mathematics is among the most frequently cited. Indeed mathematics proficiency is at the heart of national conversations about education at all levels. News headlines and policy reports warn that U.S. K-12 declining mathematics test scores portend concerns for national competitiveness[1]. "Change the Equation," the national initiative led by more than 100 corporate CEOs, underscores math proficiency as essential to achieve STEM literacy and to stimulate technological innovation and economic prosperity. The recent February 2012 report by the President's Council of Advisors on Science and Technology includes a focus on mathematics preparation as one of five key strategies to produce one million additional college graduates with degrees in science, technology, engineering and mathematics over the next decade[2].

Like the rest of the country, Boise State University has implemented research projects and initiatives to study and improve mathematics success among engineering students, with particular emphasis on freshman retention. An engineering professor who has led several of these initiatives decided to experience freshman-level calculus firsthand by re-taking Calculus 1 nearly 30 years after her own freshman days. Her instructor was the chair of the mathematics department, a professor with whom she has collaborated on numerous research projects. The evidence presented in this paper is based on the experiences of these two professors. The information presented may provide other instructors with insights of use to them in their instructional strategies.

The paper is structured in six sections. First is a collection of observations from the perspective of the engineering professor, the Learner. These were written approximately mid-semester without comment or review from the Calculus professor, the Teacher. Next is a similar section from the point of view of the Calculus professor, written in the same time-frame. This was also written without any knowledge of the engineering professor's comments or any input or feedback. The third section is a third party analysis of data collected from students and the two professors after obtaining institutional review board approval late in the semester. The next two sections are the thoughts of the two professors *after* reading each others' commentary and the third party analysis. Finally, a set of actions that have already been taken or that are planned as a result of being experiencing "both sides of the equation," are listed.

[1] The Learner's Experience

Our professor came to class today with a giant packet of exams. By now, he knows all our names, having practiced daily with index cards we created on the first day, with our name and favorite movie. I admit I am a little nervous about getting my exam back. I believe I did okay; I think it's possible that I did very well. I was able to answer all the questions, but I have a superstition about this – when I think I've done well on an exam, I often wind up with a low score (meaning, a B, instead of an A; or the rare C). So, I'm a little nervous.

My exam is now in my hands – both parts. I can't see a score readily visible; there are two stapled sections, each with multiple problems, each problem with a score. I start adding them up to see my total. My professor has a very interesting model for giving math exams. Because our class periods are only 50 minutes in duration, he allots two days for each exam, so that we have enough time to answer the problems. He gives the exam in two parts. Part 1 was given on Wednesday, in the fifth week of classes, and was worth 80% of the possible points. We read on the professor's website[3] that part 1 is designed with a strategy of being a combination of "type 1" and "type 2" questions – the sorts of questions we've been practicing on, in our homework. Part 1 is closed book; we can use any form of calculator we want, but cannot connect to the internet. There are two versions of Part 1, as the class sits in relatively close quarters. The exams are put in front of us in different colored envelopes so the professor can easily see which version you have as he distributes them across the classroom. Part 2 of the exam was given on Friday, and worth 20% of the grade. Part 2 is open book, open notes; designed to be a higher level of problem solving, or as my professor calls them, "type 3" questions. Each day, I finished within the time-frame, with time to check my work and even time beyond that; but I noticed other students used every minute of the time available.

Today is the following Wednesday, and the professor already has both parts of the exam graded. If my addition is right, I got a 95! Feeling good, I look to my friends on my left and right – some have small smiles; others look a little tense. The distribution is put on the board, and we are told the average was a little higher than normal, at 78. I have the fifth highest score in the class. What a great feeling! But, about a third of the class has a score below a 70, and these students are invited to meet with the professor, to make an appointment to see him. He tells them to “bring everything – your notes, your homework, your exam.” He will do his best to diagnose some things that they can try to do differently to help them correct their course.

Perhaps you realize by now that I am not a “normal” calculus student. I am retaking it. Not because I failed it the first time, but because when I last took calculus, Jimmy Carter was President and I was enrolled in calculus as an undecided first-semester freshman. I attended a large lecture three days a week, and a discussion section two days a week. I have zero recollection of the math professor and only slightly more recollection of the subject matter. I was the sort of student who went to all the classes, and did the assigned homework. With weekly quizzes and collected homework in my Jimmy Carter calculus class, I earned an A – which likely influenced my decision to stay in a “STEM” major.

My motivation for taking calculus under the presidency of Obama began at first with my desire to be able to help first-year engineering residents with their math homework. I’m the Engineering Residential College’s Faculty in Residence at a metropolitan university in the northwest. This means that I live in an apartment in a residence hall on campus, on a floor with 18 engineering students. And I get asked lots of questions – some of them about homework. I can generally rally on the chemistry questions, and reason through the physics, but my memory of calculus has faded dramatically over the years due to lack of use. This fuzzy memory was embarrassing to me, so I decided to retake the class. It has been an interesting experience, and I have recorded some of my observations, from the perspective of a student who has taught as a materials science and engineering professor for nearly twenty years.

Observation #1, Help Students Connect! Students don’t reach out to each other much. At first, in the class, hardly anyone spoke to each other. After hearing the professor actively encourage group study sessions, I became relatively proactive in the back of the room, and got students exchanging cell numbers with me, and with each other, and even put one student in charge of distributing the information about informal study sessions they subsequently organized. I am pretty sure that this study session, involving about 10 people from time to time would not have formed without my instigation. By the sixth week of classes, they sorted themselves into smaller groups, pairs and triplets of students who study together and routinely sit next to each other.

Observation #2, **Engaging the Class:** My math professor is an excellent instructor. He uses a variety of techniques to engage the students. He has us do in-class exercises, “warm-ups” he calls them, and says we should be able to differentiate as fast as he can write the statements on the board. He has us work in small groups with each other. He smiles in class and has a wry sense of humor; the class chuckles from time to time and students genuinely seem to enjoy class. Students feel very comfortable asking in-class questions, and he will deviate from his lecture to accommodate as many questions as we have about the homework or what we've been covering. He uses interesting analogies – for example, he refers to the rules of differentiation as "power tools." And then goes on to explain, "You have to know how and where to use them so you don't hurt yourself." And: "Practice, practice. You want to get to a point where your fingers remember even if your brain forgets!"

Observation #3, **Assumptions:** As professors, we make assumptions. For example – my professor assumed that we knew it was okay for us to work on our homework together. As a student, this was not obvious to me – and so in class, the second week, I asked aloud the question – "What do you think about study groups?" I think my professor was getting a little tired of questions from me by then, and he replied concisely, "They're good." "Why?" I replied. He elaborated – and his words were important to the class. He explained that generally, students who work in study groups, do better on exams, and that a lot of research has been done on this. In his view, it boils down to students in study groups having a more rapid feedback loop when doing homework. The key thing that he said was, "I encourage you to work with other students when you're doing homework." This was important to state aloud because the "honest" student might have imagined that it was unethical to work together; that we were expected to be doing all our own work. In fact – we were encouraged to work with each other. Having this explicitly allowed was important, and as instructors I propose that we all make a point of stating our views on study groups in the first or second week of classes. It could be put into our syllabi as being explicitly permitted and encouraged.

Observation #4, **Weekly Graded Homework Is Important**. In my calculus class, we have four homework assignments a week, one for each day of class, and they are collected on Fridays into two piles. We do not have any discussion sections – the professor is there every day. Pile 1 – a thicker stack, is for a grader (an undergraduate paid by the hour) to briefly examine and assign a grade. Pile 2 is for the professor, a shorter stack – he is careful with his time, but gives each of us a few minutes every week, as he evaluates our work. The very interesting aspect of this professor's homework grading – is that the homework grade for the week is a product of the two pieces[1]. So – you could get a 9 from the professor, and an 8 from the grader, making a grade of 72 for your homework that week. I really like this homework model, and plan to implement it next time I teach a class.

Observation #5, Women: About one-third of my calculus class is female. Nationally, about 20% of engineering degrees are conferred to females[4]. A recent, comprehensive article examining the causes behind the under-representation of women in engineering concludes that one of the underlying reasons for this is recruitment – more women need to be recruited, through outreach, into engineering disciplines[5]. I have made it a point to speak with many of the women in class; about half of us are in a large clump in the far back row. Has anyone considered students taking Calculus 1 as an engineering recruitment pool? I think it a great opportunity for an engineering professor to reach out to the calculus professors, and to let them know if they ever want a substitute one day, that they'd be delighted to speak to the class about some applications of calculus. Send a dynamic professor in, and then follow up with an invitation to an event, or to have coffee with you if they have questions about engineering as a career choice.

Observation #6, Vision: My eyesight has changed! It's a different experience, wearing readers in class. My vision has always been exceptional. This changed for me recently, and I now use fairly weak readers – but they make a crucial difference. I made two mistakes on the exam – and one was a transcription error, I didn't see one of the numbers. With about 14 million Americans aged 12 years and older having self-reported visual impairment[6], I see no reason not to use a 14 point font on exams and homework assignments.

Observation #7, Distractions: Students text in class. I couldn't believe my eyes; one student sitting next to me was routinely texting in class. I happen to know that my professor is one of the best math instructors we have at this university. I told the student this. And then on another occasion when we were working together and the student was glancing at her phone, I asked her to please put it away. Nearly all the other students seemed quite focused and did not look at their phones with any measurable frequency. This particular student unfortunately did poorly on her first exam, and I subsequently had a conversation with her about how educational theory shows that it takes several minutes to refocus after receiving a text – and how by then, another text has been received, so altogether texting while studying attending class or studying renders the time spent relatively ineffective. Next time I teach a class, I plan to show students some research on this issue. And then ask the class to agree on some ground rules for the class, because it distracts the people nearby as well as the person engaged in the activity.

Observation #8, Love of Learning; Reasonableness: I had forgotten – or perhaps never realized – that I love to learn! I enjoy solving problems. It is satisfying to correctly answer a math problem. My professor keeps asking questions about rabbits, and how the population of rabbits grows, and what rate of change the rabbit population has when there are 500 rabbits, and so forth. I find it fun to figure out the rate of change in the rabbit population. My skills in reasonableness are helpful. At some point in my college education, I learned how to judge an answer as being reasonable. When did I learn to apply this "reasonableness" judgment? At some point, I developed this expertise. How can we teach this to freshmen?

Observation #9, Self-Efficacy Matters: I am confident in my mathematical abilities now and it makes a difference in my classroom mentality. Knowing you can actually perform, given enough time is a great feeling. As instructors, the things that we can do to build up the self-efficacy of our students in terms of their ability to apply what they've been learning, are important. Here's an example of how my professor enables mastery experiences, which help shape self-efficacy[7]. He has 100% of his old exams – and their solutions – on his website, with statistics of student performance on each question. This allows students to practice problems that are similar to the types of problems they will see on their upcoming exam. Having a straightforward exam, with opportunity for practice and enough time to answer the questions by using two days for the exam gives students a fair opportunity to perform to their level of preparation.

Observation #10, Calculator Competency: Calculator competency is all over the map in terms of the freshmen at this metropolitan university. For this course, I began to use a spreadsheet application, to calculate the various secant slopes required in the homework. This was incredibly efficient, but I knew that for exams I had better start using my new TI-89, as I couldn't whip out my laptop during an exam. I painstakingly turned it on and started using it. In the Jimmy Carter days, I used reverse polish notation, and there were no graphing calculators available for use in examinations.

Now – my past experiences made me aware that there is definitely a way to store a number in a calculator. Yet how to do this was not obvious to me in peering with my readers down at the tiny notation on the calculator side-buttons. During an in-class exercise, hand-calculating a series of secant slopes, I realized how it would be useful to store the outcome of X times "e" raised to the X, where X was 1.003476. As an experienced "networker," I have realized that simply "asking one who knows" is a faster way of figuring out how to do things than reading the manual. So – I asked the person on my right, how to store a number. She didn't know. My eyebrows arose, internally marveling at this deficit. I asked the person on my left, who also did not know. And then the person to the left of left, and right of right. The people in the row in front of me. I actually got to my feet to get to more people and had surveyed almost one-third of the class before finding ONE student who would show me how to store a number in calculator memory. I shared how to store this number with all who were interested. I also shared it with the professor, who was unsurprised. Yet – this is a performance issue, and I am still astounded by the lack of calculator knowledge in our freshmen. Their calculator "know-how" is not even. This has motivated our university to develop and hold a calculator help session in spring, 2012; we may also introduce some topics in the Introduction to Engineering class that engineering students usually take concurrently with Calculus 1.

Observation #11, Networking: Students don't know how valuable a resource they are to each other. I think that this is where experience plays in, and I believe I learned some of this during my upper division education as a chemical engineering student, and the rest during the research phase of my Ph.D. I learned to ask for help, to consult with multiple people about how to approach a task that I hadn't done before. I applied my networking skills like crazy in this course. It is definitely helpful to be living on a floor of fellow engineers, because when I hit a task on my calculator that I need help with, I go down the hall looking for a student who can help me. One student in particular has been incredibly helpful to me, even though he wasn't familiar with the Ti-89, he knew that what I needed to do, could be done, and fiddled with it until he figured it out. One thing I asked him was how to program an equation in to calculate a series of numbers – this was in about week four of the class. I learned from him that it could not only do that, it could also graph it by just hitting two more keys.

You can probably guess what I did with this information – the next day I was demonstrating this to the students I sat near. Some already knew, but more didn't. One of them was dumbfounded and wished she'd known how to do this three weeks ago, before we started doing hundreds of secant slopes by hand. By showing my fellow students what I had learned from another student, I was hoping to be teaching them more than just that one new skill. I hoped that I taught them that there is a wealth of information to be obtained from your peer group. Network and ask your classmates.

Observation #12: **Thank You, Newton!** Differentiation is a LOT FASTER than taking secant slopes – even if you know how to program your calculator! By the time we were introduced to the derivative, we were definitely ready for it.

Summary: It is now the end of the semester, and grades are in. Although I didn't formally register for the class, I did take all the exams, including the final. I attended every class when I was not out on travel. I did most of the homework. My grades on the exams were pretty good: I certainly achieved my original goal of relearning calculus.

[2] The Teacher's Experience

When the engineering professor first broached this idea I was immediately attracted, but for no concrete reason that I could articulate at the time. Mostly the positive feeling arose from my respect for her dedication to improving instruction at our institution. It also helps that we have a history of productive collaborations on educational initiatives at our institution. At the same time, through, there was a sense of trepidation. Probably no teacher is immune to a little doubt or nervousness when there is a peer or professional observer watching – and what she proposed was an entire semester of it. It's also worth noting that I had a weak understanding of why she wanted to do this. I have to admit that her claim of wanting to relearn Calculus seemed odd – and this seemed like a weirdly large investment of time to achieve that end. I was a bit skeptical, and suspected that she was just as much interested in a close-up look at teaching methods, at least as practiced by one person from the math department.

Although I did not have specific expectations of the value that the experience might provide, my intuition was that it would be both useful and interesting, and well worth any potential downside. It was easy to say "yes" to the proposal.

Observation #1, A Different Audience: Teaching is at some level a performance. There is usually a real-time feedback loop running in my head in which I am asking and answering questions like "Is this working?" "Are they getting it?" The engineering professor's attendance in my class has altered my sense of the audience. While I teach, I hear questions in my head. Often it's "Did she get that?" but I also extrapolate to, "If she is not getting this, surely others are not," and, "Looks like she is getting this. I wonder how it comes off to the others."

Observation #2, New Signals: When she looks puzzled, sometimes I'm not sure why. Is this because she doesn't get the math that was just explained, or is it meta-puzzlement? Maybe the math makes sense to her but she is wondering why I chose to say it that way or how on earth I could think that freshman would understand what I just said. Sometimes I wonder if she is puzzled-on-behalf-of-others. That is, she gets it, but worries that the students don't, and she's sending a signal so that I will notice and respond with an alternate explanation. Sometimes I wonder this when she asks a question, too.

Observation #3, Student Questions: I start almost all classes by checking to see if anyone has questions. The intent is to answer questions about last night's homework, but I take anything. It is not unusual for this to garner complete silence. Some of this is probably students' anxiety or fear of being the first to ask a question. The engineering professor breaks the ice. I'm never sure if she's asking about homework she struggled with or if she is acting as spokeswoman for other students. No doubt there is a spokeswoman effect, since it is not uncommon for students to take the approach of hoping that someone else asks about the problem that they got stuck on. Sometimes I worry if she does too much of this, since I want an atmosphere in which students are taking advantage of the question period and not just waiting, hoping someone else will ask. If she does a lot of this, students may come to rely on it rather than develop the habit of asking themselves.

Observation #4, Introspection on Difficulty: I find myself introspecting more than I normally would on the level of difficulty I have designed into the course. From ungraded homework all the way to the most difficult exam questions, I find myself wondering if she thinks this is too hard for first year calculus students, or perhaps too easy. I also wonder what she thinks about where this is all going. That is, why should students bother to learn this stuff (or more accurately, why should they be required to learn it)?

Note: Observations 1-4 reflect thoughts that were fairly prominent in my mind early in the semester; less so as the semester progressed.

Observation #5, Facilitation: Sometimes I want students to do work at their desks, either for a very brief period to warm up to what I want to talk about, or for most or all of the class period. The engineering professor helps this come off better than it might otherwise. Some of her contributions:

- She always jumps into the work. Sometimes the rest of the class is a little reluctant to get going and needs some prodding.
- She immediately involves a few other students sitting near her. Frequently the rest of the class needs more than a little prodding to do this.
- At the very least, this gets some small number of students immediately working and immediately involved in some sharing and discussion. Sometimes this seems to work as a social example and get other students to do likewise.
- When the student work occupies a full class period I like to move about the room keeping tabs on what each group is managing in the way of forward progress. There are 37 students, and usually a dozen groups, so I can't make rounds fast enough. Sometimes she will jump up and circulate also. This is *very* valuable on days that are primarily centered on student work. It is also a new experience for me to have an additional, equally able, facilitator to make the rounds. I seems like there is an opportunity here for me to learn a very different approach to group work that could rely on a number of sufficiently able facilitators and serve a potentially much larger group of students.

Observation #6, Direct Feedback: She is highly complimentary of my teaching. It feels shallow to care, but I do and it's nice. She is never not-complimentary. Nor has she offered specific advice or constructive (or other) criticism. If she did, it would probably bug me a little, and then I would feel shallow about that, too.

She should do it anyway. It seems likely that she has advice that she would like to share. It's possible that she feels constrained to silence by the social dynamics or by the nature of the experiment we are conducting. However, if what we are doing could possibly be a vehicle for increasing teaching effectiveness, then it must include an open channel for advice and constructive criticism.

Observation #7, Classroom Dynamic: When I was much younger it was normal for a more senior faculty member to conduct an occasional classroom observation. My recollection of this is that students were well aware of the unique situation on the day of an observation, and that there was a tangible difference in the classroom dynamic. Interestingly, I sense no such phenomenon generated by this observer. It could be because it's more of an every-day thing. It could be that she manages to convey a non-authoritarian presence, whereas other observers did. Maybe, her consistent attendance and her participation on homework and exams made her role one of student/learner, like them, rather than observer.

Observation #8, Graded Work: It is revealing to grade the engineering professor's work next to student work. She's pretty good. She gets most everything right, but then so do a lot of other students. But there is a huge difference in the quality of communication conveyed on her exam papers. I am intrigued, but unable to pin down what's actually happening here. Whatever she is doing it is very much what I (and probably many, many other college instructors) wish students would do. It is probably worth a close examination of what she does differently and how she learned to do it.

Observation #9, Study Groups: I believe that one of her goals is to foment the formation of stable study groups. It is unclear if her presence does this (except as noted already for in-class work). If so, it is also unclear if her presence changed what would have happened.

Summary: The entire experiment – having an engineering professor in my Calculus class – gave me a stronger sense of education as a collaborative effort instead of a solo act. I don't know how much I respond or adapt to her presence, but I do find myself searching out her face each morning. Without knowing why, it seems that my first thought at the beginning of each class is whether or not she's there.

[3] Students' Perspectives Compared to the Professor and the Learner

Based upon reflections on the two professors' observations, we identified several main issues to pay attention to:

1. Importance of help-seeking and networking - Wouldn't it be important for students to provide and seek help to/from each other and to network with each other?
2. Expectations and effectiveness about collaborative homework - Are the instructor's and students' expectations/assumptions on collaborative homework same or different? Did students find the weekly graded homework as effective as intended?
3. Perceived effectiveness of instructional methods - Did students find the instructor's instructional strategies and characteristics to be effective?
4. Perceived effectiveness of multi-tasking during class - Did students understand the potentially negative effects of multi-tasking in classroom?
5. Effects of timed tests - Is our method of timed tests the best way to measure student competence?
6. Impacts of the engineering professor's presence on other students - Did her presence have an effect on students?

We then developed a survey questionnaire (see Appendix A) to measure students' perspectives on those identified issues. Thirty-seven students enrolled in this Calculus class. We administered the anonymous survey on the last day of the class without the presence of the math professor and the engineering professor in class – 30 students voluntarily submitted the survey (21 male and 9 female). The average age of the survey participants was 21 years old (*Min.* = 18, *Max.* = 32). The math professor and the engineering professor also independently completed the same survey to provide their perspectives.

Finding #1: Importance of help-seeking and networking (Q3-Q9)

Is it important for students to provide and seek help to/from each other and to network with each other? Research has shown that the answer is usually yes, although the help-seeking behavior is complex and often influenced by motivational and attitudinal factors[8].

Both the math professor and the engineering professor either *agreed* or *strongly agreed* that in order for students to be successful in this class, it would be important for them to provide help to, or seek help from, their classmates.

A majority of students (80%) also *strongly agreed* (33.3%) or *agreed* (46.7%) that it would be important to collaborate with their peers. Students perceived that about 3 or 4 members in a study group would be an optimal number (*M* = 3.43, *Min.* = 1, *Max.* = 6).

Students reported that they *occasionally* provided help to their classmates, sought help from them, and shared information with them. As shown in Figure 1, the engineering professor's observation supports the students' self-report on these help-seeking and networking behaviors, likely because of her experience as a student in class (see 2.7 vs. 3.0, 2.7 vs. 3.0, and 2.9 vs. 3.0 in Figure 1).

The more important students think it is to provide help to, or seek help from, classmates to be successful in the class,

- the more frequently they actually helped their classmates (*rho* = .606, $p < .001$)
- the more frequently they sought help from classmates (*rho* = .459, $p = .011$)
- the more frequently they shared useful information with each other (*rho* = .738, $p < .001$)

Several students reported difficulty in finding knowledgeable peers as a problem in trying to help or seek help (e.g., “they were as stuck as I was”). The engineering professor’s observation confirmed this problem; she recalled, “their fellow students may not have been able to help them. For example, when I was seeking help with my calculator, I had to consult with about 8 students before I found one who could help me.” The math professor also observed a similar issue during small group activities in class: “[I] did not have sufficient opportunity to observe students seeking help from each classmate. The only chances I had were during small group work in class. On those days my time is mostly spent on groups that are completely stalled.”

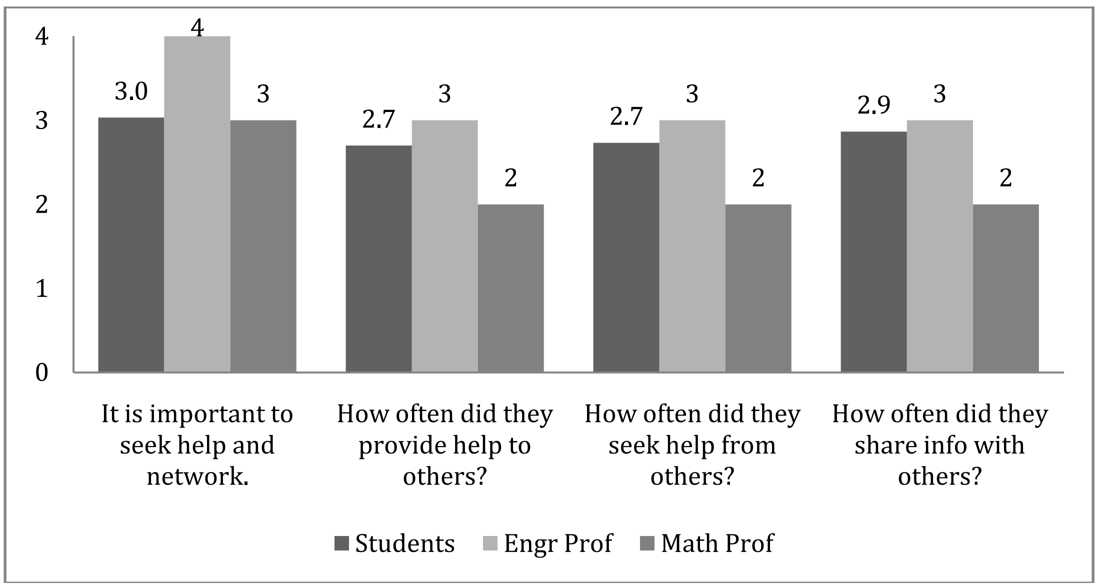

Figure 1. Importance and frequency of help-seeking and networking.
(1 = strongly disagree or never, and 4 = strongly agree or frequently)

Finding #2: Expectations and effectiveness about collaborative homework (Q14-Q18)

Students were assigned to complete weekly homework which was counted toward 16.7% of the total grade. All students except one thought that the weekly graded homework assignments *tremendously* (66.7%) or *fairly* (26.7%) helped them improve their knowledge in Calculus, which was supported by the math professor’s expectation and the engineering professor’s observation.

However, students tended to work in a study group for only *some* of the homework assignments (on a 4-point scale of none, some, most, and all), although they thought that it was *fairly effective* to work with classmates in a study group to compete their homework assignments. Students experienced difficulty in scheduling for meeting and not having anyone in their team capable of solving difficult problems.

Another possible reason is a lack of understanding of the instructor's expectation. A meta-analysis on the effects of small-group learning on undergraduate science, math and engineering courses has shown positive outcomes on academic performance, and attitudes toward and persistence in learning[9]. Supported by the meta-analysis, the math professor's expectation was that students were allowed to work with their classmates to complete their homework assignments. However, both the engineering professor and the math professor predicted that in the beginning of the class, students might have not been sure about whether they had to complete the homework assignments alone or collaboratively.

Indeed, the student survey showed that eighteen students (60%) knew from the beginning that they could collaborate with classmates to complete homework, but three students (10%) were not sure about the expectation and nine students (30%) thought they had to do it alone. In other words, 40% of the students (the last two groups) did not clearly understand the instructor's expectation in the beginning.

Overall, the students and the engineering and math professors shared similar perspectives about the frequency and effectiveness of collaborative work in a study group when completing homework assignments (Figure 2).

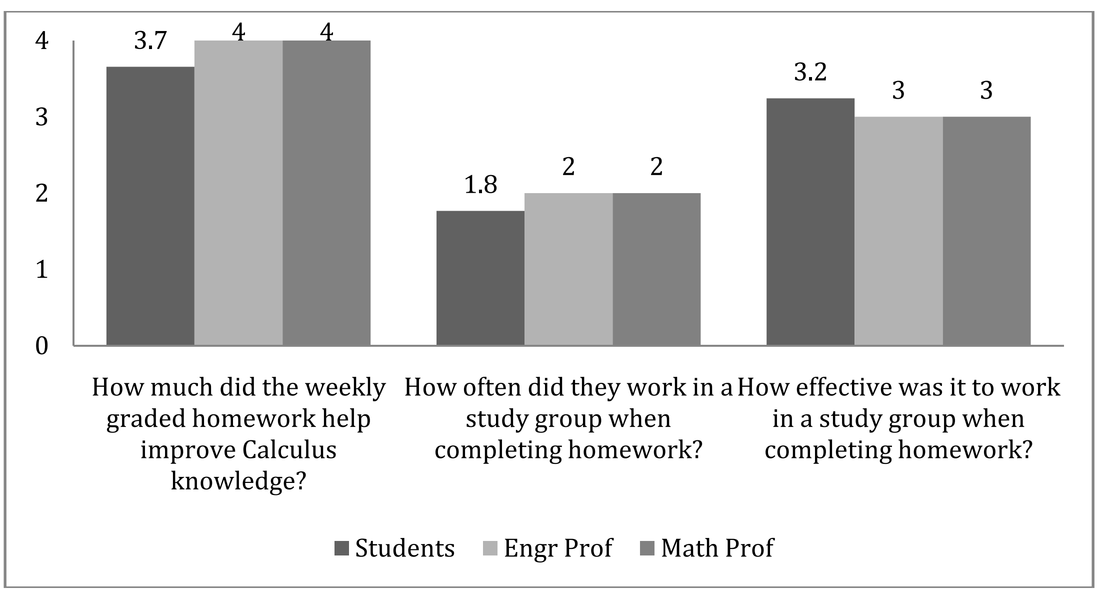

Figure 2. Frequency and effectiveness of collaborative work in a study group.
(1 = none or not effective, and 4 = tremendously, all homework, or extremely effective)

Finding #3: Perceived effectiveness of instructional methods (Q10-Q13)

What do the students think about the instructor's teaching strategies and characteristics? During the class, the engineering professor observed the math professor using several effective instructional methods – e.g., having a Q/A session in the beginning of each class, providing warm-ups exercises, showing a sense of humor, and having students work in small group activities. The students confirmed in the survey that those instructional methods were helpful (Figure 3). Encouraging students to ask questions in each class helps them self-monitor their understanding of Calculus and develop self-regulative behaviors, which likely facilitate higher learning outcomes[10]. Research has also shown positive outcomes of using small group activities in learning math[11, 12].

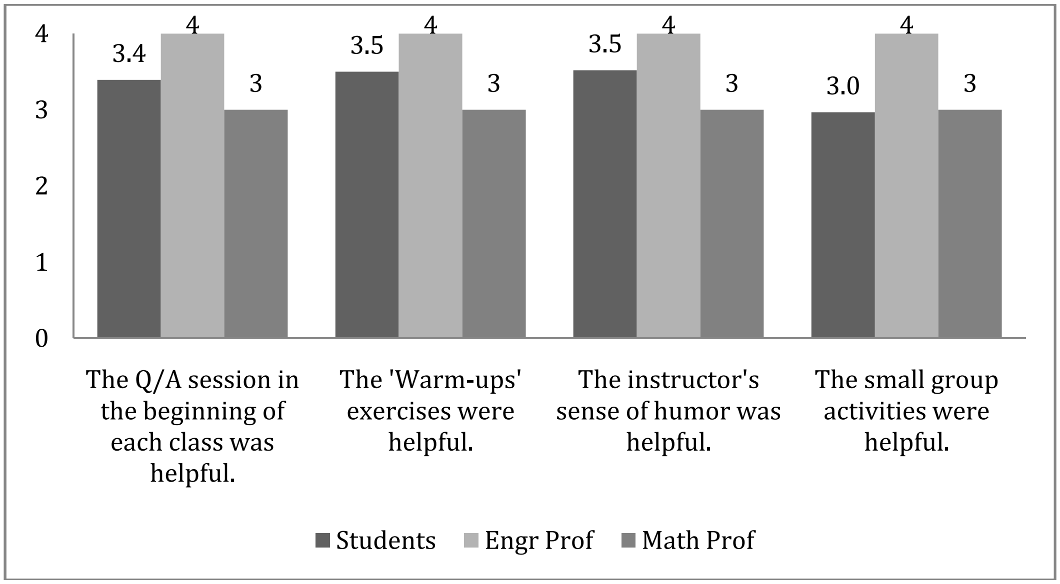

Figure 3. Perceived effectiveness of instructional methods.
(1 = strongly disagree, and 4 = strongly agree)

Finding #4: Perceived effectiveness of multi-tasking during class (Q19-Q21)

The engineering professor observed that some students engaged in off-tasks during the lecture such as reading and sending text messages, and she strongly believed that multi-tasking during the class would reduce learning effectiveness, as supported by research[13]. This multi-tasking behavior in class was confirmed by students' self-report, although students also shared the same belief that multi-tasking could reduce learning effectiveness (Figure 4). More than half of them (51.7%) either *frequently* (3.4%), *occasionally* (17.2%), or *sometimes* (31.0%) engaged in multi-tasking such as reading/sending text messages during the lecture, even though all students *strongly agreed* (75.9%) or *agreed* (24.1%) that it would be important to focus on lecture without multi-tasking.

The more confident in learning Calculus students were, the more strongly they agreed that it would be important to avoid multi-tasking during the class (*rho* = .432, *p* = .019).

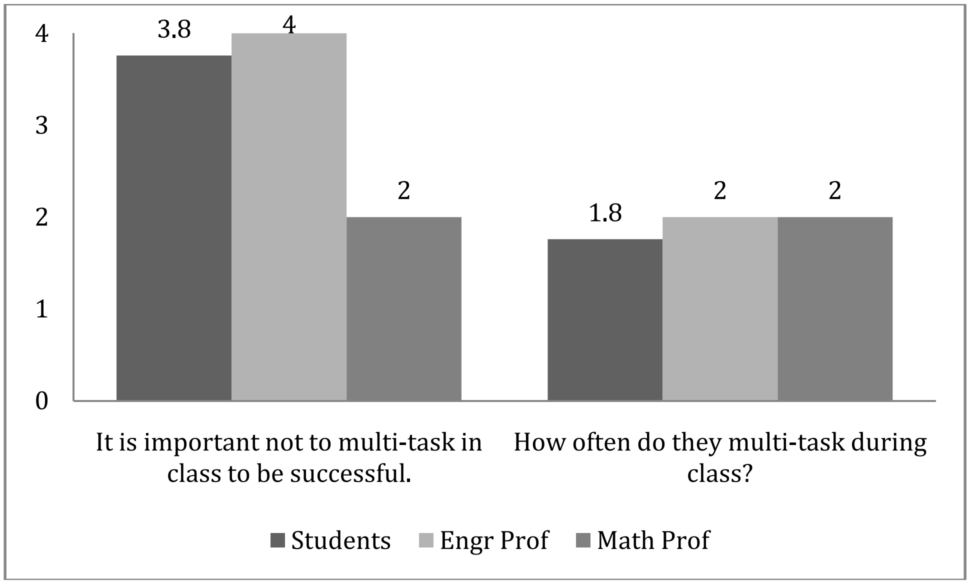

Figure 4. Perceived effectiveness of multi-tasking during class.
(1=strongly disagree or never, and 4=strongly agree or frequently)

Finding #5: Effects of timed tests (Q22-Q24, Q26)

Is timed testing an appropriate measure of student competence? Sixteen students (57.1%) reported that they *sometimes* had to submit a test without completing all questions because they ran out of time (also see Figure 5). Not surprisingly, the more strongly they *disagree* that speed is an important measure of student competence in Calculus, the more strongly students think they could produce a better score if they were given more time during the test (*rho* = -.457, $p = .013$). Also, students who experienced lack of time during the test tended to expect to receive a lower grade (*rho* = -.396, $p = .037$). Research seems to support these outcomes; one study has shown that both low- and high-achieving students performed better on their statistics exam under the 'no time limits' condition than under the 'timed' condition, and that students with high anxiety (often lower performers) had greater benefits from the untimed condition[14].

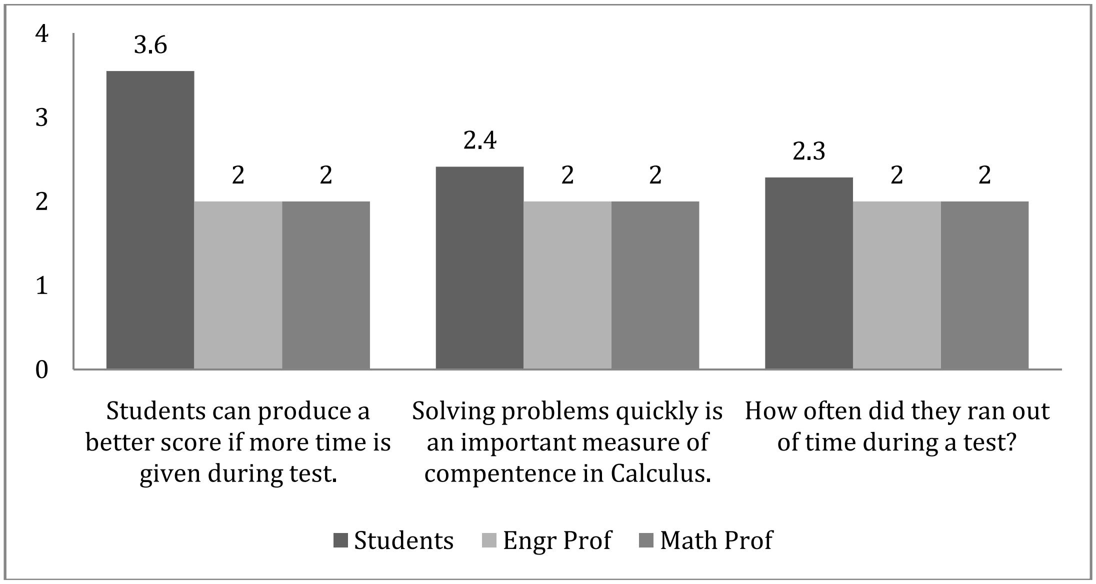

Figure 5. Effects of timed tests.
(1=strongly disagree or never, and 4=strongly agree or frequently)

Finding #6: Impacts of the engineering professor's presence on other students (Q27)

The engineering professor attended the Calculus class and engaged in class activities as other students did. She did not introduce herself as an engineering professor until the last week of the class. Did her presence have effect on other students in class?

Forty-eight percent of students reported that they thought she was just one of the students, and that her presence did not directly affect them. However, students reported various benefits they gained from her presence in class (Figure 6). Most of all, students acknowledged that her active involvement in class helped them ask more questions in class (n = 11). This was one of things that the engineering professor was hoping for, and the math professor also pointed it out as one of the impacts that she made in class.

Students reported that her presence also helped them want to do homework with others (n = 6), want to seek help when they needed help (n = 6), pay attention to class (n = 5), not to multi-task during the class (n = 3), want to help other students when they needed help (n = 3), and use their calculator more effectively (n = 2).

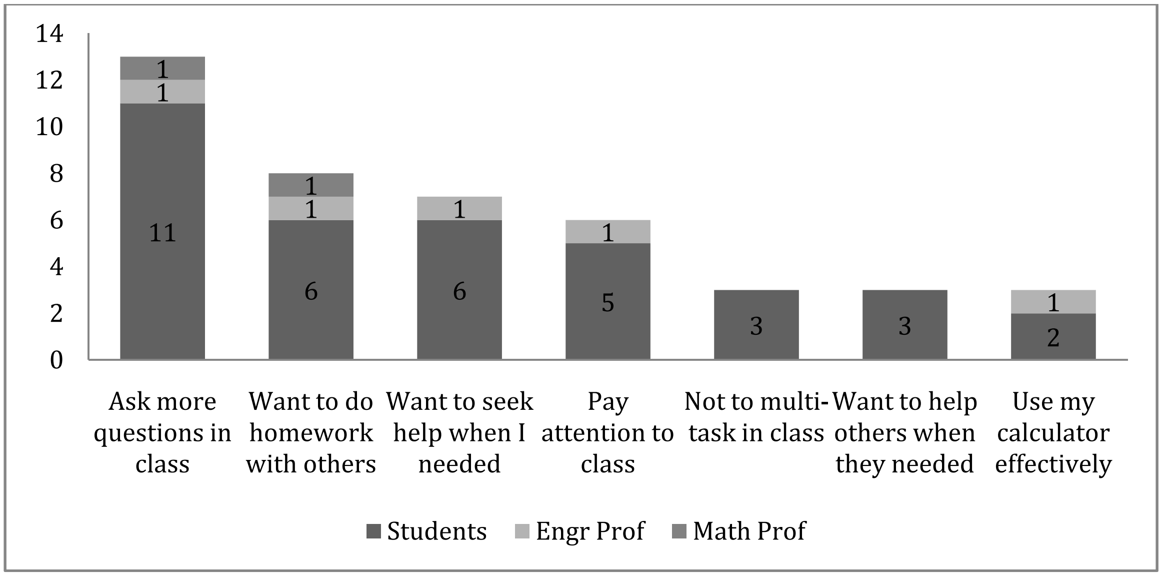

Figure 6. Impacts of the engineering professor's presence.

[4] The Learner's Reflections

In fall, 2011, I went to math class four days a week, from 8:40 to 9:30 a.m. I sat in the back of the class with other students who became my friends. Beyond content remastery, I gained an incredible perspective. I learned some new teaching tricks while also reconnecting with what it feels like to be a student. It is scary to be a student. We don't know some basic skills – how to network, how to use our calculators, how to study, how to focus, when it is and is not okay to study with others. As professors, we can help students with these skills, by actively talking about them. That is – in math, we can help students form study groups. We can encourage them to network when they get stuck on something. We can help them use their calculators. We can learn their names.

When asked during the semester what it was like to take calculus, and also in retrospect, I have this to say: "It was the favorite part of my day."

[5] The Teacher's Reflections

I could write another paper solely in response to thoughts generated by reading the comments from the engineering professor in section [1]. For brevity's sake I will restrict myself to a few of the more important lessons I believe I learned from this experience.

- I was a much more reflective teacher this semester, due wholly to the sustained presence of this colleague in my class.
- On a deeper level, my exposure to her written thoughts, and then subsequent iterations of my response to them and ensuing dialog, have proved to be much more valuable to me as an instructor than any amount of self reflection could ever be.
- The biggest single item from her observations that I plan to add to my teaching practices is to be deliberate about instructing students in successful habits.

Specifically, the formation of collaborative groups and an understanding of the value of homework. I've always known I should say more about these things, but semester after semester I end up not saying much. This needs to change.

- I am quite surprised by the how closely her and my answers to survey questions track with the average student responses. My *a priori* assumption was that students would display significantly different opinions and perceptions. I am particularly happy to see that students value homework.
- The two answers where there was significant distance between the students and the professors stand out. The first is the issue of time allowed on exams, which both professors felt was mostly irrelevant, while students felt it strongly affected performance. On this point I have no illuminating comment, other than that it is good to know their collective opinion.
- The other survey question with divergent answers was about student's multitasking during class. I felt it was largely irrelevant, but students and the observer felt it was very damaging (Even more than time pressure on tests!). I had long held a firm opinion about this – I assumed it was irrelevant because I think very little of what I say in lectures has much to do with student learning. I believe learning lives in the homework. What I do with my 50 minutes each morning is entirely aimed at inducing the doing of homework. For many students, direct instruction in how to work a homework problem is a pretty useful inducement, so I do a lot of that. On the other hand, there are many other ways for a student to find their way through a homework set. I don't presume to know whether or not watching me do examples is the thing that works for any particular student. If someone chooses to text instead of watching me, I would assume that this is because watching me work examples isn't what gets them traction on homework problems.

  However, the student response to this question is too extreme for me to remain comfortable with my prior belief. Clearly this bears additional thought. And perhaps it calls for a change in tactics in the classroom.

[6] Specific Actions Taken

As a result of reflecting deeply on both sides of the equation, learner and teacher, and the vast number of unintended positive consequences associated with the semester's experiences, we recommend and/or are institutionally adopting the following specific actions:

- A calculator "help session" will be offered in spring, 2012.
- We plan to support mathematics instructors who may be interested in taking a follow-on course, e.g. physics with calculus, in order to foster deeper relationships between instructors in different fields, student-centric teaching, share and learn teaching best practices and to help teach how mathematics is applied to solve problems in different fields.
- The chair of mathematics and the engineering professor hosted an informal brown-bag seminar, to discuss and disseminate their observations in December of

2011. It was the best attended STEM brown-bag lunch held at this university, with 21 attendees.

- The engineering professor wrote and published in the engineering student newsletter, "Become an Instigator (Instigator: leader, mastermind, troublemaker)." It shared her experiences as a calculus student and actively encouraged them to "become instigators in their classes." To set up study groups; to speak with the person sitting next to them in class[15]. A follow-on student article is planned.
- Calculator functionality will be incorporated into the Introduction to Engineering class, which is taken concurrently with Calculus I at this university.
- The engineering professor plans to skip Calculus II and go right into Calculus III in spring, 2012. Wish her luck!

Acknowledgments

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Appendix A. Student Survey

Please respond to each question with your best answer. It is an anonymous survey and your name will not be associated with your responses.

Q1. I am __ male __ female.

Q2. I am ___ years old.

Please think about this class when you answer the following questions.

Q3. I think that it is important to provide help to, or seek help from, my classmates, in order to be successful in this class.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree

Q4. How often did you provide help to your classmates in this class?
__ Frequently __ Occasionally __ Sometimes __ Never

Q5. How often did you seek help from your classmates in this class?
__ Frequently __ Occasionally __ Sometimes __ Never

Q6. How often did you share useful information with your classmates?
__ Frequently __ Occasionally __ Sometimes __ Never

Q7. When you need help with using your calculator, how likely are you to ask a classmate or friend for assistance?
__ Very likely __ Somewhat likely __ Somewhat unlikely __ Very unlikely

Q8. How many members in a study group do you think works best?
_____ members (including yourself)

Q9. If any, what problems did you experience while seeking help from your classmates?

Q10. My instructor's question/answer section in the beginning of each class was helpful.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree
__ I don't remember having a question/answer section in the beginning of each class.

Q11. My instructor's "Warm-ups" exercises were helpful.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree
__ I don't remember the "Warm-ups" exercises he used.

Q12. When my instructor showed a sense of humor during the class, it helped me learn the material better.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree __ It did not matter.

Q13. The small group activities in this class helped me actively engage in learning.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree
__ I don't remember having small-group activities in class.

Q14. In the beginning of the semester,
__ I thought I had to complete the homework assignments alone.
__ I knew I could complete the homework assignments with classmates in a study group.
__ I was not sure about whether I had to complete the homework assignments alone or if I could complete them with classmates in a study group.

Q15. How often did you work in a study group to complete your homework assignments?
__ All homework assignments
__ Most of the homework assignments
__ Some of the homework assignments
__ None of the homework assignments

Q16. How effective do you think it is to work with classmates in a study group to complete homework assignments?
__ Extremely effective __ Fairly effective __ Fairly ineffective __ Not effective at all

Q17. If any, what problems did you experience while working with classmates in a study group to complete homework assignments?

Q18. How much do you think the weekly graded homework helped you improve your knowledge in Calculus?
__ Tremendously __ Fairly __ Just a little bit __ Not at all

Q19. I feel confident about my ability to learn Calculus.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree

Q20. I think that in order for me to successfully learn in this class, it is important that I focus on the instructor's lecture without multi-tasking such as reading/sending text messages.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree

Q21. How often do you multi-task, such as reading/sending text messages, while listening to this math instructor's lecture?
__ Frequently __ Occasionally __ Sometimes __ Never

Q22. In this class, I think I can produce a better test score if I am given more time during the test.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree

Q23. Solving problems quickly is an important measure of student competence in Calculus.
__ Strongly agree __ Agree __ Disagree __ Strongly disagree

Q24. How often did you have to submit a test without completing all questions because you ran out of time?
__ Frequently __ Occasionally __ Sometimes __ Never

Q25. After solving a problem, how often do you ask yourself, "Is this a reasonable answer? Does it make sense?"
__ Frequently __ Occasionally __ Sometimes __ Never

Q26. I think my grade in this class at the end of the semester will be:
__ A __ B __ C __ D __ F

Q27. In what way did the engineering professor's presence in class affect you? Select all that apply.

Her presence in class helped me:
__ pay attention to class
__ ask more questions in class
__ want to help other students when they need help
__ want to seek help when I need help
__ not to multi-task during the class
__ want to do homework with others
__ use my calculator more effectively
__ Others – please describe:

Q28. Please describe your experience in having the engineering professor in your class, and provide any suggestions.

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AC 2012-5540: USING ONLINE ASSESSMENT AND PRACTICE TO ACHIEVE BETTER RETENTION AND PLACEMENT IN PRECALCULUS AND CALCULUS

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Using Online Assessment and Practice to Achieve Better Retention and Placement in Precalculus and Calculus

Abstract

In the fall of 2008 Boise State University began using an online assessment tool, ALEKS[1], as an initial assignment in Precalculus and Calculus courses. This paper reports on the effectiveness of the ALEKS assessment as a self-placement tool, used in conjunction with standard placement tests and prerequisite courses. The benchmark levels of 40% and 70% of knowledge space in the ALEKS course: *Preparation for Calculus* for Precalculus and Calculus courses were used. The paper looks at the effectiveness of the assessment with these benchmark levels as a first student assignment, both as a tool for student success, and as an instrument for making efficient use of the university's resources. Although there are no hard answers, and although much information is anecdotal, we introduce a statistic that is pertinent to these questions and show that it indicates partial effectiveness of the ALEKS assessment.

Introduction

Placing students into the proper mathematics course is challenging; across the United States colleges and universities employ a wide array of strategies. A study conducted at Merrimack College in Massachusetts by Rueda and Sokolowski[2] provides a literature review, citing works by Cederberg[3], Cohen, et al.[4], Krawczyk and Toubassi[5] and others. In looking over placement rubrics there does not appear to be consensus on any one particular strategy for placement. Many mathematics departments use a combination of ACT/SAT; others have developed home-grown tests that are used with reasonable success. Some use a combination of ACT/SAT, home-grown tests, and commercial placement exams (such as COMPASS). When available, prerequisite courses are also used.

For universities that enroll significant populations of students who have stopped out of school for a period of time, placement is particularly challenging. When there is a significant time lag between when a prerequisite course is taken and when then the next registration occurs, students may fail to retain adequate material from the prerequisite course. Placement exams can also be problematic. For example, COMPASS exams are designed to be taken without preparation, but often students do prepare for them, or take them several times, skewing the results. Also, the timing of a placement exam can result in improperly placed students. At Boise State it is not uncommon that newly enrolled freshmen took their ACT or SAT one time only, in their junior year of high school, because their scores at that time were sufficient for admission to the university. Most students in STEM majors (science, engineering, technology and mathematics) go on to take a subsequent math course, resulting in more knowledge than revealed by their ACT or SAT scores. Other students, not destined for STEM majors, may choose to not enroll in mathematics in their senior year of high school, resulting in lack of knowledge retention by the time they enter the university and eventually enroll in a required mathematics course.

When a student is placed in a class that is too easy it is a waste of time and resources, but the situation will often right itself after one semester. If a student is placed in a class that is too difficult there are two serious deleterious effects:

1. The student may perform poorly, and fail the course or earn a grade damaging to the student's grade point average. There is a documented connection between first year GPA and graduation rate, so placement is crucial to student success.
2. The student will need to retake the course, occupying a seat that another student could have had. At Boise State University, Precalculus and first semester Calculus have been identified by the Enrollment Management Committee as bottleneck courses, i.e., courses that can be hard to enroll in and then to pass, but are needed as prerequisites for other courses in a major program. In an era where budgets grow slower than enrollments bottlenecks are bound to occur, but that does not excuse the systemic bottlenecks often encountered in these two courses. Any mitigation of bottlenecks would be worthwhile.

For the most part, placement at Boise State University is via a single event approach: a prerequisite course or a placement exam determines which course the student will be in, and that result is not revisited. Students who are new to the university are generally placed in courses based on their ACT or SAT results. It has become increasingly apparent that that approach is inadequate. Frequently, mathematics instructors will schedule an exam that cannot be graded before the deadline to drop a course, which at Boise State is the end of the 6th week of class. Unfortunately, by that time it is too late either for the student to switch to a lower level class that might be more appropriate, or for that student's vacated seat to be occupied by someone else.

It is worth exploring additional activities designed to refine or confirm placement. Such activities would take place early in the term so that students identified as unlikely to succeed would have a realistic option to move to different courses. An additional benefit would be that seats vacated by students exiting this early in the term would be available to other students waiting to gain access. Boise State University allows students to add or drop classes freely throughout the first week of instruction. This fact, together with the broad informal agreement that students should not miss more than a week of a semester-long math class, led us to focus on placement activities that can give a signal to the student no later than the end of the first week of class.

The two courses used in this student were Precalculus, which in the fall of 2011 had 10 standard sections with 376 students receiving a grade (including W), and first semester Calculus, which in the fall of 2011 had 10 standard sections with 378 students receiving a grade.

ALEKS

ALEKS (Assessment and LEarning in Knowledge Spaces) has been described in detail elsewhere[6,7], but briefly it is a battery of online adaptive tools that permit a student to work problems in a given course of study and get immediate feedback. It was designed to be used as a learning tool, and when used in this mode it includes a periodic assessment component that the student completes as part of their online learning.

ALEKS may also be used in an assessment mode only. This paper reports on results from this mode only. The first use of ALEKS in an assessment mode (separate from the learning mode) to help assure proper student placement into a Precalculus or Calculus course occurred in fall of 2007 at the University of Illinois; the second university to use ALEKS in this mode was Boise State Universit, which deployed it in fall of 2008[6]. Boise State University adopted the same implementation strategy as the University of Illinois, which involved requiring a benchmark score during an unproctored ALEKS assessment. Achievement of the benchmark score by the end the add/drop cycle constituted 10% of the student's grade in the upcoming course. If this benchmark was not achieved, the underlying assumption was that students would self-select to a lower level course, rather than receive a zero for this rather heavily weighted first assignment. The benchmarks used initially by the University of Illinois and by Boise State were 40% and 70% for Precalculus and for Calculus, respectively. Boise State has retained these benchmark levels; however after a couple of years, the University of Illinois shifted theirs to the current levels of 50% and 70%[8]. Since these first implementations of ALEKS in assessment mode, a number of other universities have also implemented ALEKS in some manner as an assessment strategy; for example, Arizona State University[9], University of Arizona[10] and the University of Montana[11].

At Boise State University, ALEKS is used both as a confirmation of placement and as a learning tool, but the modules that were used in the courses described in this paper were mainly the assessment and reassessment modules for **Preparation for Calculus.** Different modules are used in different courses and settings. Further details of this implementation are given in Bullock, et al[6].

ALEKS at Boise State University

Since fall of 2008, the ALEKS Preparation for Calculus assessment (APFC) has been required for both Precalculus and Calculus I. The assessment has been required for summer courses as well as fall and spring. Data for summer classes has been omitted from this paper because there are not very many students in the summer classes, and because there are other inconsistencies between summer and regular terms that complicate comparisons. The chief source of these inconsistencies is the fact that summer term is 8 weeks long while fall and spring are 16 weeks long, which makes distinguishing between the first week versus the first two weeks difficult.

Students who attend orientation functions that precede the regular semesters are told about the assessment requirement and encouraged to take it as soon as possible. All enrolled students receive email reminders of the requirement in the weeks that lead up to the start of each semester. A few students assess well in advance, but most wait until the start of classes or the week before to attempt the assessment.

The APFC assessment is graded pass/fail, but weighted approximately equal to a midterm exam. Since the assessment is given online with no proctors present there is the potential for cheating. No studies have been done at Boise State to examine the extent of the possible cheating, but the spread of scores indicates if extensive cheating is going on, it is limited in its effectiveness. As Table 1 shows, only about half of the students generally attempt the assessment before the semester begins. It is worth noting that while early drops are about the same in Precalculus and Calculus I, the overall success rate for the assessment in Calculus is generally higher, while the overall percentage of students taking and passing the assessment by the first day of class is generally higher in Precalculus. While this might be interpreted to mean the assessment had more impact in Precalculus, there are other possible explanations. There may be a lot more students in the Precalculus course who already have seen a lot of the material and for whom the assessment is relatively routine. It may also be easier to cheat on the Precalculus assessment, where a score of on 40% suffices, contrasted with the 70% required for Calculus I.

There have been some efforts by individual faculty to try to correlate final grades with scores on the APFC. While there may be useful pedagogical information to be found in that statistic, it is not clear that it is the most fruitful approach to discovering if the APFC is useful as a confirmation of placement. In order to investigate that question, data from fall and spring semesters from fall 2003 through fall 2011 were collected and analyzed. Since the APFC assessments began in fall 2008, this meant ten semesters without using APFC and seven semesters using AFPC. Among others, the following statistics were gathered:

- The percentage of students who dropped the course. This included early drops, defined as drops no later than the 10^{th} day, which thus cause the course to be removed the transcript; drops between the 10^{th} day and 6^{th} week, which are recorded as a W on the transcript; and complete withdrawals from the university, which if they are done after the 10^{th} day are recorded as a CW on the transcript.
- The percentage of students who must take the course again; that is all drops as described above, plus all D's and F's.
- The percentage of students who drop before the first day of class.
- The percentage of students who are early drops, as described in the first bullet.
- The ratio of early drops to all students who must take the course again, i.e., the ratio of the number of students who drop before the tenth day to the total drops plus D's plus F's. This last statistic has been dubbed the Early Drop Index or EDI in this paper.

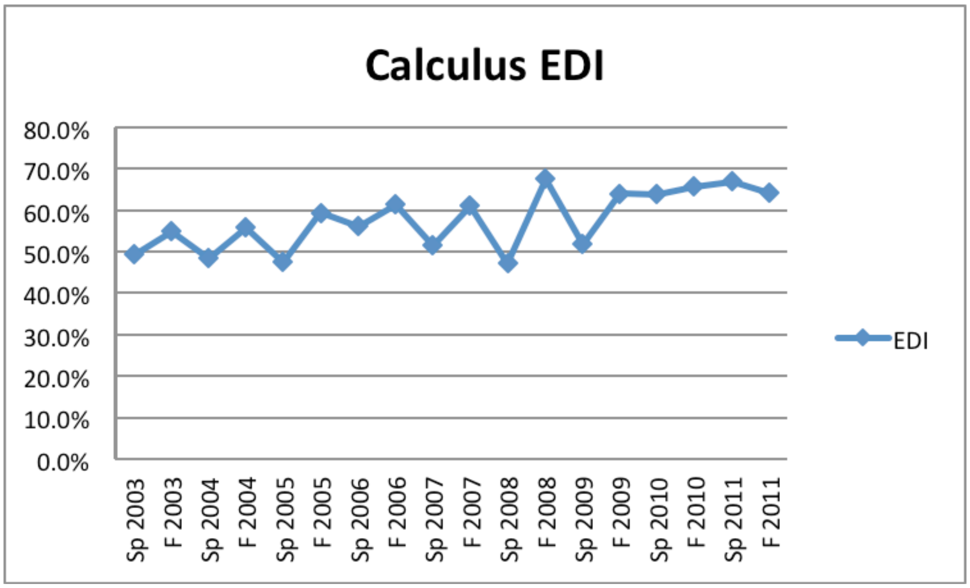
The EDI is proposed as an indicator of the effectiveness of any regime of placement. A high EDI indicates that most people dropping the class are doing so before they have a substantial investment in the course and suggests that the placement regime is good at allowing students to either select an appropriate course or make an early adjustment to a more suitable course. A low EDI indicates that many people are failing to master the material even after a significant investment in the course. Our hope is that adding a confirming assessment to the traditional one shot placement mechanism will achieve a higher EDI.

The EDI is an imperfect measure of placement effectiveness, as there are many reasons why a student might perform poorly in a class, even if placed correctly. The instructor in the class may be ineffective; the student may experience a significant or traumatic event outside of the class and have to readjust priorities; the student may re-evaluate his or her goals mid-semester and lose interest in the course; or any number of other things. However, given a single university environment with a stable cadre of instructors, many of these issues will balance out over time. Since placement is the only aspect of the course that has had a major overhaul during the study period, it is reasonable to expect that any change in the EDI over this period is at least partly attributable to the addition of a confirming assessment after the initial placement.

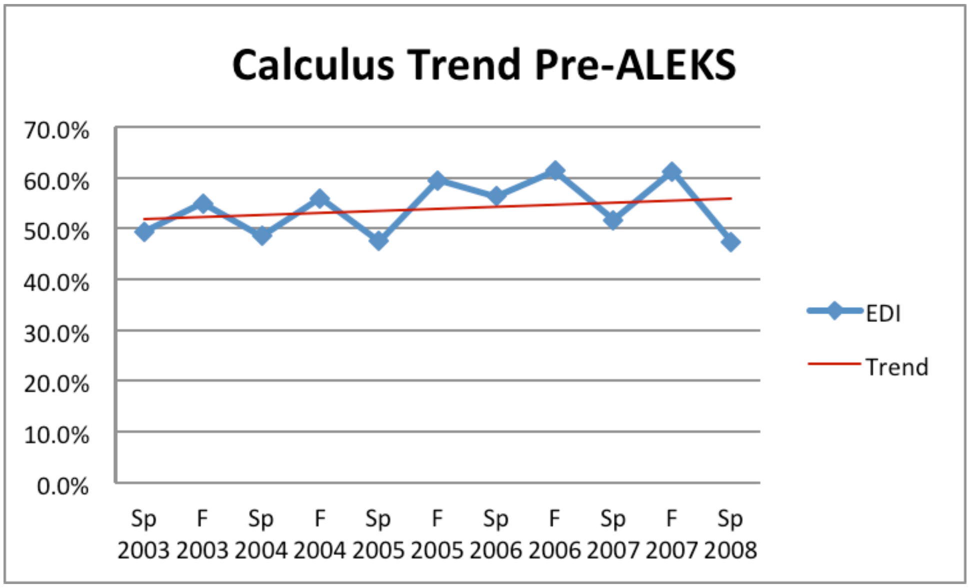
Ultimately, students who withdraw from a course do so of their own volition. Students have access to academic advisers and sometimes discuss the decision to drop with their instructor. We have no data on how much of this discussion goes on and how much it influences a student's decision to withdraw or continue with the course.

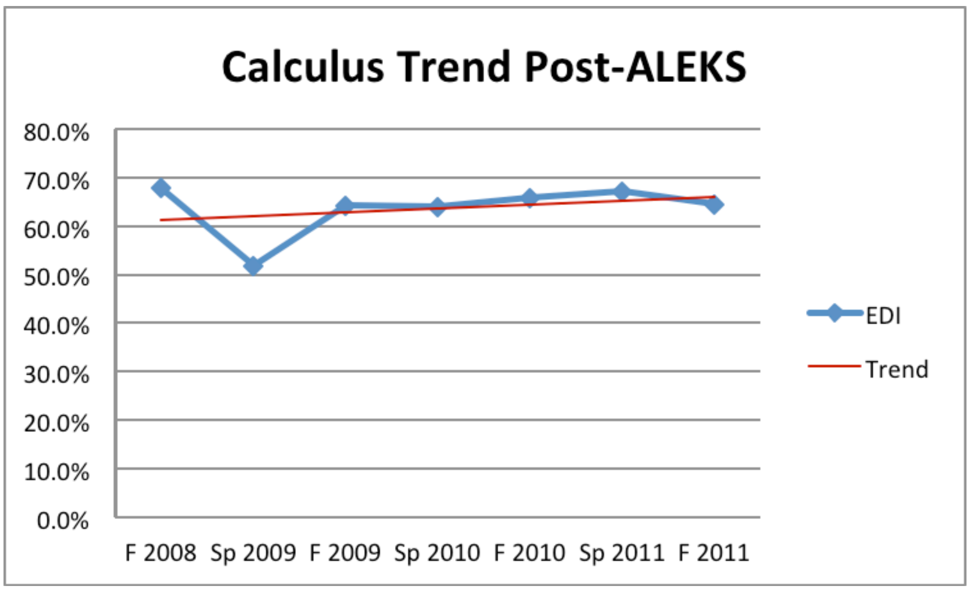
Results

All results described here are extracted from the data attached in Tables 1 and 2 (appendixes). Our central question is what, if any, changes to EDI occurred when ALEKS was implemented in fall 2008. Here is EDI for Calculus graphed against time, from fall 2003 to fall 2011.

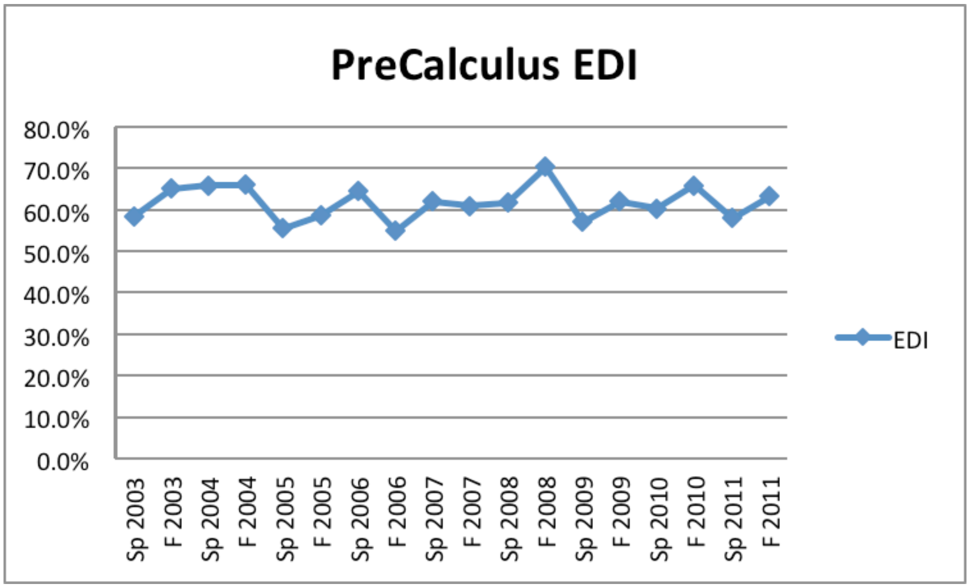


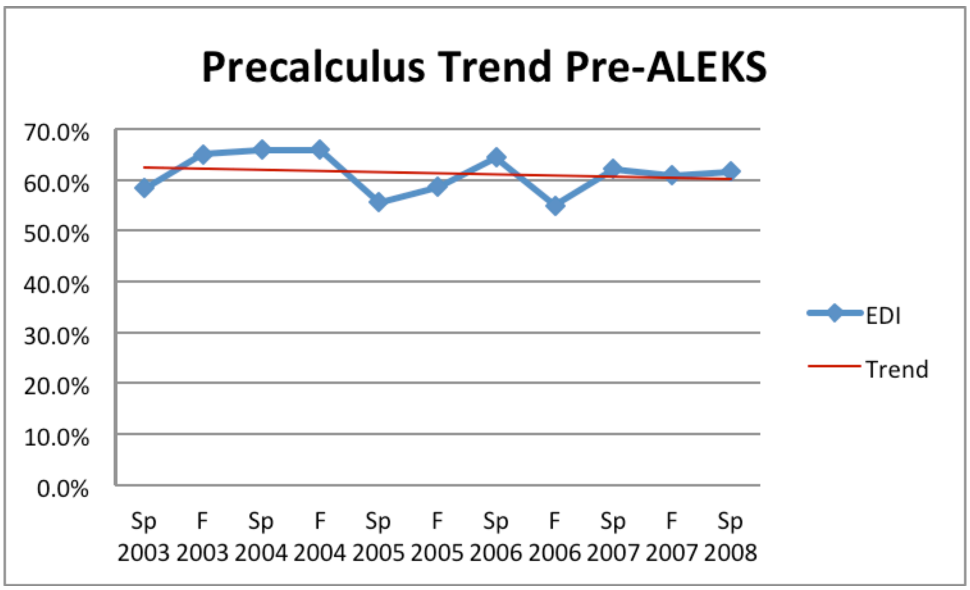
There is a notable jump in EDI in the fall 2008 semester, then a return to relatively low EDI one term later, followed by a much less volatile sequence of semesters with fairly high EDI. The change from pre-ALEKS to post-ALEKS is more evident in the next two graphs: one shows the linear trend in EDI up to spring 2008, and the other shows trend after 2008.

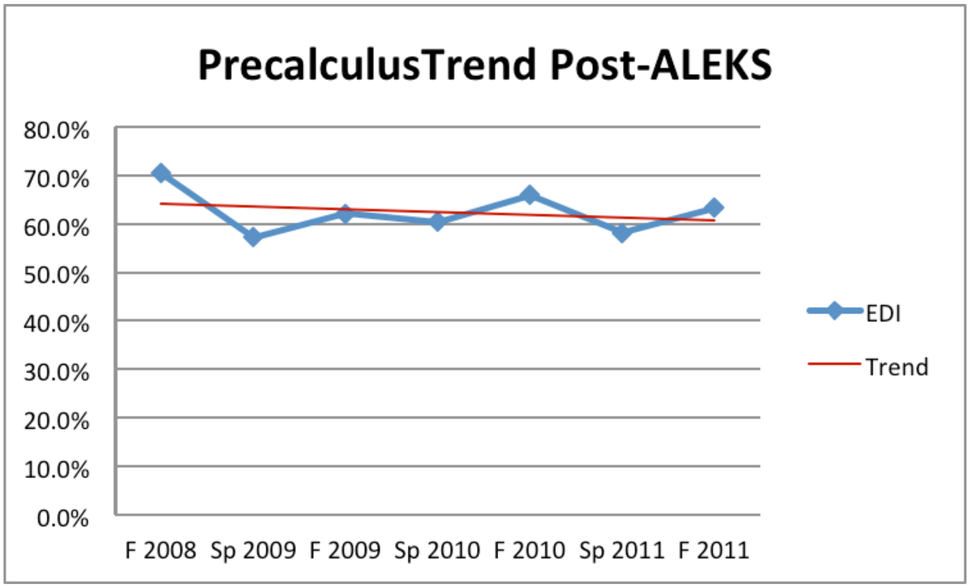




There is a clear difference in behavior before and after ALEKS. Through spring 2008 the EDI trend was nearly flat at about 54%. After ALEKS the trend is again nearly flat, but jumps to about 63.5%. The corresponding graphs for Precalculus show much less impact. There is a similar upward jump in EDI in the implementation semester (fall 2008), but general behavior as shown in the before and after trends does not seem much affected. The pre-ALEKS trend averaged an EDI of 61.3%, while the post-ALEKS trend averaged 62.5%.


PreCalculus EDI
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Sp 2003
F 2003
Sp 2004
F 2004
Sp 2005
F 2005
Sp 2006
F 2006
Sp 2007
F 2007
Sp 2008
F 2008
Sp 2009
F 2009
Sp 2010
F 2010
Sp 2011
F 2011
EDI


Precalculus Trend Pre-ALEKS
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Sp 2003
F 2003
Sp 2004
F 2004
Sp 2005
F 2005
Sp 2006
F 2006
Sp 2007
F 2007
Sp 2008
EDI
Trend



Analysis

The data provided here show almost no difference in EDI at the break point in fall 2008 for Precalculus, but there is a reasonable difference for Calculus. Students and instructors seemed to be on board with the idea of the assessment, since by the deadline at the end of the first week, at least 98% of the students had completed a satisfactory assessment. This indicates relatively modest pushback – few instructors were letting students off without taking the assessment, and relatively few students were trying to get away without taking it. It is reasonable to assume that some of the students who did not take the assessment by the deadline had already decided to drop the class, but had not formally done so yet. Anecdotal evidence suggests that there are always some students enrolled in these classes who have no intention of finishing but put off dropping the class, or never bother to do it at all.

The assessment appears to be providing some value for Calculus and has been retained for spring 2012. After using the assessment from fall 2008 through fall 2011, it was judged that the effectiveness for Precalculus was not worth the costs and inconvenience.

Further Study

Data is presently being gathered for a longitudinal study for these same students to see how they performed in Calculus 2 and possibly some non-math courses that use Calculus. Pass rates and grades are also being examined to see what kind of predictive effect a score on the assessment might have.

Acknowledgments

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Appendix.

The following two tables contain the grade and drop data for the study period for Precalculus and Calculus. Column O contains the EDI or Early Drop Index, described above. "Pre drops" are drops that occur before the first day of class; "10 day drops" are drops that occur during the first 10 days of class. "10 days" is a term of art at Boise State – it usually means 2 weeks. Fall 2008 is the first time that assessments were used, so there is shading change starting in that row. All of these data reflect only genuine drops – formally, when a student switches sections it is recorded as a drop followed by an add. Those drops are not included in the data.

Precalculus															
Term	**A**	**B**	**C**	**D**	**F**	**Pre drop**	**10 day drop**	**W, CW, Late**	**Total**	**Drop %**	**Drop+F+D %**	**Pre drop %**	**Pre+10 Dr %**	**EDI**	**Pass Rate**
Sp 2003	50	80	86	48	99	166	78	26	633	42.7%	65.9%	26.2%	38.5%	58.5%	55.5%
F 2003	55	77	91	53	93	232	94	29	724	49.0%	69.2%	32.0%	45.0%	65.1%	56.0%
Sp 2004	26	69	73	41	41	136	61	20	467	46.5%	64.0%	29.1%	42.2%	65.9%	62.2%
F 2004	35	80	93	37	72	185	76	25	603	47.4%	65.5%	30.7%	43.3%	66.1%	60.8%
Sp 2005	23	36	63	21	85	110	53	24	415	45.1%	70.6%	26.5%	39.3%	55.6%	48.4%
F 2005	36	49	63	35	130	180	87	23	603	48.1%	75.5%	29.9%	44.3%	58.7%	44.0%
Sp 2006	33	47	54	27	61	141	58	22	443	49.9%	69.8%	31.8%	44.9%	64.4%	54.9%
F 2006	42	79	70	23	83	142	62	61	562	47.2%	66.0%	25.3%	36.3%	55.0%	53.4%
Sp 2007	27	62	37	25	53	123	59	33	419	51.3%	69.9%	29.4%	43.4%	62.1%	53.2%
F 2007	69	113	87	32	84	179	65	40	669	42.5%	59.8%	26.8%	36.5%	61.0%	63.3%
Sp 2008	34	52	52	26	72	127	67	22	452	47.8%	69.5%	28.1%	42.9%	61.8%	53.5%
F 2008	72	89	77	36	77	203	98	13	665	47.2%	64.2%	30.5%	45.3%	70.5%	65.4%
Sp 2009	23	47	58	20	105	132	72	28	485	47.8%	73.6%	27.2%	42.1%	57.1%	45.6%
F 2009	35	70	96	44	112	246	68	36	707	49.5%	71.6%	34.8%	44.4%	62.1%	51.1%
Sp 2010	20	66	52	33	77	128	56	11	443	44.0%	68.8%	28.9%	41.5%	60.3%	53.3%
F 2010	51	58	76	42	106	212	92	9	646	48.5%	71.4%	32.8%	47.1%	65.9%	54.1%
Sp 2011	31	53	60	24	87	119	64	21	459	44.4%	68.6%	25.9%	39.9%	58.1%	52.2%
F 2011	49	72	95	35	94	216	63	33	657	47.5%	67.1%	32.9%	42.5%	63.3%	57.1%
Totals	**711**	**1199**	**1283**	**602**	**1531**	**2977**	**1273**	**476**	**10052**					**61.7%**	**54.7%**

Calculus 1															
Term	**A**	**B**	**C**	**D**	**F**	**Pre drop**	**10 day drop**	**W, CW, Late**	**Total**	**Drop %**	**Drop+F+D %**	**Pre drop %**	**Pre+10 Dr %**	**EDI**	**Pass Rate**
Sp 2003	26	39	51	31	45	63	24	13	292	34.2%	60.3%	21.6%	29.8%	49.4%	56.6%
F 2003	44	79	65	35	90	140	41	24	518	39.6%	63.7%	27.0%	34.9%	54.8%	55.8%
Sp 2004	42	36	47	13	88	85	26	17	354	36.2%	64.7%	24.0%	31.4%	48.5%	51.4%
F 2004	37	48	41	30	73	149	31	39	448	48.9%	71.9%	33.3%	40.2%	55.9%	47.0%
Sp 2005	24	37	39	27	71	87	25	26	336	41.1%	70.2%	25.9%	33.3%	47.5%	44.6%
F 2005	35	47	64	24	72	151	41	35	469	48.4%	68.9%	32.2%	40.9%	59.4%	52.7%
Sp 2006	28	31	47	17	54	98	23	23	321	44.9%	67.0%	30.5%	37.7%	56.3%	53.0%
F 2006	37	57	58	18	64	131	36	23	424	44.8%	64.2%	30.9%	39.4%	61.4%	59.1%
Sp 2007	25	62	56	19	59	80	18	14	333	33.6%	57.1%	24.0%	29.4%	51.6%	60.9%
F 2007	75	79	67	32	62	155	44	32	546	42.3%	59.5%	28.4%	36.4%	61.2%	63.7%
Sp 2008	58	46	45	36	71	76	39	21	392	34.7%	62.0%	19.4%	29.3%	47.3%	53.8%
F 2008	69	91	69	25	67	168	61	17	567	43.4%	59.6%	29.6%	40.4%	67.8%	67.8%
Sp 2009	40	56	72	28	64	81	34	15	390	33.3%	56.9%	20.8%	29.5%	51.8%	61.1%
F 2009	77	92	65	33	76	186	51	24	604	43.2%	61.3%	30.8%	39.2%	64.1%	63.8%
Sp 2010	42	72	84	33	72	148	59	12	522	42.0%	62.1%	28.4%	39.7%	63.9%	62.9%
F 2010	80	118	84	31	71	181	62	24	651	41.0%	56.7%	27.8%	37.3%	65.9%	69.1%
Sp 2011	36	69	97	22	51	150	48	24	497	44.7%	59.4%	30.2%	39.8%	67.1%	67.6%
F 2011	51	76	104	47	79	211	56	22	646	44.7%	64.2%	32.7%	41.3%	64.3%	60.9%
Totals	**826**	**1135**	**1155**	**501**	**1229**	**2340**	**719**	**405**	**8310**					**57.7%**	**58.4%**

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Coherent Calculus Course Design: Creating Faculty Buy-in for Student Success

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Coherent Calculus Course Design:
Creating Faculty Buy-in for Student Success

Abstract

This paper recounts the process used and results achieved as first-semester Calculus at Boise State University was transformed over a period of approximately 16 months from a collection of independent, uncoordinated, personalized sections, into a single coherent multi-section course. During the process of this transformation, section size and the instructor pool remained relatively constant; however, profound changes were made across all sections in terms of pedagogy, homework, timing of course content, grade computation and exam content.

The motivation for focusing on Calculus I arose from a five-year National Science Foundation Science Talent Expansion Program grant that was awarded in 2010 to a multi-disciplinary team that spanned engineering, mathematics and science. A major grant objective was to raise first-semester, full-time retention of students in STEM majors. The projects supported several year-long faculty learning communities (FLCs) of about 10 instructors each. With significant involvement from mathematics faculty, the first two FLCs prepared the ground for pedagogical reform of calculus. In 2013-14, a final FLC was created with the express purpose of implementing consistent, student-learning focused strategies across several section of calculus.

The specific approach used to design a coherent calculus course was tied to a decision made by the FLC to use identical homework assignments, with common due dates and times. The FLC structure facilitated buy-in and rapid communication and feedback between instructors, who as they came to agreement on the exact homework exercises, also came to agreement on learning goals and content for each individual lesson. Although there was no explicit attempt to have all instructors adopt the same pedagogy or classroom practices, because FLC discussions frequently turned to pedagogy, all members of the FLC chose to adopt a similar pedagogical approach which included devoting class time to solving problems, working in small groups, facilitated by the lead instructor and a learning assistant. In subsequent semesters, all calculus instructors have opted in to the common, coherent approach to the course (except for those teaching online or honors sections).

Pass and withdrawal rates pre and post implementation reveal an increase in pass rate of 13.4% and a drop in withdrawal rate of 3.9% as a result of the project. Results from anonymous faculty surveys show that faculty in the project changed their teaching practices in Calculus, that they observed positive effects of this in their classrooms, that they took advantage of the FLC to learn from their colleagues and that their experiences with Calculus will have spillover impacts in their other classes. Results from student surveys show, among other things, that students were aware of the pedagogical difference in terms of their classroom experience, with some expressing discomfort in terms of working in groups to solve problems in class and not receiving a traditional lecture experience and others reporting group work as a valuable aspect.

Introduction: Why Design *Coherent Calculus*?

Over the past six years, STEM enrollment at Boise State University grew from 2,421 to 3,778, representing more than half of the university's undergraduate enrollment growth over that time. This growth exposed three major problems that had been lurking in the Calculus sequence.

First, there was a lack of alignment of content, despite the presence of a guiding master syllabus and a common textbook. Second, there was a lack of alignment in terms of assessment. Pass rates varied widely from instructor to instructor, creating a strong sense in the minds of students and faculty in other departments that success in calculus was dependent on luck of enrollment: "Who you took" mattered more than "What you learned." Third, the average pass rates were quite low. As a reference point, the average pass rate across the 2005-2006 academic year for Calculus I was approximately 51%.[1]

Calculus I at Boise State University serves a wide range of students. For most, the course is a gateway requirement for their degree. Much of the content in the typical Calculus I offering was selected and focused toward mathematicians, rather than selected for relevance to the study of engineering or science. Too often, and for too many, Calculus functioned as an artificial barrier to progress, resulting in dissatisfaction among constituent departments and their students. These are not unique to Boise State University. Calculus as a barrier and its apparent lack of relevance are well known and longstanding problems. Potential solutions have been identified at many other institutions. Our efforts at reform were heavily influenced by a successful first-year engineering program at Wright State University[2] and informed by research summarized by Bressoud, et. al.[3]

Boise State University's efforts have been successful because we identified and capitalized on two important sources of momentum: 1) efforts to reframe calculus content and 2) faculty development supporting calculus instructors. In 2010 we were awarded a National Science Foundation Science Talent Expansion Program grant, specifically aimed at increasing STEM graduates by improving first-time, full-time student retention. One of the elements of the project was the support of three, year-long, STEM-specific faculty learning communities (FLCs) (e.g. see Cox, 2001).[4] Based on interest from numerous math faculty, coupled with Calculus I leadership by one of the co-PIs on the grant, the latter two FLCs became exclusively focused on Calculus. These FLCs were connected to work done by one calculus instructor to reframe his calculus content toward that needed by future engineers and scientists.

This manuscript first describes the activities that led to the creation of Coherent Calculus at Boise State University. This is followed by presentation of the impact of the reformed calculus on students, student success, the faculty involved, and campus culture in general.

Activities Leading to Coherent Calculus

The faculty member who emerged as the Calculus I project leader had been refining how he taught his section over approximately 10 semesters, guided by the principle that Calculus should have relevance for students in constituent departments. Between 2004 and 2012 he taught

Introduction to Engineering, had an engineering faculty member sit in on his calculus course,[5] participated heavily in developing ways for students to prepare themselves through on-line approaches and more. A revised Calculus I course emerged over time, with content reshaped to adhere to these principles:

- Whenever possible, students work with data sets and/or continuous models selected from actual physical, biological, financial or other applied models.
- Whenever possible, Calculus concepts are introduced and motivated by application to these models and data sets.
- Whenever possible, content is presented using notation, language and conventions of the disciplines from which the models are taken.
- As much as possible, content will be relevant, recognizable, and applicable in subsequent STEM coursework.
- All content will be accessible from an intuitive or practical viewpoint. In particular, the level of abstraction will be significantly less than typically found in Calculus I.

This approach stands in contrast to traditional calculus which is more abstract, more devoted to a formally rigorous foundation based on limits and continuity, and lightly dusted with applications. Thematically the revised Calculus I class is focused on three outcomes:

- Develop geometric and physical intuition for derivatives and integrals.
- Master the standard rules for symbolic computation of derivatives and some basic integrals.
- Apply both intuitive understanding and rules mastery to solve problems.

As the Calculus I faculty leader was reshaping the content he was also moving to a more active-learning pedagogy. The course design that eventually emerged had the following features:

- Many short homework assignments with immediate computer driven feedback/assessment, typically due on a two-day cycle.
- Each assignment designed along learning cycle principles to target one or two specific learning goals.
- The vast majority of class time devoted to students working in small groups on these homework assignments.
- In-class work facilitated by lead instructor and peer learning assistant.
- Additional and more involved weekly work with written feedback.

The redesigned course was effective, but it was only one section of approximately a dozen taught each semester. Its impact on student success was therefore muted, and, because it was limited to a single faculty member, any benefits were not institutionalized.

In parallel with this focus on calculus content, we had begun engaging STEM faculty to consider course design and evidence based instructional practices. This engagement was done primarily through a faculty learning communities (FLCs) strategy. An FLC is a type of community of practice in which a group of 8-10 faculty "engage in an active, collaborative, yearlong program with a curriculum about enhancing teaching and learning and with frequent seminars and activities that provide learning, development, the scholarship of teaching, and community building."[4, p. 8] As described in the literature, these groups generally draw faculty from multiple disciplines. The underlying logic of using an FLC to promote faculty change is that "undergraduate instruction will be changed by groups of instructors who support and sustain each other's interest, learning, and reflection on their teaching."[6] Indeed, studies have shown that faculty participation in FLCs increases interest in the teaching process, enhances understanding and influence of the scholarship of teaching and learning, increases reflective practice, and promotes exploration of new teaching strategies.[7-13]

The Center for Teaching and Learning at Boise State University has had an FLC program since 2007. With the 2010 STEP grant funding, one STEM-focused FLC was launched in the 2010-2011 academic year. The next two, held in 2012-13 and in 2013-14 were focused on calculus only, as a result of intense interest by math faculty. The first calculus-focused FLC was different than other FLCs that had been supported and differed from FLCs described in the literature in important ways. Because it involved faculty from a single discipline focused on a single course, the line between individual course-based teaching projects and a collective effort to improve a multi-section course was blurred. This is reflected in the goal that participants crafted for themselves:

> The purpose of this FLC is to explore and experiment with strategies at both the individual and institutional level in order to make recommendations about:
>
> 1. practices that will substantively impact student learning and success in calculus
> 2. structures within which these practices can occur

Throughout the experience, the community held in tension the practice of a "traditional" FLC, in which the focus was on mutual support of individual projects and exploration, and that of a group focused on consensus-driven, deliverable changes to calculus. Teaching projects and pedagogical exploration were focused at the individual instructor/section level, with the FLC meetings spent discussing the projects and their ongoing results and assessment. At the same time, meetings enabled a collective exploration of course content (e.g., scope and emphasis of various calculus topics). Likewise, a decision was made in the second semester of the FLC to co-author three questions that could be placed on all final exams, creating an opportunity to "try on" and de-brief the practice of common exams across sections. The FLC also allowed the group to place their exploration in the context of STEM student success at the institutional level. Despite the goals the group set for itself, no clear recommendations for common practices or institutional structures emerged at the end of this FLC. However, the seeds for the next step, the collaborative creation of a coherent calculus course, coordinated across sections, were planted.

Creating Coherent Calculus

In the 2013-14 academic year these two paths converged. The FLC structure was used to bring together a group of Calculus instructors with the concrete goal of delivering a multi-section calculus course with agreed upon common materials. This group was led and facilitated by the instructor who had developed the revised calculus content and related pedagogy. His course materials formed the starting point for adoption of a consensually agreed upon course structure.

Recruitment for the Coherent Calculus FLC took place in October of 2013 via email to 18 instructors who were either scheduled for a section of Calculus I for spring 2014 or known to be otherwise interested in this or similar projects. Several had already participated in one of the previous STEM or Calculus FLCs. Participants were offered a course reduction or an additional stipend for participating and were expected to dedicate ~4-5 hours/week across 20 weeks of the project. The recruitment email stated the objective:

> *"[The project] is about designing a Calculus I course that can be adopted by a reasonably large number of instructors with a high level of coherence. Ideally this means that all sections assign the same homework, all sections use the same quizzes and other assessments, all sections take the same or similar exams, and --- most importantly --- all sections are focused on the same learning outcomes and are using similar methods (pedagogy) to get there. [It is] loosely defined … since no such course design yet exists."*

The recruitment email also detailed the expectations and commitments required of participants.

> *If you participate in [the project] your duties would be:*
> - *Teach at least one section of [Calculus I] in the spring of 2014.*
> - *As much as possible, conduct your section coherently with all the other participants in this project. The extent to which this is possible is highly dependent on the next bullet item.*
> - *Join the course design and development team. The team will meet weekly, starting this semester and continuing until the end of Spring 2014. The team will be charged with designing, testing, and refining all of the homework and assessments that will eventually be the agreed upon course materials. It is expected that the course materials will be tested in your sections of Calculus I in the spring of 2014. The aim is to have something ready for potential broader use in the Fall of 2014.*

The invitation was extended to instructors of all ranks. Six made the full commitment to join the design team: one tenured math faculty member (project lead), three full time lecturers, and two adjunct faculty. Four others (2 tenured and 2 lecturers) participated in peripheral roles, providing occasional input or feedback. The core group of six instructors had been assigned to 8 of the 13 sections of Calculus I in the spring 2014 semester. The design team convened in November of 2013 to begin work.

The task for the FLC was to agree on as many elements as possible for a common course structure that would be rigorously defined and locked in for all sections. The FLC quickly arrived at consensus on some key structural elements. They agreed to adopt a common syllabus that locked in specifications on grading, weighting of homework, quizzes and exams, final letter grade cutoffs, and typical policies. More importantly, they agreed to two critical principles of the course design:

- All sections would use identical homework assignments, both daily computer graded work and weekly instructor graded work.
- All sections would use identical timing of homework. All dues dates were identical and synchronized to class start times for individual sections.

The agreement on homework and timing of due dates meant that the team would also be unified on the content and topics to be covered in every class session, because class sessions would necessarily be built around that day's homework.

They then began their most important task: agreeing on the exact homework exercises, and therefore the learning goals and content, for each individual lesson.

This was the critical element. The entire project is based on the principle that coherence and course coordination are best achieved by agreement on homework, which in turn creates a common understanding of the learning goals targeted by each homework assignment. The FLC took as a starting point the materials already developed by the project lead in previous semesters. These were debated, vetted, and revised as needed until the group was comfortable with the content. This level of detail was applied to the first five weeks of course material. At this point the group had to commit to the use of less carefully reviewed material, because the spring semester was about to start.

As a point of strategy, there was no explicit attempt to have all instructors adopt the same pedagogy or classroom practices. However, FLC discussions frequently turned to pedagogy, partly because the pre-existing course materials had been built with a particular model in mind but also due to professional interests of the FLC members. As implementation rolled out, all members of the FLC chose to adopt something similar to the model used by the lead instructor: class time devoted to solving problems, working in small groups, facilitated by lead instructor working with learning assistant (described earlier).

During the spring 2014 semester, while the FLC members were all teaching their courses, the weekly meetings continued. Approximately half of the time was spent on continuation of the process of examining and vetting remaining course material. All remaining time was consumed by logistical issues and in discussions of how to create, deliver and grade exams.

Most homework was computer graded with no instructor input or feedback. However, each week there was one written assignment graded by hand. All of these assignments were identical for all sections, but each instructor graded these independently. There was much discussion about the value and costs of attempting a standardized rubric. It was judged that the benefits were not worth the time and work required to come to such detailed agreement. Instead, each instructor used these assignments to define the expectations for written work that would be applied on their exams.

Instructors created separate exams. However, each exam cycle included a full review of every exam by every other FLC member. This review, plus the common base of homework and learning goals, led to a reasonable amount of similarity among exams. Instructors then graded their own exams.

One of the project goals was to build a structure that could be scaled up in future semesters. For the fall 2014 semester four of the six design team members were assigned Calculus I sections by the department chair. All chose to continue using the material and continue working as a team to refine the course. In the weeks before the fall 2014 semester all other instructors assigned to a Calculus I course were offered the option of opting into the common course structure. The offer was made with no conditions – so an instructor could simply take all of the developed material and modify it to suit their tastes. What actually happened was that every instructor (except for online and honors sections) chose to adopt all of the prepared material, all of the structure in the common syllabus, and some form of the active learning pedagogy. There were no extra incentives for this, so their decisions were presumably made on the basis of perceived intrinsic value. The result was a team of 8 instructors for fall 2014, four with prior experience from the

FLC**.** The instructor pool was too large to have a regular common meeting, but the new instructors were able to find support and mentoring when needed by consulting with the returning instructors.

The Impact of Coherent Calculus

The impact of Coherent Calculus at Boise State University can be measured in several ways. Below we describe the impact of the project based on student success and from the perspective of students and faculty involved as seen from course instructor surveys completed by students and by surveys on the course completed by faculty who elected to participate in an anonymous survey. We also describe the impact on faculty and comment on the degree of institutionalization of the project. All data discussed below were collected and analyzed after the conclusion of the fall 2014 semester, so the described effects represent the impact of two semesters of Coherent Calculus.

Student Success as Measured by Course Grades

Figures 1 and 2 capture the effect of the project on Calculus I as seen by enrollment and final grades across all sections over time. Figure 1 details the portion of Calculus I enrollment that, over time, has been affected by the adoption of the new Calculus materials and pedagogy.

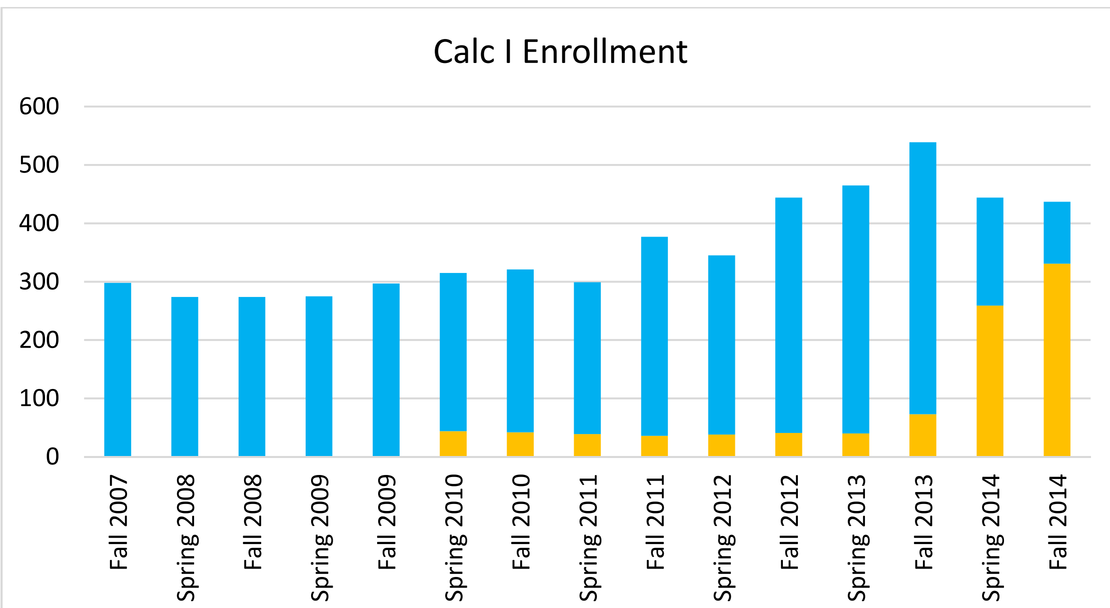

Figure 1: Total student enrollment in Calculus I (blue) and students enrolled in coordinated Calculus I (yellow).

Figure 2, using the same time axis, shows the university-wide pass rate in Calculus I, (number of A, B, C grades divided by total 10th day enrollment.) The results show a clear correlation between the implementation of Coherent Calculus across multiple sections (beginning in Spring 2014) and improved pass rates in the course.

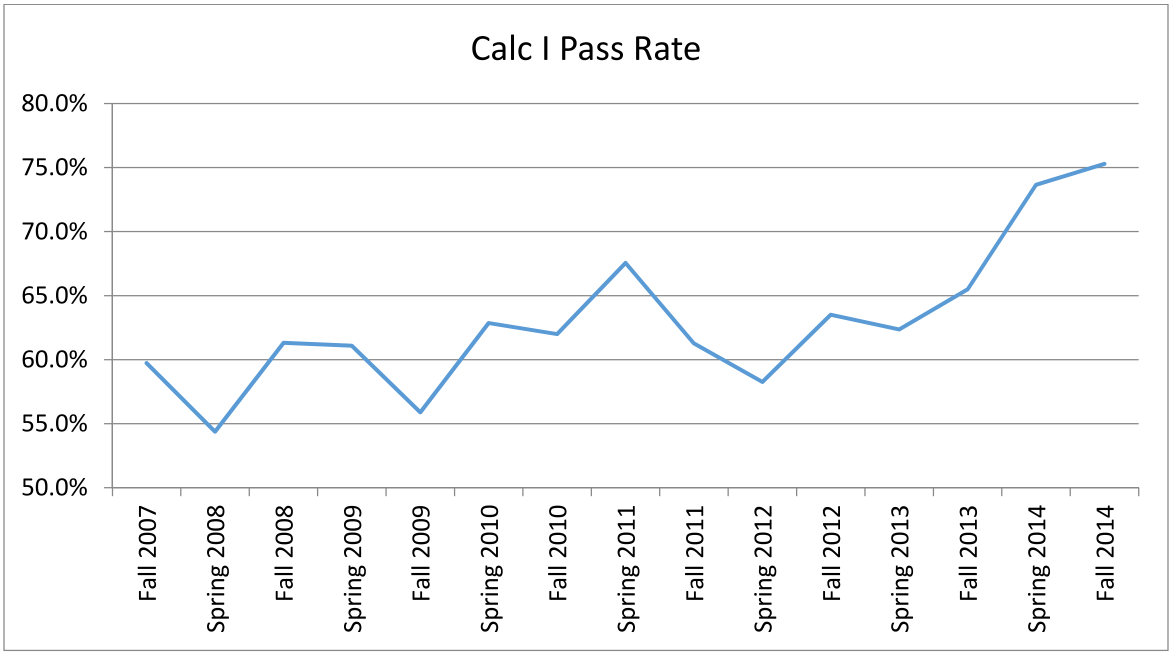

Figure 2: Pass rate in Calculus I as a function of time, spring and fall semester.

In order to further drill into student grades, an examination of course grades across six semesters was conducted. This included four semesters prior to the beginning of the project and two following the formal course coordination. Across the ten faculty who participated, six had taught Calculus I both before and after the transition, see Table 1.

The following observations are noted from this data. First, five out of six instructors showed marked increases in pass rate. The pass rate across all six instructors increased from a weighted average of 60.5% to 73.9%. Of particular note: the percentage of students earning A and B grades increased from 34.4% to 48.6%, an increase of 14.2%. Finally, the withdrawal rate decreased markedly, from 4.7% to 0.8%, a drop of 3.9%. Put in student numbers, for these 590 students following the transition, 23 more students persevered to complete the course with a passing grade than might have otherwise, and 84 more students received grades of A or B.

Table 1: Student grades awarded in Math 170 before and after the transition, by instructor.

Before the transition, four semesters

Instructor	N	A	B	C	DF	W	CW	%AB	%ABC	%W	%W+CW
1	192	44	50	52	40	5	1	49.0%	76.0%	2.6%	3.1%
2	39	5	9	11	11	2	1	35.9%	64.1%	5.1%	7.7%
3	168	8	22	62	57	12	7	17.9%	54.8%	7.1%	11.3%
4	215	27	43	42	93	7	3	32.6%	52.1%	3.3%	4.7%
5	78	17	11	13	32	3	2	35.9%	52.6%	3.8%	6.4%
6	115	18	24	30	31	9	3	36.5%	62.6%	7.8%	10.4%
Total	**807**	**119**	**159**	**210**	**264**	**38**	**17**	**34.4%**	**60.5%**	**4.7%**	**6.8%**

After the transition, two semesters

Instructor	N	A	B	C	DF	W	CW	%AB	%ABC	%W	%W+CW
1	97	23	31	27	13	1	2	55.7%	83.5%	1.0%	3.1%
2	34	9	5	14	6	0	0	41.2%	82.4%	0.0%	0.0%
3	64	6	21	19	17	0	1	42.2%	71.9%	0.0%	1.6%
4	38	9	11	4	13	1	0	52.6%	63.2%	2.6%	2.6%
5	141	25	44	31	40	0	1	48.9%	70.9%	0.0%	0.7%
6	33	7	5	5	12	1	3	36.4%	51.5%	3.0%	12.1%
Total	**590**	**119**	**168**	**149**	**140**	**5**	**9**	**48.6%**	**73.9%**	**0.8%**	**2.4%**

Student Perspectives

After two semesters of implementation, participating faculty were invited as part of this study to share their student end-of-course evaluation results (see Appendix A for details). Of particular interest were comparisons of comments pre and post-implementation of Coherent Calculus. Of the ten instructors who participated in the Calculus project, four had no record of teaching calculus prior to their participation. Of the remaining six, three elected to have their comments included as part of the analysis for this project. For these three instructors, all had taught at least one semester of Calculus prior to the project, and two of the three had taught the Coherent Calculus course for two semesters. Fall 2014 (second semester after the project began in spring of 2013) to Fall 2012 or Fall 2013 comparisons were made.

An analysis was conducted, using four of the end-of-course evaluation questions. Q1: Tell us about this course. What aspects of the course were most valuable? Q2: Tell us about this course. What barriers to learning, if any, did you experience in this course? Q3: Course Items. What suggestions do you have for improving the course? The last question analyzed was: Q4: What other comments, if any, do you have about the instructor, this course, or about this survey?

We were particularly interested in comments that would elucidate student responses to the significant changes in the course with respect to content focus and pedagogy. A thematic analysis revealed students comments in the following categories: WebAssign/homework, Pedagogy, Teacher, Learning Assistants, Exams, and Textbooks.

The overwhelming majority of comments centered around pedagogy. For most students, the opportunity to work in groups in a math class was quite different from what they would have experienced in the past. For example, while pedagogy was not mentioned in pre-Q1 responses, it was mentioned multiple times for two of the three instructors in post-Q1 (What was the most valuable aspect of the course). One student indicated: “Being led to connections of big ideas rather than told what they were…i.e. the instructor asked questions that would lead students to their own conclusions.” Another mentioned: “Building understanding of larger concepts by gradually adding together many smaller concepts.” Many others, for two instructors, described group work as being a valuable aspect (post): “I really learned the value of teamwork. I worked hard to get a good grade in class, but I wouldn’t have done nearly as well as I did if it wasn’t for the people I sat with and worked with everyday.” Not surprisingly, many students also put comments about pedagogy under post Q2, barriers to learning experienced in the course. These all had a similar theme, describing discomfort with not being taught, or getting direct answers to questions. For example, “At times I felt like I wasn’t being taught, I was just told to read the notes and learning goals.” Or, “The instructor didn’t really teach…it seemed like we taught ourselves.” And, “A barrier I noticed was lack of time spent on actually learning the material…it seems that we spent most time just doing homework in class.” One was particularly blunt: “I hate the workshop theory, let teachers lecture and pass on what they know. Quit asking students to “discover” the concepts.” Some students were self-aware in terms of their own comfort level: “Sometimes I was frustrated that we didn’t lecture longer rather than doing homework, but as the semester continued I kind of got used to it.” The quantity of students who commented positively on pedagogy (post) (15) was approximately the same as those who commented negatively on pedagogy (post) (16).

Pedagogy emerged again in terms of suggestions for improving the course (post-Q3) – a few students expressed a desire to have more lecture and less learning on their own: “Instruct the material in class instead of making the students figure it out on their own time.” It was also mentioned by a few students in terms of post-Q4 which asked for any other comments: “This is the second time I’ve taken calculus and it was a lot easier to understand because of the way it was taught,” and, “The whole concept of making the students figure it out on their own for this kind of class seems like it goes against the whole point of why we are taking this class and why I’m paying money for an instructor to teach the class when they don’t actually teach the course material.”

Only minor and typical comments were made about teachers (e.g. teacher availability, clarity, etc.), textbooks (waste of money, not used enough) and exams. One instructor received a few comments about their exams, with one student suggesting they use the standard ones from the department – indicating an awareness of students across sections that such an option was available.

WebAssign/homework was the only other significant emergent theme from student comments. Many students had been exposed to ALEKS, an online learning tool that provides very articulated example problems, and expressed a wish for WebAssign to include examples of how problems are worked, for example, “It does not adequately assist students in learning. There is no immediate feedback when doing assignments other than a right or wrong answer. This makes it very difficult to understand what you have done wrong.” Complaints about WebAssign were numerous (N=13). One student however, remarked (post): “The homework was strangely satisfying and helped a lot with understanding topics.”

These comments suggest that many students were aware of the pedagogical difference in terms of their classroom experience. That so many students remarked on pedagogy indeed indicates a significant shift had taken place. It is not uncommon for there to be resistance from students when there is a significant change in pedagogy from lecture format to a format where students are active in the classroom.[14-16] In the future, faculty in the project could be better supported to help students transition to this more active learning environment.

Interestingly, the analysis of course evaluations did not reveal any comments which indicate that students noticed or appreciated the shift in content focus. This is likely because most students didn’t have any basis for comparison; they didn’t know how it had been different before. It is also the case that the shift in content focus was eclipsed by the shift in pedagogy for most students in the course.

Faculty Perspectives

After two semesters of implementation, the effectiveness of the project from the perspective of the faculty involved was assessed as part of this study using a five-question survey conducted at the end of the fall 2014 semester (see Appendix A for details). The survey was distributed to the nine faculty members who participated in the Coherent Calculus project in fall 2013, spring 2014 and fall 2014. The lead faculty member was left out of this survey. Eight of the nine responded. The survey asked about participant motivation, whether and how participation changed their teaching practices, participant’s opinions about student engagement, and what benefits there may have been to participating in the project. Most of the questions were open-ended so as to capture instructor perceptions without influencing their responses. The relevant survey questions are included in Appendix A.

Instructors were asked, **“What motivated you to participate in the coordinated calculus project? Select all that apply. Select all that apply.”** Results are shown in Figure 3. Most participants were motivated by an interest in exploring new

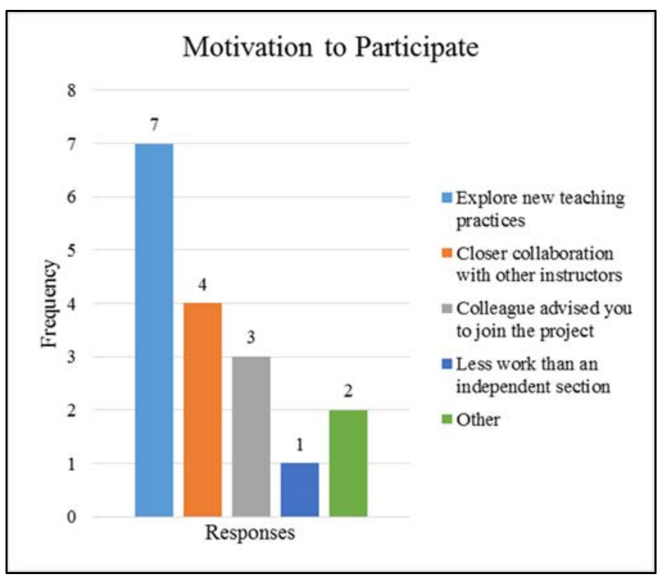

Figure 3: Instructor’s motivation to participate in coordinated calculus project.

teaching practices. Approximately half the participants were motivated by factors that related to connections with colleagues. Two colleagues indicated "other" responses: one deemed it politically wise to join, and one replaced an instructor who had originally been in the project.

When asked, **"Did your participation in this project change your teaching practices in any significant way?"** seven out of eight respondents indicated that it had. For respondents who answered "yes," they were asked, **"Please describe at least one feature of your teaching practice that is different as a result of your participation."** Six responded that they spent less time lecturing, with students spending time in class working on material on their own, e.g.: "Giving time in class for students to do more work and less just watching me do problems." One commented here about increased student engagement: "The collaboration opened up different ways to present the same material. The major effect is that more students are staying engaged in the class. I started with 26 students and 25 took the last exam. I expect all 25-26 to participate in the final exam."

Importantly, the impact on pedagogy appears to extend beyond Calculus I, Figure 4. When asked if participation impacted teaching practice in other courses, five instructors selected "As a result of the project, I have considered changes to my practices in other classes" and two faculty indicated "As a result of the project I have implemented changes in other classes." Only one respondent indicated that "Any changes were restricted to the Calculus course taught as part of the project. I have not changed my practices in other classes."

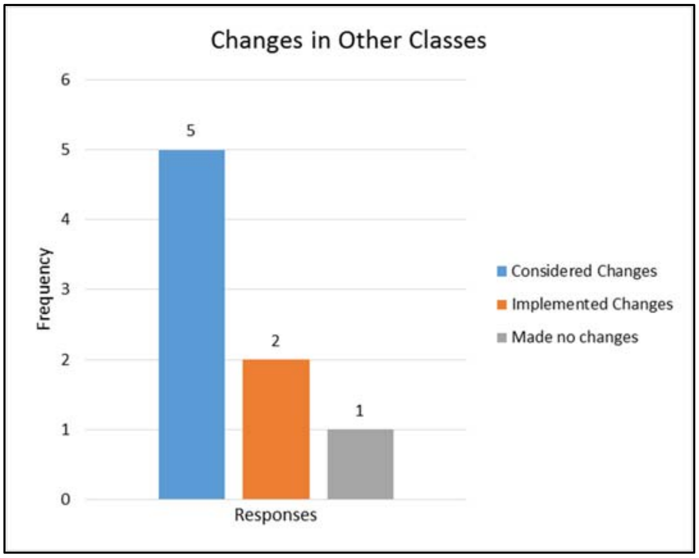

Figure 4: How has participation in the Calculus I project influenced your teaching practices in other courses?

The next question asked, **"In your opinion, was student engagement significantly different in your Calculus course, compared to your overall impression of student engagement in other courses you have taught?"** Five out of eight respondents affirmed a significant difference. The follow on question asked **"Please describe at least one thing that was different."** Two instructors commented on aspects related to attendance: "Since work was being done in class I had better attendance because students saw a direct impact in attending class," and "This was different in the past where students didn't engage in classroom lectures, ask questions and eventually stopped showing up in class." Three respondents discussed aspects associated with working in class. One remarked, "By working in groups the students were able to discuss freely the concepts and algebraic simplifications. When they do it, they remember it. If I do it on the board, they have not gone through the whole thought process and often get lost." Another indicated: "They are actively working, not just trying to pay attention."

Finally, “Some of the groups of students spent more class time reasoning through the material rather than listening and taking notes.”

Taken as a whole these survey responses indicate that faculty in the project changed their teaching practices in Calculus, that they observed positive effects of this in their classrooms, that they took advantage of the FLC to learn from their colleagues, and that their experience with Calculus will have spillover impacts in their other classes. These conclusions are reinforced by one last survey question.

The last survey question inquired, “Were there any other benefits to participating in the calculus project?” All eight respondents answered in the affirmative and described these benefits, which are summarized in Table 2.

Table 2: Other Benefits from Calculus Project

It showed not only a willingness to work together in a teaching community but an actual desire to share one’s own thoughts about teaching calculus as well as listening to others. As a result all the participants were active in sharing their experience and improving the quality of their course.
It was helpful to have someone else's perspective on the tests that I had written. They were open to discussion and revision (for the better). Also helped me with my weakness of typos (form and from, sign and sing, etc).
Being able to bounce ideas off of other instructors and gain from each other's strengths.
Opportunity to see others' [instructors] assignments and how they approached course topics.
Better instructor interaction and student understanding.
Group protection from student complaints.
Better instructor interaction and student understanding.
I think that the students who really bought into the program were able to reason through test questions better. They were better able to detect when they were making mistakes and had some ideas on how to fix them.
Getting to know students on a more personal basis rather than just as a name...believe it helped with classroom interaction and participation.

Faculty Development and Institutionalization

The project had the specific goal of creating and delivering a multi-section course using common course materials. This was achieved. However, there were deeper institutional and faculty development goals, including the following

- Shift the teaching culture to be accepting of the trade-off of autonomy versus common course material and common grading standards.
- Shift the culture of practice from individual responsibility for single sections to team-based responsibility for a multi-section course.
- Create and sustain conditions for regular exchange of ideas and practices between instructors.
- Move faculty towards more active learning or student-centered pedagogy.

That we have moved toward the achievement of these goals is evidenced by the fact that in the two semesters following the project launch (fall 2014 and spring 2015) all Calculus I instructors have been invited to voluntarily join the team and to have their sections use common materials. So far 100% of instructors outside of online or honors calculus have opted in. Every new instructor has opted into every feature of the coherent course. Also, in the last two terms the common structure has moved to include some common exams in fall 2014, and entirely common exams in spring 2015.

Discussion and Summary

In summary, the project succeeded in creating a sustained culture of collaboration for the purpose of delivering Calculus I. It had significant impact on the pedagogy used in Calculus I and it had the spillover effect of shifting faculty teaching practices outside of Calculus I. The changes had a large effect on student success, including a profound impact on student success as measured by grades, as seen by an increase in pass rates, by an increase in the percentage of students receiving course grades of A and B, and by a decrease in withdrawal rates. The pedagogical emphasis on group work/collaboration, noted in student comments as being a "valuable aspect of the course," may have profound and long-term impact on student persistence and success in major, due to the increased student engagement with one another and a possible future outside-the-classroom effect on student behavior.[17, 18]

It demonstrated a viable path to course transformation that does not rely on a top down command structure. We could have designated someone to coordinate and manage the course. That person might have enforced a common syllabus and grading policy, chosen an appropriate content focus and trained faculty in a particular pedagogy with requirements for their implementation. This paper describes an alternative strategy of cultivation, support and collaboration. It begins with a decision to agree on the homework. Next comes agreement on and definition of the skills and objectives that the homework is designed to deliver. This in turn gives definition to exams, grading practices, standards and course level objectives. Consensus on the higher level aspects flowed readily from the initial decision to agree on the homework. The end result was a significantly changed course that was consensually agreed upon and rapidly adopted by essentially all instructors without any coercion.

Future Work

While we view the positive outcomes as strong indicators of success, we also recognize that we must continue to attend to the sustainability of the project. We have identified a second faculty member who will share managerial workload with the original project leader. Together they can support instructors who come into the project. Additionally, we will continue to provide faculty development both through the department and through the Center for Teaching and Learning to be sure faculty have what they need to teach the Coherent Calculus. This will allow us to help faculty address areas of concern such as the fraction of students who perceive it would be better to be lectured to during class.

The Calculus I project scaled up more quickly and achieved wider buy-in than was initially thought likely. While this is a clearly positive development, it means that Calculus II is now an urgent priority. The same project lead, working closely with one member of the FLC team, is currently launching a Calculus II project with the same general plan but an accelerated time line:

- Spring 2015: reframe and revise content.
- Fall 2015: form an FLC to review and revise content, creating buy-in along the way.
- Spring 2016: Launch a multi-section course.

Although the Calculus I project originated in a move away from the traditional course content, there is one clear advantage to the traditional content – when students move on the Calculus II their instructors will expect a traditional Calculus I background. While we are happy with the new Calculus I content as a self-contained course, there is a real possibility that it aligns poorly with the traditional Calculus II content that students encounter one semester later. We have recently begun building tools to measure the effects of Calculus I reform on Calculus II students, as well as other downstream courses. Future work will be informed by this data. This is likely to impact both the Calculus II content currently being built and to indicate areas where the Calculus I content can be strengthened. Finally, the coordinator and instructors of Calculus III and Differential Equations are becoming interested in the Calculus I and Calculus II projects, so there will be additional collaboration up and down the entire course sequence.

Awareness of the project has already spurred interest from those who teach outside the Calculus sequence. There are nascent efforts to apply the coordination model to Business Calculus and General Statistics. Two instructors of Linear Algebra have already run a course using common homework. And the group that oversees our multi-section Scientific Computing course is considering a similar approach. If successful, these efforts would achieve full coordination of the entire suite of service courses across every STEM or related discipline.

Acknowledgments

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Appendix A:

A.1 [[http://hausdorff.boisestate.edu/14spring170/gen_syllabus.htm]]

Survey

Email 1: Script for email for recruitment of instructors:

You are being asked to participate in the following research study, "The Effectiveness of the Implementation of a Coordinated Calculus Course and its Impact," because you were part of the coordinated calculus project. Your feedback will be used at an aggregate level to evaluate the effectiveness of the implementation and its impact.

Participation is voluntary and anonymous. X and Y are the co and principal investigators on this project. Results will be aggregated and shared with the calculus course coordinator and the Director, Center for Teaching and Learning who have worked on the project and some of the results may be reported in a publication or to the National Science Foundation.

Because this study involves human subjects, informed consent is required. The first page of the survey contains an online consent form. Participating in the study should take approximately 30 minutes. The study has five questions, some of which are open response questions.

Go to Survey [hyperlink]

A.2 Faculty Participant Survey:

This survey has 5 questions, some of which are open response questions. It is expected to take approximately thirty minutes to complete.

1. What motivated you to participate in the coordinated calculus project? Select all that apply:

Less work compared to an independent section

Opportunity for closer collaboration with other instructors

Opportunity to explore new teaching practices

A colleague advised you to join the project.

Other: (written response)

2. Did your participation in this project change your teaching practices in any significant way? Yes/no

2a. (If yes from 2) Please describe at least one feature of your teaching practice that is different as a result of your participation.

2b. (If yes from 2) Please select the statement that you most agree with:

A. Any changes were restricted to the Calculus I course(s) I taught as part of the project. I have not changed my practices in other classes.

B. As a result of the project, I have considered changes to my practices in other classes.

C. As a result of the project, I have implemented changes in other classes.

3. In your opinion, was student engagement significantly different in your Calculus I course, compared to your overall impression of student engagement in other courses you have taught.   Yes/no

3a. (If yes to 3) Please describe at least one thing that was different.

5. Were there any other benefits to participating in the calculus project? Yes/no

5a. (If yes to 5) Please describe.

Thank you for participating in this survey.

A. 3: Email 2: Request to Instructors Regarding Course Evaluation Data:

To understand how student perceptions of the course may have changed as a result of the coordinated calculus project, we would like to access your course evaluation data for Math 170. This data would be accessed without needing your assistance, (other than your permission) and rendered anonymous so that no identifying comments or other features can permit identification of you as the instructor (e.g. if a student would write, “Dr. Jones speaks too softly,” we would replace that with “Instructor 1 speaks too softly.”) Please indicate what level of access to your course evaluation data you would permit by reply email (reply A, B, C or D).

A. No access to the course evaluation data collected for my course.

B. Access to numerical data for these four specific questions only:

1. The WebAssign homework was valuable to my learning.
2. The weekly written homework was valuable to my learning.
3. The in-class warm-up exercises were valuable to my learning.
4. Class time used for students working on problems was valuable to my learning.

C. Access to all numerical data, but not to student comments, from any semester going back three years.

D. Access to the all course evaluation data going back three years.

Instructional Faculty Development and Student Success

Janet Callahan, Doug Bullock and Susan Shadle

Boise State University, Boise, Idaho

Introduction

Boise State University is a metropolitan public university in the state capital and the largest institution of higher education in Idaho. The College of Engineering was established in 1997 in response to regional demand for engineering education from Micron, Hewlett Packard, and other industry leaders. Despite the formation of the college, which in less than 20 years now has an enrollment of nearly 3000 students, there remains a steady and ongoing need for technically trained STEM graduates. For example, as of fall, 2015, the regional demand for computer scientists is estimated at 10:1 or more as a result of the rich computer science industry located in Boise, with hundreds of small startups and dozens of mid-sized software companies. Thus, the project's overarching goal was to increase the number of high quality, technically trained engineers and computer scientists who graduated with B.S. degrees.

As a result of this demand for technically trained graduates, STEM enrollment at Boise State University has grown steadily for more than a decade. Between 2002 and 2008, STEM enrollment grew by ~1400 students; this growth was again seen between 2008 and 2014. This relentless growth exposed several issues that the Idaho Science Talent Expansion Program (Idaho STEP) grant's effort helped address. One of these was a very low first-year retention rate. At the time the grant was submitted, we had seen first-time freshmen retention for the 2008-2009 academic year of only 57% for STEM majors.[1] We had a pretty good idea why we had low retention – we had low pass rates in mathematics classes, for example. An example of this, was an average pass rate in 2005-6 in Calculus I of only 51%.[2] Thus, during the grant funding period, we had issues to resolve: we had very low retention in STEM majors, we had low pass rates in Calculus and other mathematics, engineering and science courses, and on top of it all, during the five-year grant funding period, the number of STEM majors kept steadily growing, fueled by the roaring regional demand for STEM professionals.

Approaches Used and Results:

Our project used three main approaches; we had a diverse leadership team, we focused on instructor development using year-long faculty learning communities, and we supported several, targeted curricular, extra-curricular and co-curricular activities for students. These are described below, with an emphasis on the first two strategies which distinguish this project from many others.

<u>A. Diverse Leadership Team:</u> A highly diverse project team was selected by the Principal Investigator (PI Callahan) to participate as co-principal investigators (co-PIs) in the project. Our leadership team consisted of five members who brought diversity both

in terms of the position they held at the university as well as in their field of specialization. This diversity brought strength. It was enormously impactful, for example, to have the Chair of Mathematics and the Director of the Center for Teaching and Learning as co-PIs; this combination resulted in very significant engagement of mathematics faculty in professional development activities. One of the co-PIs was a lecturer in physics; with increasing numbers of entry level math and science courses being taught by full-time instructors, he brought a valuable perspective to the project. All of the leadership team routinely taught first year courses and had a strong affinity for student success. In forming the team who wrote the proposal, the PI selected three of the investigators based upon whom she had observed voluntarily showing up on a routine basis for summer orientation events. The team transcended college boundaries – while the PI was associate dean of the college of engineering, only one other co-PI was from engineering, with two other members of the team coming from the college of arts & sciences (physics and mathematics). Finally, our Director of the Center for Teaching and Learning, Shadle, as a chemist and an expert in STEM pedagogy was a natural fit for the project. The program activity she directed, providing three year-long STEM focused faculty and instructor learning communities (FLCs) proved to be highly impactful. Additional details about the FLCs, including details on implementation and how they evolved over time, are presented in the following section.

<u>B. STEM Faculty and Instructor Leaning Communities (FLCs)</u>: In reflecting on the Idaho STEP project at its conclusion, the leadership team believes the most impactful activity, by far, was the prolonged exposure to evidence-based instructional practices (EBIPs) that STEM instructional faculty experienced during this project; this is therefore the primary focus of this paper. We accomplished this professional development through the use of year-long instructional faculty learning communities (FLCs) that provided training on innovative teaching strategies. Faculty learning communities are a unique kind of community practice[3] that have been shown to impact teaching practice.[4] More than just a seminar series or faculty task force, FLCs have the potential to transform institutions into learning organizations. Three such FLCs were held, impacting thirty STEM faculty across five years.

We approached this activity by holding our first STEM focused FLC, FLC-I in the first program year, fall 2010. Eight faculty participated, from chemistry, physics, mathematics, materials science and mechanical engineering; they had either tenure line or lecturer appointments. FLC-I began with a two-day retreat before the fall semester commenced, and continued across the fall and spring semesters with a meeting every other week. Each meeting lasted for 1 hour and 45 minutes and used discussion, presentation, reflection, sharing and readings to engage in deep exploration of various teaching and learning topics. To incentivize participation, FLC-I participants received $1500 to use for their research, and a one-course buyout that was funded by the grant.

During the FLC-I meetings, facilitated by Shadle, participants rotated the responsibility for leading meetings. Each meeting focused on a specific topic relevant to STEM teaching and learning, including best-practice pedagogies, frameworks for student development, strategies for assessment, dealing with student misconceptions, a discussion of institutional student success data, and how STEM disciplines frame the context for teaching and learning.

The next year we spent assessing FLC-I by conducting an analysis of participants' teaching logs or journals. A content analysis exposed trends and themes; the result of this analysis is presented elsewhere.[5,6] One conclusion drawn from this analysis related to the critical importance of reflection: being a member of FLC-I played a critical role in supporting reflection, and likely primed participants for future changes in their teaching.[7,8]

FLC-II and FLC-III were conducted in the third and fourth year of the grant. As a result of the STEM enrollment growth, the departments of physics and chemistry were unable to contribute faculty because they would not be able to meet student demand for their course offerings. Consequently, we focused FLC-II exclusively on the subject of Calculus I. Thus, FLC-II was a single-discipline group, comprised of ten members, focused on a single set of courses. Outgoing department chair, co-PI Bullock had been laying the groundwork for focused revision of this particular course for some time. While Shadle continued to facilitate FLC-II, Bullock's leadership and effort was critical to the recruitment of faculty and also to its success.

We funded this FLC slightly differently; four "core" faculty members received either summer salary or a course reduction buyout to focus on a project aimed at improving success in calculus. Six affiliate faculty attended meetings and contributed to the development and completion of the core faculty projects. The goal of FLC-II was to explore and experiment with strategies at both the individual and institutional level in order to make recommendations about practices that would substantively impact student learning and success in calculus, and structures within which these practices could occur.

A critical development occurred; faculty in FLC-II who were teaching Calculus I piloted the use of three common final exam questions. The questions were co-written by the FLC-II faculty. This yielded several insights: 1) the grading on the questions was very similar between instructors, despite no agreed upon rubric; 2) students in all sections struggled with similar difficulties; 3) there were some student difficulties that seemed to stem from wording of the questions. The experiment was very useful in helping people to see potential value and challenges in having common exams, and laid the groundwork for improved assessment practices.

The exploration conducted in FLC-II laid the ground work for Bullock to propose a common calculus offering. This was partially supported through an internal, provost-funded grant in spring 2013. This offering involved some of the same calculus instructors as in the FLC-II, as well as some additional faculty. FLC-III continued to focus on Calculus I. Instructors worked together in the fall 2013 term to develop a common course design. In the spring 2014 term all instructors taught the course from the common materials. While adoption of common pedagogy was not required, an active, problem-solving focused approach emerged through discussion and practice and all faculty teaching Calculus I now use this approach. FLC-III participants included: 3 tenured faculty; 5 full time lecturers; and 2 part time adjuncts. The common calculus offering has now been delivered every semester since spring of 2014.

As of 2016 we now have a coordinated calculus offering that employs common homework questions, has common exams, uses similar pedagogy and has a common final exam. We presented the method by which we facilitated the reform of this course at ASEE in 2015;[9] this approach was sufficiently compelling that the paper received the mathematics division “Best Paper Award.” Table 1, below, summarizes the key strategies taken to accomplish the development of our coherent calculus course. We learned that reform starts from within – it is not a result of a course coordinator being appointed, or a common syllabus of final. A coordinate course is based on common homework problem. Having common homework problems leads to natural commonalities on quizzes. A dialogue on how to weight course elements works better than imposing a common syllabus. Over time, we found course instructors naturally wanting common exams and finals; these were not imposed. We measured pass rates, percentages of students earning certain grades; some of these results are presented below, with full details presented elsewhere.[9]

Table 1: How to create a coordinated course with a common pedagogical approach

Adoption Strategy – NO – Do not do this!	**Adoption Strategy – YES – Do this:**
Appoint a course coordinator.	Start with common homework. Build consensus agreement on every exercise.
Combine 12 small sections into 2 huge sections.	Do the same thing with quizzes.
Impose a common syllabus.	Agree on basic weighting of all Hw, Qz, Ex, Final and letter-grade cutoffs.
Impose a common final.	Build consensus on exam content – eventually reach common exams.
Impose common midterm exams and final	Eventually adopt a common final
Impose a pedagogical model.	Along the way, allow the course content to shape pedagogy with the strategic goal, for example, of active learning.

Some of our results from the coherent calculus course:[9]

- The pass rate across the six instructors who taught in fall, 2014 increased from a weighted average of 60.5% to 73.9%
- The percentage of students earning A and B grades increased markedly, from 34.4% to 48.6%, an increase of 14.2%.
- Student course survey results commented heavily on the pedagogical approach. For many students, the opportunity to work in groups in a math class was quite different from what they had experienced in the past.

- Instructors reported spending less time lecturing, as they surrendered time in class for students to work instead of watching them solve problems.
- All but one instructor in this curricular reform project reported this impacting their pedagogical approach in other courses.
- Instructors reported improved attendance and many other benefits.

C. Curricular, Co-curricular and Extra-curricular Activities: In addition to the FLCs for instructional faculty, a number of different activities were conducted across the grant period. The first of these, accomplished in the first summer of the grant period, was to deliver a coordinated STEM summer orientation session.[1] The very act of proposing and coordinating "STEM" orientation was our first step taken on campus to establish a "STEM Identity."[10] Other projects focused on undergraduate research experiences, on-line mathematics learning/review and the Introduction to Engineering course.[1,11-13]

In addition, we developed an outdoor STEM Summer Adventure experience for entering freshmen; we took them rafting on the Payette river in Idaho for a multiple day, extended field program.[14] This is an institutionalized activity which grew from a handful of participants in 2010 to over 30 in 2015.

Idaho STEP Program – Original Outcome Statements:

By the end of the grant funding period, the Idaho STEP program will:

(1) Have increased the first year retention level of first-time freshmen from 57% by 10 to 15% for STEM majors, with a target level of 70%. This represents an annual gain of approximately 35 retained first year STEM students.

(2) Have increased STEM undergraduate degrees by 22% (reference data: there were 163 STEM graduates in 2007-8).

(3) Have increased the first year retention rate of at-risk freshmen engineering students by 10%.

(4) Continue to post gains in women engineering enrollment from 9.8% in 2005 to 12.8% in 2008, attaining 15% or more by program's end.

(5) Have institutionalized freshman STEM Learning Communities and Orientation programs.

Idaho STEP Program – Actual Outcome Results:

(1) Significant progress: We increased first-year retention of first-time freshmen by 7%.

(2) Accomplished: We increased STEM undergraduate degrees from 234 graduates in 2009-10 to 454 in 2014-15, an increase of 93%.

(3) [note, because of institutional data restrictions, we had to reframe the goal] We increased the percentage of underrepresented minority students in engineering and computer science from 10.6% in fall 2009, to 13.5% in fall 2014.

(4) Accomplished: We increased the percentage of women in engineering and computer science from 12.8% in fall 2008, attaining 15.6% by program's end.

(5) Accomplished: We institutionalized the freshman STEM Learning Communities and Orientation programs.

Broader Impacts

We report that the strategy of focusing on student retention in the first year, led by a diverse team with a two-pronged approach that included faculty development as well as student programming, has yielded positive results. This project aimed to increase overall STEM graduation numbers while also increasing the percentage female and underrepresented minority (URM) students enrolled in engineering and computer science. Across the grant period, STEM enrollment increased from 2,238 in fall of 2008 to 3,778 in fall of 2014 while first-time, full-time retention of STEM students increased by 7% (attaining 64%) and graduation numbers grew from 163 in 2007-8 to 402 in 2013-2014, an increase of more than 200%. Along with these very strong outcomes, we additionally increased female enrollment which attained 14.9% (up from 12.8% in fall, 2008) and increased Hispanic enrollment from 8.1% to 13% across the same time frame. This compares favorably with the %URM in the state (14.3%, of which 11.6% are Hispanic). We attribute this increase to improved retention, resulting from improved student learning experiences.

One of the broader impacts of this grant involved drawing attention to the needs of undergraduate STEM students. We essentially created a "STEM Identity" on campus.[10] One of the long-lasting inter-institutional outcomes realized was an awareness of the fact that STEM students had lower retention levels in STEM than the general student population, despite being among the best students admitted. Our STEM students were capable; they needed improved first-year experiences to remain STEM majors.

Finally, this award helped show the need for coordination of STEM initiatives and for the need for reliable data. In January of 2015, the Institute for STEM and Diversity Initiatives was formed; the Institute's aim is to build a diverse community of students, faculty, and others involved and invested in STEM. The primary goal of the Institute is to promote a culture of active inclusive excellence in STEM at Boise State University. The Institute fosters diversity in STEM through (a) advocating for and nurturing underrepresented student and faculty success inside and beyond the classroom, (b) strengthening avenues of communication and collaboration among University and external partners, and (c) conducting and catalyzing STEM educational research.

Intellectual Merit

Across the five years of funding and a no-cost extension, between January 2010 and December 2015, ten papers were published. These included two journal publications[5,15] and eight proceedings of the American Society for Engineering Education.[1,6,9-13,16] In addition, five posters, two workshops and one panel discussion were presented at the National Science Foundation annual STEP meetings.[17] Finally, two webinars were presented through STEP Central (now STEM Central).[18] In addition, numerous internal communications, press releases, seminars, learning communities and more were conducted in order to broadly disseminate our program results.

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Longitudinal Success of Calculus I Reform

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Longitudinal Success of Calculus I Reform

Abstract

This paper describes the second year of an ongoing project to transform calculus instruction at Boise State University. Over the past several years, Calculus I has undergone a complete overhaul that has involved a movement from a collection of independent, uncoordinated, personalized, lecture-based sections, into a single coherent multi-section course with an active-learning pedagogical approach. The overhaul also significantly impacted the course content and learning objectives. The project is now in its fifth semester and has reached a steady state where the reformed practices are normative within the subset of instructors who might be called upon to teach Calculus I. Gains from the project include a rise in the pass rate in Calculus I, greater student engagement, greater instructor satisfaction, a general shift toward active learning pedagogies, and the emergence of a strong collaborative teaching community.

Project leaders are seeking to expand these gains to other areas of the curriculum and to broaden the community of instructors who are fully accepting of the reforms. Common concerns expressed by faculty resistant to the overhaul include suspicion that pass rate gains might reflect grade inflation or weakened standards, and that altering the traditional content of Calculus I might leave students unprepared for Calculus II. External stakeholders also have a vested interest in ensuring students receive a solid preparation in Calculus I. In this paper we develop a response to ensure solid evidence of Calculus II readiness that we hope will be useful to change agents and campus leaders in many other settings.

We address concerns about Calculus II readiness by conducting a natural experiment, tracking two cohorts of students through Calculus I and into Calculus II. The "treatment" cohort consists of students who reach Calculus II after passing the reformed Calculus I. The "control" cohort consists of students who reach Calculus II after passing non-reformed Calculus I at Boise State University. The experiment has no designed randomizing, but enrollment data shows that both cohorts spread out across all sections of Calculus II with apparent randomness. Our research question is: "Does the treatment cohort perform any worse than the control cohort in Calculus II?" Data on pass rates and grades in Calculus II will show that the answer is "No."

Introduction: History of the Calculus I reform project.

Boise State University has been experiencing growth in STEM enrollment every year since the formation of its college of engineering in 1997. In fall, 2015, STEM enrollment included nearly 4,000 students. Accompanying this growth came a demand for increased capacity in Calculus I, which has grown in enrollment by 74% over the past decade (from 244 in fall, 2006 to an enrollment of 433 in fall, 2015).

With the increased demand for calculus instruction came several undesirable consequences. These included a lack of coherence between instructors in terms of content. Related to this was a lack of agreement in terms of what exactly students were expected to be able to do by the end of the course. In fact, that topic – the learning outcomes of the course – had not been addressed; each instructor instead carried their own learning outcomes. In nearly all instances, these outcomes were not actually articulated into a statement such as, "By the end of this course (chapter, section, unit), students will be able to…," but rather were internalized; each instructor had their own sense of what should be taught in calculus, which guided their teaching, assignments and examinations.

There was agreement about what c text should be used, and a common syllabus was on file. Yet, as a result of both growth and lack of coordination between instructional faculty, a situation had developed by 2005-6 which students, the mathematics department, and others recognized as being problematic. At that time, from a student's perspective, it appeared to matter more, "who you took," than "what you learned" in terms of their chances of passing the course.[1] This was supported by pass rate data; the average pass rate in 2005-6 was 51% and ranged from 30% to 90% depending on who taught the course.[2] The variation in pass rate was a confounding problem in post-requisite courses such as Calculus II; students had highly variable preparation, and the Calculus II course also had no framework of common learning outcomes.

In part as a result of an externally funded Science Talent Expansion Program (STEP) grant from the National Science Foundation in which the Chair of Mathematics was a co-investigator with the Director for the Center for Teaching and Learning, but also motivated internally by the mathematics department, and by the Office of the Provost, an initiative was launched to tackle calculus. This effort has been described elsewhere[1] and is briefly summarized below.

Our efforts at reform were influenced by a successful first-year engineering program at Wright State University[3] which focused on engineering applications of mathematics and also informed by faculty development research summarized by Bressoud, et. al.[4] In our reform, we were able to use STEP grant funding for year-long STEM-focused faculty learning communities (FLCs). [5] We held three FLCs across the five-year grant, with the last two cohorts exclusively focused on calculus instruction. These FLCs were facilitated by one calculus instructor who had reframed his calculus content into an application-based focus oriented to help future engineers and scientists appreciate why they need to learn calculus. A brief overview on the course is given below; full details on the FLCs, and how our "coherent calculus" course was developed, supported and implemented are presented elsewhere.[1]

Coherent Calculus -- overview

The "Coherent Calculus" course contains the following elements, outcomes and pedagogical approach:[1]

- Whenever possible, students work with data sets and/or continuous models selected from actual physical, biological, financial or other applied models.
- Whenever possible, Calculus concepts are introduced and motivated by application to these models and data sets.
- Whenever possible, content is presented using notation, language and conventions of the disciplines from which the models are taken.
- As much as possible, content will be relevant, recognizable, and applicable in subsequent STEM coursework.
- All content will be accessible from an intuitive or practical viewpoint. In particular, the level of abstraction will be significantly less than typically found in Calculus I.

Thematically the revised Calculus I class is focused on three outcomes:

- Develop geometric and physical intuition for derivatives and integrals.
- Master the standard rules for symbolic computation of derivatives and some basic integrals.
- Apply both intuitive understanding and rules mastery to solve problems.

The course design has the following pedagogical features:

- Many short homework assignments with immediate computer driven feedback/assessment, typically due on a two-day cycle.
- Each assignment designed along learning cycle principles to target one or two specific learning goals.
- The vast majority of class time devoted to students working in small groups on these homework assignments.
- Additional active learning assignments that occur in-class with real-time formative assessment (these were added in 2015-16).
- All in-class work facilitated by lead instructor and peer learning assistant
- Additional and more involved weekly work with written feedback.

Table 1 summarizes the successful adoption strategy. Long-lasting change for us was derived from an approach driven by the faculty, based on homework. That is, our reform was driven by what the calculus instructors agreed that students needed to be able to do, not from any sort of imposed model or framework.

Table 1: How to create a coordinated course with a common pedagogical approach	
Adoption Strategy – NO – Do not do this!	**Adoption Strategy – YES – Do this:**
Appoint a course coordinator.	Start with common homework. Build consensus agreement on every exercise.
Combine 12 small sections into 2 huge sections.	Do the same thing with quizzes.
Impose a common syllabus.	Agree on basic weighting of all Hw, Qz, Ex, Final and letter-grade cutoffs.
Impose a common final.	Build consensus on exam content – eventually reach common exams.
Impose common midterm exams and final	Eventually adopt a common final
Impose a pedagogical model.	Along the way, allow the course content to shape pedagogy with the strategic goal, for example, of active learning.

The Impact of Coherent Calculus

Enrollment and Pass Rates: We now have two full years of pass rate data since the Calculus I project scaled up, see Figure 1. There was an initial jump from approximately 65% to 75% that has not been sustained, but the overall pattern is still good. Prior to scale up the historical pass rate was about 60% -- improving, but struggling to ever exceed 65%. Since the scale up pass rate has averaged 72% and only once dipped below 70%. Although participation in the project remains purely voluntary, all instructors continue to opt into the project except for those teaching honors or online sections.[i]

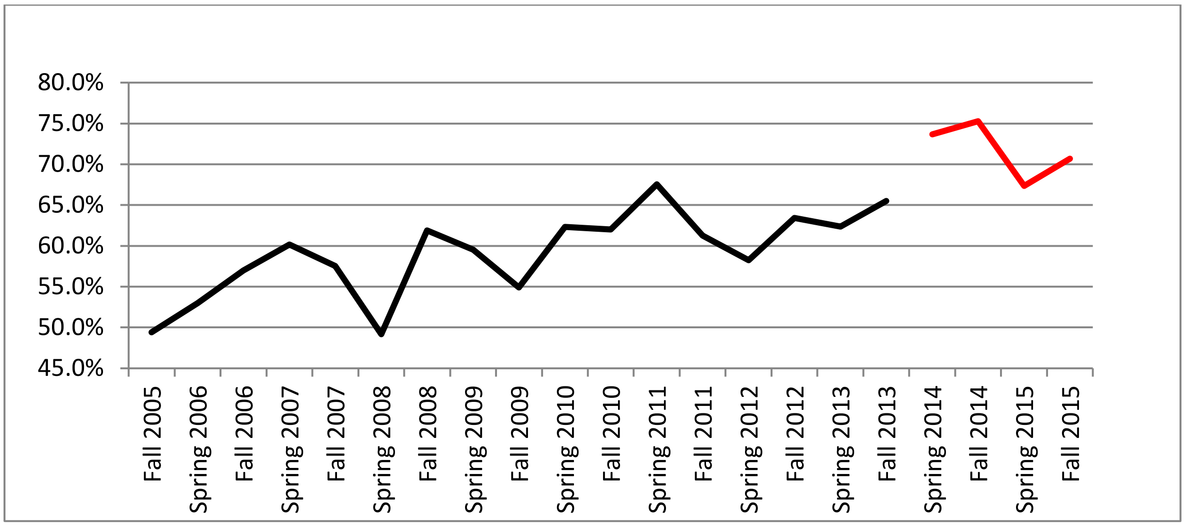

Figure 1: Calculus I pass rate as a function of semester. Red line corresponds to pass rates after the Coherent Calculus model was implemented in spring, 2014.

[i] In summer 2014 it was not possible to join, since materials were not ready. One summer 2015 instructor declined to participate. Pass rate and enrollment data here do not include summers. Summer does not much alter the overall averages or trends, but it is much more volatile.

The total number of students served by the reformed Calculus, compared to more traditional Calculus, is shown in Figure 2. During the initial scale up term, spring 2014, there were roughly equal numbers of reform and other Calculus sections. Presently, the only non-reform sections are (1) honors, (2) online, or (3) face-to-face but taught in parallel with the online section.

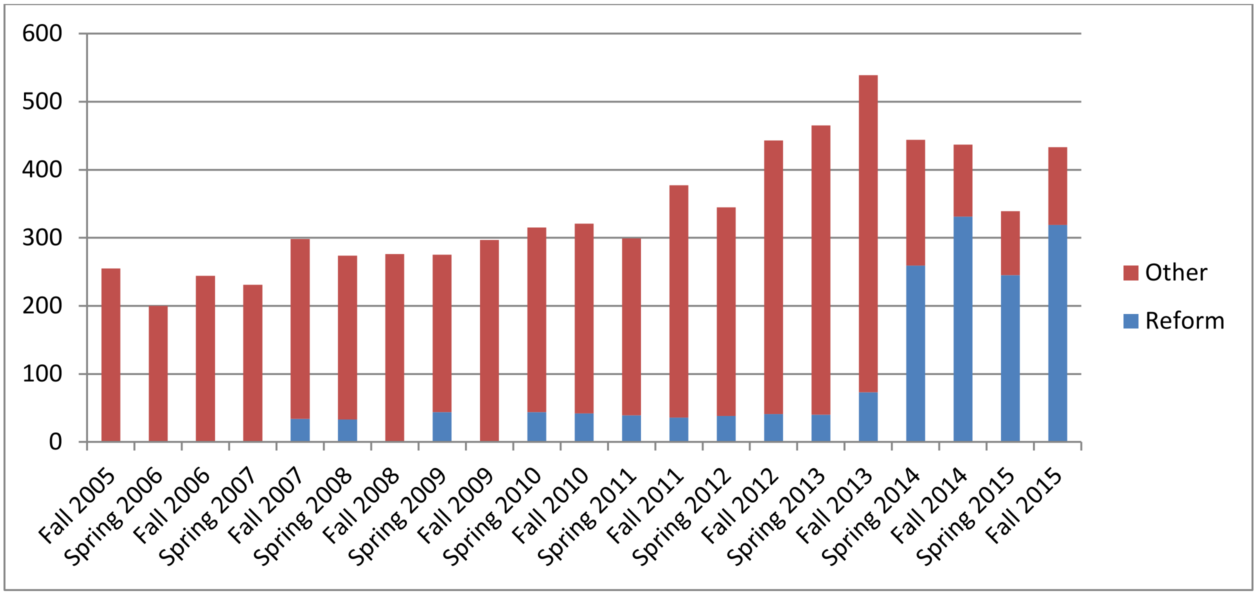

Figure 2: Calculus I enrollment by semester.

Total students "captured" by the reform project, as a percent of enrollment is shown in Figure 3. It appears to be stabilizing in the low to mid 70's, which currently reflects the portion of calculus that Boise State University has chosen to offer as honors, online, or face-to-face but parallel to online.

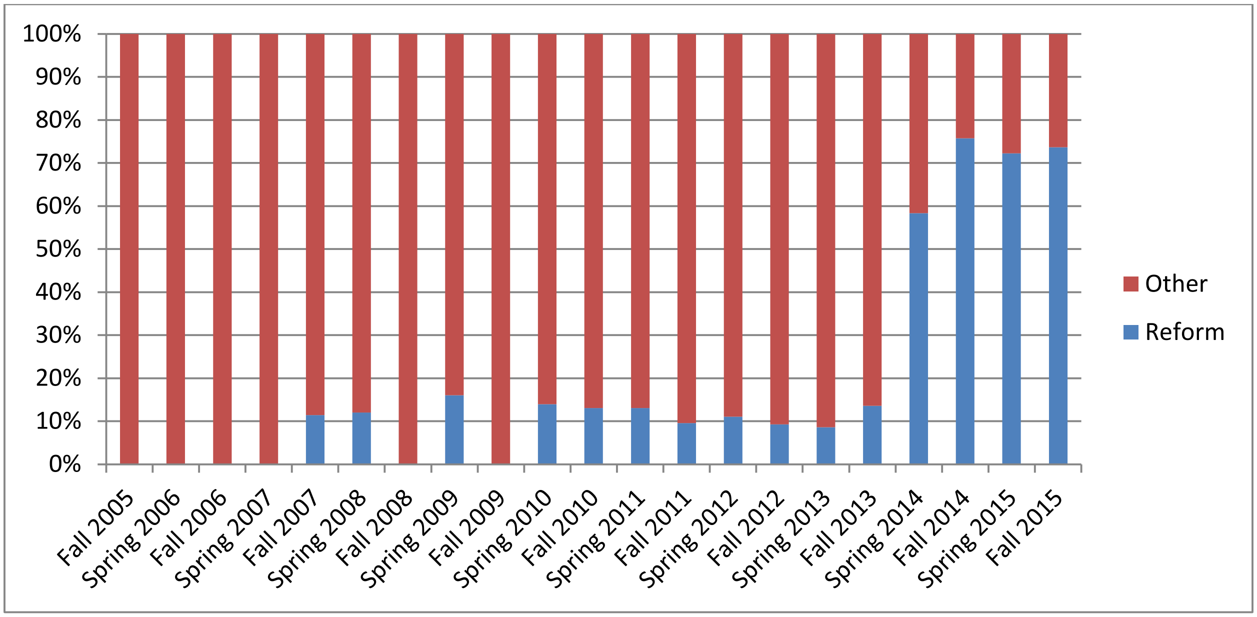

Figure 3: Enrollment in reformed Calculus I expressed as a percentage of enrollment.

The focus of this paper

Pedagogical reforms are subject to criticism for many reasons and from many quarters. As a leader or change agent, one must be prepared to address such concerns. This paper presents a rigorous, data driven technique for refuting such. Resistance takes the form of critiques such as "pass rate gains are probably grade inflation," or "important content is missing or mishandled in the new Calculus I". Any such claim is a testable hypothesis. If the claim is that the inflated grades or missing content result in harm to Calculus II students, then this is a claim that the null hypothesis, "Students do equally well in Calculus II, regardless of the Calculus I reform" is rejected in favor of the alternative hypothesis, "Students coming from reform Calc I do worse in Calculus II."

This paper reports on a natural experiment that provides a rigorous statistical test of the above hypothesis. We find that you cannot reject the null hypothesis, which means Calculus I reform causes no damage to students in Calculus II.

Additional resistance or continued skepticism is entirely possible. In particular, one cannot conduct this experiment without making many choices about how to select and interpret data. In anticipation of such we will explore several alternative choices. In this process we will sometimes see negative treatment effects, but never at any statistically significant level and never sufficient to reject the null hypothesis, even when the data are carefully chosen to make the treatment look as bad as possible. We will also see some positive treatment effects, meaning there are subsets of the data in which the treatment group out performs the control group in Calculus II.

Effects of Reformed Calculus I on Calculus II

To analyze student performance in Calculus II, we created two cohorts, further sliced by term, defined as follows:

<u>Term X Treatment Cohort</u> consists of all students who passed reformed Calculus I during term X at Boise State University, and then immediately enrolled in Boise State University Calculus II.

<u>Term X Control Cohort</u> is all students who passed non-reformed Calculus I during term X at Boise State University, and then immediately enrolled in Boise State University Calculus II.

We define "immediately" to mean without skipping a regular term, so spring-to-fall succession is immediate. This means that a spring cohort will contain students who took Calculus II in either the subsequent summer or fall. No student can be in more than one cohort. We capture only the first instance of a student passing Calculus I, and then only the immediate enrollment in Calculus II. We care only about passers of Calculus I because our dependent variables are performance metrics in Calculus II, which requires Calculus I as a prerequisite. We wish to compare the efficacy of reformed to non-reformed Calculus I, so we do not consider transfer Calculus I credits or other routes into Calculus II (CLEP, AP, etc.).

Cohorts are defined by term so that we can conduct time series analysis. However, the analysis in this paper will primarily deal with aggregated cohorts. Unless specifically stated otherwise, both the treatment and control cohorts will be aggregated from spring 2013 to summer 2015, inclusive. We refer to this as **the baseline cohort**.

For each cohort, we measure two dependent variables: Calculus II pass rate and average Calculus II grade.[ii]

Calc II Pass Rate = (Number of A's, B's, C's) / (Cohort size – Audits – Incompletes)

Grades are converted to grade points on the usual 4-point scale. Boise State University uses +/- grades, so the conversion is A+ = 4.0, A = 4.0, A- = 3.7, B+ = 3.3, etc. We count W's and CW's as 0.0, along with F's. This is consistent with DWF used more generally as a student success metric, in that it considers an F and a W to be equally unsuccessful.

Calc II Grade = (Total grade points) / (Cohort size – Audits – Incompletes)

It is possible that there one cohortcould be better prepared for college level work than another cohort. So we tracked four control variables for each cohort:

> GenACT: About 70% of our students have either an ACT or an SAT Math score. SAT Math scores are converted to ACT using published concordances.[7] If this results in two scores (some students have both ACT and SAT) we take the higher. This is averaged across members of the cohort that have at least one score.
>
> CumGPA: This is the cumulative grade point average for each student at the conclusion of the cohort term, averaged across the cohort. Data limitations at Boise State University force us to use a value of Cum GPA that may have become slightly inflated by grade replacement in the semesters since the cohort term. The effect is small, but nonetheless unfortunate. In a future paper we hope to replace this variable with a more rigorously controlled GPA.[6] We continue to use this variable because it is the only independent variable for which we have data for all students, and because it is our only independent variable that directly measures ability to do college level work.
>
> HSGPA: High school GPA. Along with SAT/ACT scores this is recognized as one of the strongest predictors of college success. We have data for approximately 85% of our students.
>
> AdIndex: Boise State University calculates an Admission Index based on HS GPA and composite ACT/SAT. Also recognized as a strong predictor of college success. Since it uses composite test scores rather than just Math, it is not redundant. We have data for approximately 55% of our students.

[ii] "Dependent" in this context means only dependent on control vs treatment. Although there are also "independent" variables, we will not conduct regression analysis in this paper.

The Data

Here is the complete time series for all variables and both cohorts:

Variable	Cohort	Spring 2013	Summer 2013	Fall 2013	Spring 2014	Summer 2014	Fall 2014	Spring 2015	Summer 2015	Grand Total
Cohort Size	Control	156	46	173	72	38	48	34	31	**598**
	Treatment	14		30	114		162	100	25	**445**
Calc II Pass Rate	Control	76%	67%	63%	61%	63%	71%	65%	48%	**66%**
	Treatment	57%		33%	61%		74%	66%	48%	**64%**
Calc II Grade	Control	2.35	2.16	1.88	1.71	1.75	2.20	1.89	1.56	**2.01**
	Treatment	1.95		1.04	1.76		2.27	1.93	1.67	**1.94**
CumGPA	Control	3.16	3.23	3.20	3.04	3.03	3.19	3.17	3.20	**3.16**
	Treatment	3.41		3.19	3.06		3.13	3.13	3.16	**3.13**
GenACT	Control	24.7	22.3	25.6	22.7	24.9	24.3	25.0	23.4	**24.6**
	Treatment	23.5		24.2	24.0		25.4	24.4	25.7	**24.7**
AdIndex	Control	58.2	49.2	64.6	58.5	54.7	65.7	64.4	53.7	**60.9**
	Treatment	64.0		56.5	58.6		63.7	59.2	55.5	**60.9**
HSGPA	Control	3.38	3.29	3.45	3.31	3.26	3.41	3.43	3.24	**3.38**
	Treatment	3.49		3.24	3.30		3.40	3.38	3.39	**3.36**

Table 2: Time series for all variables and both cohorts, spring 2013 – summer 2015.

The scale up semester, spring 2014, is highlighted in Table 2. The last column shows the aggregate measures. Statistical analysis follows, but we have found that this or similar charts are useful as descriptive data. Some observations:

- In aggregate, the treatment cohort scores slightly worse on both measures of Calc II performance. (Neither group is doing all that well in Calculus II, but that is a separate topic.)
- The four independent variables suggest, in aggregate, that the difference in academic preparation between the treatment and control cohorts is negligible. It is difficult to judge whether a 0.1 gap in average ACT Math score matters compared to a .03 gap in GPA, or a 2% loss in pass rate. This is what statistical tools are for.

Although the aggregate data looks at first glance worse for the treatment group, the time series from the scale up term onward actually shows treatment groups did slightly better in Calculus II. Not by much, but consistently and on both measures.

Analysis

We apply standard statistical tests to compare Calc II Pass Rate and Calc II Grade for each cohort.

We also conducted statistical tests to compare the average values of the independent variables for each cohort. For all independent variables, the hypothesis we are testing is whether the cohorts are actually any different. The results are summarized in Table 3.

Table 3: Effects and *p*-values for baseline cohort				
Values	**Control**	**Treatment**	**Effect Size**	***p*-Value**
Cohort Size	598	445		
Calc II Pass Rate	66.4%	64.3%	-2.1%	0.239
Calc II Grade	2.01	1.94	-0.07	0.215
CumGPA	3.16	3.13	-0.03	0.836
GenACT	24.60	24.70	0.11	0.359
AdIndex	60.88	60.91	0.02	0.493
HSGPA	3.38	3.36	-0.02	0.671

The effect size column recaps what was observed in Table 2. We see a small negative effect of treatment. However, the *p*-values are large, meaning the effect is of no significance. We cannot reject the null hypothesis, so we conclude that Calc II pass rates and grades for the two cohorts are not meaningfully different. The *p*-values for independent variables show that the two cohorts are not meaningfully different on *a priori* academic measures.

Discussion

The overarching purpose of this analysis is to provide change agents, campus leaders, and curriculum reformers the ability to present a persuasive and rigorous argument to potential resisters. Resisters may legitimately claim that these statistical results are influenced by choices made by the study designers, such as which terms to aggregate, whether to consider only immediate follow-on to Calculus II, etc. It is true that results are sensitive to such choices. Table 4 presents the results from aggregating only the three regular semester cohorts since scale up began.

Table 4: Baseline cohort restricted to regular terms from scale up to present				
Values	**Control**	**Treatment**	**Effect Size**	***p*-Value**
Cohort Size	154	376		
Calc II Pass Rate	64.9%	68.1%	3.2%	0.756
Calc II Grade	1.90	2.03	0.13	0.839
CumGPA	3.12	3.11	-0.01	0.575
GenACT	23.69	24.77	1.08	0.010
AdIndex	62.33	61.24	-1.09	0.695
HSGPA	3.36	3.37	0.00	0.492

This choice converted negative treatment effects into positive effects. However, it also created a larger and much more significant gap between the cohorts' ACT Math scores. This *p*-value is small enough to suggest that for this subpopulation more analysis, with some device to control for effects of this variable might be appropriate. However, for the purpose at hand, it is enough to note that the effects of treatment are non-negative. It is not possible for any positive treatment effect to lead to rejection of null hypothesis. What the finer analysis is really saying is that it would be a mistake to attribute the positive effects to the treatment itself.

Since the goal is to provide good evidence that Calculus I changes do no harm in Calculus II, , the best approach is to rerun the analysis with as many different reasonable modifications as possible. If the null hypothesis is true, then it will be unlikely that any modification results in data that calls for rejecting the null hypothesis. Moreover, if you have looked at the data in many ways, as we have, you can present as the baseline case a viewpoint that is least favorable to your project. Then repeated attempts to probe your data will mostly show better results for your project.

We have done this for the data set here. Some modifications that we have studied:

- Start at the scale up term, as in Table 4, but remove summers. This makes the treatment cohort look better than it does in the baseline case of Table 3.
- Put the summers back. Now the treatment cohort looks a lot better. In fact it is almost good enough (p =0.06) to reject the null hypothesis in the other direction, suggesting that the non-reformed Calc I is hurting students in Calc II.
- Restrict to subsets of students for whom we have complete data on independent variables. For example, Table 4 could be modified to look only at students who actually have ACT Math scores. (Oddly, the positive Calc II effects vanish – so they were not after all caused by the ACT scores. This sort of odd artifact is not uncommon when studying data sets in which the measured effects are statistically insignificant.)
- The control group contains all of the honors calculus sections. We could remove them. Unsurprisingly, this makes the treatment cohort look better.
- We included only students who took Calc II immediately after Calc I. There are good technical reasons for this choice, but someone could argue it was done to massage the data. To test this we added delayers to the data. This, too made the treatment cohort look better than before.
- Combinations of the above. All of them result in treatment results that are better than Table 3. Many of them result in positive treatments effects.
- At least one perfectly reasonable combination -- begin at the scale up term, keep all subsequent terms, and remove honors sections -- results in very attractive treatment effects: Table 5.

Table 5: Baseline cohort, from scale up to present, without honors

Values	Control	Treatment	Effect Size	*p*-Value
Cohort Size	206	401		
Calc II Pass Rate	59.7%	66.8%	7.1%	0.957
Calc II Grade	1.72	2.00	0.29	0.994
CumGPA	3.07	3.11	0.04	0.202
GenACT	23.25	24.79	1.53	0.000
AdIndex	56.72	61.14	4.42	0.015
HSGPA	3.28	3.37	0.09	0.026

Here we see large positive effects of treatment. However, it is statistically irrelevant to the specific experiment and research question addressed in this paper. This is because we set out to test a one-sided hypothesis – that the treatment cohort does no worse. If the observed result is that the treatment cohort does *better* then you cannot reject the null hypothesis. What this means is that, if we had chosen to design an experiment to test a different hypothesis – say that treatment improves Calc II outcomes, there is a good chance that we would have a positive result. We will focus on this in future work.

The larger point is that the decisions defining the baseline cohort are probably sound decisions, and are certainly defensible against any claim of massaging data in favor of positive treatment outcomes.

Here is a summary of the reasons for originally settling on our choice of baseline cohort.

- The time series from spring 2013 to summer 2015 is well balanced temporally. It includes 2 regular semesters prior to scale-up, during which control cohorts were larger than treatment cohorts. It includes 2 regular semesters after scale-up, when treatment cohorts were larger than controls. And it includes the scale-up semester itself, in which the cohorts were most nearly equal in size. Also, this time series achieves a nice middle ground between the desire to have large overall *N*, and to have *N* split into roughly equal sized treatment and control cohorts. (Older start dates clearly increase *N*, but older terms are also overwhelmingly control cohort. More recent start dates, like spring 2014 both lower *N* and skew the cohort size towards treatment.)

- We chose a cohort definition that restricts us to students who take Calculus II immediately after passing Calculus I. This lowers *N* by about 10%. (Approximately 90% of students who ever continue on to Calculus II do so without delay). Including delayers would skew towards older cohorts, since only the older cohorts have had enough time to delay and then eventually take Calculus II. This in turn means that control cohorts will have more delayers than treatment cohorts. Delayers may have different personal characteristics that influence Calculus II performance, which would be disproportionately present in the control cohorts.

- We stop with summer 2015 because it is simply the most recent term in which we could have any longitudinal data for students continuing to Calculus II (fall 2015).

- We left honors students in the control cohort because, if ever in doubt, do not restrict data in ways that obviously promote your result.

Effect of treatment on female and underrepresented students

This section examines how women and under-represented minority students perform in Calculus II as a result of the reform of Calculus I. Table 6 presents results for the baseline cohort restricted to female students.

Table 6: Baseline cohort, women only.				
Values	**Control**	**Treatment**	**Effect Size**	***p*-Value**
Cohort Size	108	87		
Calc II Pass Rate	67.6%	64.4%	-3.2%	0.318
Calc II Grade	2.06	2.08	0.01	0.528
CumGPA	3.29	3.26	-0.02	0.621
GenACT	24.27	24.83	0.56	0.187
AdIndex	66.22	66.60	0.38	0.447
HSGPA	3.53	3.60	0.07	0.147

We see the usual results: small effects in Calc II, none significant, and we cannot reject the null hypothesis. The treatment group here has an edge over controls in the independent variables, so if there were anything of interest in the Calc II effects it would be best to control for the effects of ACT and HSGPA.

This is the first slicing of the data that even hints at possibly weaker performance in Calc II. Calculus II course grades are up a tiny bit; pass rate is down a larger amount; but both *p*-values indicate this not significant. Control variables are split. Mostly this data says female treatment and control groups perform the same in Calculus II.

Table 7 presents the results for underrepresented minority students (URM). URM is defined here using our Data Warehouse IPEDS Ethnicity field. We would prefer to have more nuanced information, but working with this we exclude all students who are classified as White, Asian, Non-resident Alien, Two or More Races, or Unknown. This drops our *N* from over 1000 to 108.

Table 7: Baseline cohort, URM only.				
Values	**Control**	**Treatment**	**Effect Size**	***p*-Value**
Cohort Size	52	56		
Calc II Pass Rate	53.8%	66.1%	12.2%	0.904
Calc II Grade	1.79	1.88	0.09	0.624
CumGPA	3.13	3.08	-0.05	0.673
GenACT	23.37	22.88	-0.49	0.701
AdIndex	56.19	57.74	1.54	0.361
HSGPA	3.36	3.36	0.00	0.485

Unsurprisingly, with such low *N*, the difference between pass rate and GPA between the treatment and control cohorts is not statistically significant. Nor are the differences in academic ability of the two groups. However, the pass rate effect is large, indicating that further study may show that treatment causes pass rate to go up for some groups. That the reformed Calculus I resulted in increased pass rate for URM students is not unexpected; the literature clearly shows that active learning strategies, such as is deployed in the reformed Calculus I (group work, etc.), have a positive influence for students who are part of an under-represented minority group. For example, Klingbeil's longitudinal study[3] showed that graduation rates were tripled for URM students who took their engineering-based introductory mathematics course (a hands-on, application focused course) compared with those who did not, and that URM students had higher graduation rates than the control group.

Statistical Significance of Calculus I Effects

Our treatment is an intervention in Calculus I. It is not surprising that it shows no statistically significant effect in Calculus II. It was designed to achieve effects in Calculus I. It does. This is what statistically significant data look like:

Table 8: Calculus I Effects.

Values	Control	Treatment	Effect Size	*p*-Value
Cohort Size	1540	994		
Calc I Pass Rate	67.3%	72.8%	5.5%	0.001
Calc I Grade	2.03	2.21	0.18	0.001
CumGPA	2.95	2.93	-0.02	0.774
AdIndex	59.78	59.73	-0.05	0.524
HSGPA	3.33	3.33	0.00	0.559

The treatment and control cohorts are indistinguishable on independent measures of academic ability. But the treatment effects of increased pass rate and grade are significant at the strongest levels of *p*-value used in any experimental studies.

Summary

Calculus I reform has produced sizable, sustainable, and statistically significant gains in Calculus I pass rates and grades. The course pair report is a rigorous, data driven response prepared to consider claims that student success in reformed Calculus I is the result of grade inflation or weakened standards. It also addresses claims that content in Calculus I cannot be altered for fear of degrading Calculus II performance. Our data persistently shows that reform Calculus I students do no worse in Calculus II than their peers who came through traditional Calculus I. This presentation is strongly resistant to any claims of data massaging, since nearly every adjustment makes the treatment look better than the baseline case we began with. No interpretation of the data comes anywhere close to rejection of the null hypothesis in favor of the alternative that Calc I reform harms future performance in Calc II. Both groups perform equally which should soundly refute any accusation that standards or content in the reformed Calculus I are any sort of danger.

These results are unsurprising when taken in light of the vast body of work done on active learning and its impact on STEM learning. Our reformed Calculus I contains active learning strategies including group work. The literature overwhelmingly supports the importance of group work/collaboration in terms of student persistence and success in major, due to their increased engagement with one another.[8,9] A recent metaanalysis of undergraduate STEM literature shows that active learning leads to increases in examination performance that would raise average grades by half a letter.[10] Our results show an increase in pass rates by approximately that level.

Future Work

As a result of numerous positive outcomes associated with Calculus I reform, the reform is now spreading into Calculus II, with a roll-out point of spring, 2016. Faculty perceptions seem to be generally positive from the Calculus I reform[1] and we will continue to monitor this. We will continue to rigorously analyze student performance by looking at course grade performance and post-requisite course performance. We plan to also begin to monitor student performance in certain engineering courses for which Calculus I or II are prerequisites (Statics and Dynamics).

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Support Model for Transfer Students Utilizing the STEM Scholarship Program

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Lessons Learned from S-STEM Transfer Student Scholarship Program

Abstract

This paper describes how the College of Engineering at Boise State University utilized a National Science Foundation S-STEM award from 2011 to 2016 to support transfer students in their path toward graduation. The need for this support was a result of both Boise State University College of Engineering's transition from a 2-year pre-engineering program to the establishment of Bachelors of Science in Engineering Degree programs in 1997 as well as the establishment of the College of Western Idaho as a regional community college in 2007. Both of these factors led to an increase in the numbers of incoming engineering students transferring from other institutions of higher education to complete their degree. These students were generally ineligible for most Boise State University scholarship programs which are mainly aimed toward students entering college directly from high school.

In this paper we describe how our program connected transfer students with university staff, faculty and resources. To date, this program has a 100% retention rate, with the exception of one student on an official leave of absence, and a projected 100% graduation rate with 91% of the students already graduated. In addition, approximately 22% of scholarship graduates are pursuing graduate degrees.

Introduction

Boise State University's College of Engineering was founded nearly two decades ago in response to regional demand for engineering education from industry leaders. The College of Engineering student body now comprises approximately 3,000 students, reflecting approximately 15% of the university's enrollment. In 2014-15, 52% of the university's students were eligible for Pell Grants, and 19% of the engineering graduates were first-generation college students. Despite the overwhelming need for financial support, the young alumni base of the college has not yet yielded substantial endowments so as to provide a significant scholarship base. Also, our students are highly debt-averse, and often delay graduation because of financial needs. Funding from the National Science Foundation has therefore been critical. Across eleven years between fall, 2004 and fall, 2015, a total of 326 unique students have received scholarships from CSEMS and S-STEM programs at Boise State. This paper reports on one of these programs targeted toward transfer students.

For the first ten years of its history, Boise State University offered only Associate's degrees through the College of Applied Technology. In 2007, the College of Applied Technology was separated to become a regional community college, located approximately 20 miles to the west. Because of this opportunity to begin their studies at a community college, the number of students transferring into Boise State's College of Engineering has substantially increased. Transfer students have unique financial needs; as many exceed their financial aid allocation before they graduate due for a number of reasons including entry mathematics level, which may be substantially below calculus. Also, most of our university level scholarships are focused on traditional students.

Along with financial need differences, transfer students' life demands and needs differ from traditional students. These students are balancing college classwork with part time employment and raising families, and struggle to incorporate additional activities and time commitments. As a result, the focus of the transfer cohort activities reported in this paper has been on a combination of support services geared to student success with social activities for helping students make connections among their cohorts, other students and faculty.

Scholarships Awarded

An analysis of the transfer student body revealed that approximately 55% of our graduating students in 2009-10 graduated with transfer credits. Of transfer students enrolled at that time, 32% came with fewer than 15 credits, 21% with 15 to 29 credits, 27% with 30 to 60 credits and 20% with 60 or more credits earned. Based on the wide variation in credits earned, we designed an array of programming events designed to meet the needs of transfer students, described later on in this report in the section on programming.

Over a five-year period, awards were made to 54 qualified full time students pursuing degrees in engineering, computer science or mathematics. In addition to demonstrating financial need, qualifications included that the students be United States citizens or permanent residents and show academic talent with a minimum GPA of 3.0. Based on Boise State University's determination of unmet financial need from FAFSA records, NSF scholarships for students ranged from $1,500 to $3,000 per semester. These amounts varied from the originally proposed funding levels of $1,000 to $5,000 per semester due to the large number of qualified applicants, increased financial need and time on scholarship. Students were supported for between one and four years, with the average length of award being four semesters. Eligibility requirements also required students have between 6 and 60 transfer credits from a community college or university.

Programming and Programming Results

Mathematics Programming

Prior work has shown the importance of early success in mathematics in terms of earning a good course grade. Moreover, an analysis of retention predictors showed that success in a student's first math course taken is a very strong predictor of retention in STEM.[1, 2] Analysis of our transfer students in fall 2011 showed that approximately 60% of transfer students in engineering, computer science or mathematics entered at the Calculus level or lower. Based on this data and prior work, the programming for this grant was designed to include an initial focus on math review and support.

To facilitate individual, independent review of mathematics at pre-calculus levels and below, we promoted the ALEKS online mathematics review program during transfer orientation. The program was made available to students at no charge through the Idaho NSF STEP program (2010-2015).[3,4] During this same time, significant pedagogical improvements were underway in Calculus I funded through two other grants (NSF STEP and WIDER) that also helped with

Calculus I outcomes. Profound changes were made across all sections of Calculus 1 in terms of pedagogy, homework, timing of course content, grade computation and exam content with the objective to raise first semester, full-time retention of students in STEM majors. Year-long faculty learning communities (FLCs) were developed which focused on active learning pedagogy, common homework, and common due dates and times. The FLC structure facilitated buy-in, communication and feedback between instructors who came to agreement on learning goals and content for each individual lesson. All members of the FLC chose to adopt a similar pedagogical approach which included devoting class time to solving problems, working in small groups; these were facilitated by the lead instructor with a learning assistant. Pass and withdrawal rates pre- and post- implementation revealed an increase in pass rate of 13.4% and a drop in withdrawal rate of 3.9%. Results from anonymous faculty surveys showed that faculty changed their teaching practices in calculus and observed positive effects of all their classes.[5]

One Stop Shop

In order to provide convenient and continuous support for transfer students and for other students receiving funding from the NSF S-STEM programs, we developed a "One Stop Shop." The genesis for this concept was based on what had been learned in our first S-STEM grant, where a single point of contact was found to be critical. This concept was continued and expanded in this grant to assist students. A Scholarship Coordinator was designated who provided students with a single point of contact to assist with academic, career and financial needs. After one year, the location of the coordinator was moved to the College of Engineering Advising Office (CEAO). This location proved to be instrumental as it was very convenient for upper division students to make contact with the Scholarship Coordinator (a licensed professional engineer) and to drop-in informally from time to time. Also, the CEAO provided a convenient location for offering some of the social events during the grant. Academic advising routinely takes place in the CEAO, and other professional staff members are also present, providing a sense of community and belonging to all students, not just the scholarship recipients.

Flexible Programming

Over the grant period the Coordinator organized semi-regular monthly events that were held both on and off campus during a variety of flexible times to allow for variety in class schedules, commuting time, family and work commitments. These activities were in addition to dedicated advising and career support and consisted of a combination of social events such as rock climbing and yoga to career development workshops covering resumes, internships, research and career fair preparedness. These events were frequently conducted in collaboration with other STEM groups including faculty and staff. Given the proximity and unique access to a local hands-on science center, events were opened to the transfer student cohorts and their families providing special access to local and national exhibits. A website along with emails to the transfer cohort were used to communicate information and details on event programming; in addition a Facebook group was created to facilitate communication in the final years of the grant. Surveys of students following years 1 and 3 of the grant confirmed students' desire for a combination of informal social and network activities as well as more career focused opportunities. Results from a post grant survey sent to all scholarship recipients rating the benefit of various programming to the students was compiled for eleven activities, many of which were repeated over the grant period. Students were asked to rate only the programs for which they participated in on a scale ranging from no benefit, to some or very beneficial with a neutral option included. Overall for those that participated in the various events, 54% reported an event as being very beneficial with 75% of the respondents reporting events providing some benefit. The highest rated events are listed below in Table 1.

Table 1 – Rating of top 5 programming events by grant participants

Programming	% reporting very beneficial	% reporting some benefit
finals and holiday socials	63%	72%
internship & resume workshops	61%	92%
academic & career advising	60%	73%
networking luncheons w/faculty & staff	60%	80%
yoga and rock climbing	55%	67%

Scholarship Results

The initial goals were to provide 40 students with funding. Due to the significant need for scholarships for transfer students, funding was provided to 54 students for as few as one year and as many as 4 years. On average students were on scholarship for 2 years. This grant, like previous College of Engineering S-STEM programs, was successful in recruiting student participation from the two largest underrepresented populations in the region - persons of Hispanic descent and women. These student populations were targeted by both email and phone calls to ensure they knew of the program and encourage their application. In addition, outreach to staff within other university programs that support these minority groups was also made including to the College Assistance Migrant Program (CAMP), the Center for Multicultural Education Opportunities TRIO Student Success Program as well as the Louis Stokes Alliance for Minority Participation (LSAMP). As a result, the pool of students who received support was significantly more diverse relative to the college and university, with 22% (12/54) of the scholarship recipients being female and 15% (8/54) Hispanic. These values may be compared with 12.7% female and 9.5% Hispanic, for engineering and computer science majors, and with 53% female and 7% Hispanic at this university (2011-2012 basis).

One of the most significant results of this program has been its 100% retention rate with the exception of one student who is on an extended leave. This is a very strong result, given that internal data shows that for a variety of financial, personal and other reasons, between 2008 and 2012, 17% of all STEM juniors stopped out of the university for a semester or more. Hence, receiving the scholarship likely enabled students to continue toward their degree without stopping out. A comment received during the post-survey, “I was able to focus my efforts and make school and my graduation my priority,” highlights the importance of the financial support of the program. A survey conducted during year one of the program reinforced the post survey results – that the scholarship lessened financial pressure including the benefit of reduction in work hours followed by a reduction in student loans.

In addition to retention, this program has delivered very strong graduation results, with an overall expected STEM degree attainment level of 100% with 91% (49/54) of the students having graduated as of fall 2015. This may be compared with university-wide six and eight-year graduation rates of 28% and 34% (respectively, 2004 basis), and a six-year graduation rate for engineering and computer science majors (2006 basis) of 29%. S-STEM transfer programs at Arizona State University have had similar success with graduation rates ranging 90-95% for students supported by the development of a transfer students support program.[7]

Further evaluation of the average cumulative grade point average (gpa) of transfer S-STEM graduates shows an increase over non supported students. The average cumulative for graduates of the program is 3.49 compared to an average cumulative gpa of 3.19 for the control group consisting of transfer students with the College of Engineering at Boise State who also graduated between fall 2011 and spring 2015. Additional analysis of students' final cumulative gpa upon graduation which includes transfer credits as well as those taken at Boise State compared to only their Boise State gpa reveal an average decrease of only 0.03 points. This minor drop occurred during the students first semester at Boise State but is less than the typical drop often referred to as "transfer gpa shock" experienced by upper division transfer students of as much as half a grade point.[7] Studies done at Boise State of transfer students enrolled at Boise State between fall of 2000 and summer of 2009 showed a drop of 0.15 for a similar transfer cohort.[8]

The average number of degree-enrolled years for graduation of the S-STEM transfer cohort was 4.7 years compared to 4.9 years for the 293 transfer students that graduated with an engineering or computer science degree between fall 2011 and spring 2015. The post-survey confirmed the impact the scholarship had on students' time to graduation as shown in Table 2. Students were asked to rank 5 statements related to the benefit of the scholarship to them. While no single statement was overwhelming selected by respondents, a review of the top three statements by students demonstrates that a majority reported that the scholarship both prevented them from attending part time and reduced their time to graduation. Students reported that the financial support was critical to their retention and graduation. A comment included in the survey by one student indicated: "If not for the scholarship I would have dropped out." A majority reported that the scholarship accelerated their time to graduation, freeing up time for them to study and take additional classes.

Table 2 – Post survey ranking of statements by students

My scholarship…	**% ranking statement #1**	**% ranking statement #1 or #2**	**% ranking statement #1, #2, #3**
prevented me from going part time.	27%	61%	78%
reduced my time to graduation.	22%	44%	72%
allowed me to increase my credit load.	16%	39%	50%
prevented me from taking a semester off.	16%	27%	50%
allowed me to take summer classes.	11%	22%	53%

<u>Graduate school:</u> Of the 49 students that have graduated, 11 students (22%) are pursuing graduate school. This is more than double the college of engineering average rate of undergraduates that go on to graduate school of approximately 10% based on exit surveys. While other universities have attained a higher percentage of students moving on to graduate school (50-60%) this scholarship rate is more on par with the national average and a significant increase over non-scholarship students at Boise State.[7, 9]

Challenges and Lessons Learned

A variety of programming was offered throughout the five years of the grant. One of the issues encountered was that without a formal time-slot associated with a weekly meeting such as other programs offer, it was a challenge to offer events that transfer students could readily attend.[10, 11] While our program required students to only attend events 3-4 times a semester or 6-8 times a year, many of the transfer students had family and work obligations that prevented them from participating. One student reported in the post-survey, "My biggest obstacle in the program was finding time for the activities. I am a single parent and between jobs, school, family, internships and clubs, my time was very limited and I was not able to be involved in all the activities."

To overcome the scheduling challenges, we offered a wide variety of programming opportunities and times. We accomplished this by working closely with other entities on campus such as LSAMP, and other student success oriented activities including those offered by the career center. These efforts to create a variety of events and times to work with students schedules was found by students to sometimes to be overwhelming. In reviewing the literature, other grant programs have operated in a more structured manner, requiring students to register for a specific class (e.g. Academic Success and Professional Development) related to their scholarship. This model has proven a successful strategy not only for scholarship recipients but for others who enroll in the course as well.[9] This model is something to consider for future scholarship projects as it simplifies scheduling and affords a structured timeslot.

The implementation of a Scholarship Coordinator as part of a "one-stop shop" for the grant was instrumental for connecting to students to resources. We were able to have a dedicated staff member as a result of the college of engineering supporting the remainder of the staff member's position on soft money. The coordinator's location in the CEAO allowed her to be readily available to S-STEM students while also supporting the advising office. As an advisor, the Scholarship Coordinator was able to provide knowledge about university policies and procedures including those related to financial aid. Other programs have the importance of enhanced advising in successful retention, including the use of both peer and faculty mentoring as part of their structure.[10] For this scholarship program, informal mentoring by students and faculty proved to be successful but could be expanded upon in the future. The Scholarship Coordinator as a licensed professional engineer was able to represent a number of disciplines and provided connections to local professional and technical societies for opportunities outside of the university.

Summary and Suggestions for Future Scholarship Programs

The NSF S-STEM transfer scholarship program at Boise State University has been very successful. In particular students on the scholarship have been retained with the exception of one students on an extended leave, and we are projected to have a 100% graduation rate with 91% of the students already having graduated before the grant's end. Students who received this scholarship are pursuing graduate degrees at approximately double that of non S-STEM transfer students. The post-survey showed that students report the scholarship as being critical to their timely graduation.

While students enjoyed the informal, active social events, and requested flexible programming, we recommend a more structured approach to event programming in order to facilitate a formal cohort development in a streamlined structure. Such a structure might include a class or set meeting time; this has been shown by Anderson-Rowland, et al. for example, to be effective.[11]

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Assessing the STEM landscape: the current instructional climate survey and the evidence-based instructional practices adoption scale

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Abstract

Background: The efficacy of active learning within STEM education is clear, and many institutions are working to help faculty adopt evidence-based instructional practices (EBIPs) which can promote active learning. In order to know the current status of our campus regarding these goals, measures of current instructional climate and the adoption of evidence-based instructional practices (EBIPs) are desired.

Results: Using a campus-wide online survey approach with remuneration for faculty participants, the 28-item current instructional climate survey (CICS) and the 6-item EBIP adoption scale were developed. When CICS and EBIP adoption scale outcomes are compared, patterns emerge which reflect the climate, conditions, and personal characteristics of faculty at different stages of EBIP adoption.

Conclusions: Although not causal relationships, understanding both climate and personal change characteristics can be helpful to campus change agents in assessing the current STEM landscape of faculty practices.

Keywords: Instructional climate, Measurement, STEM, Evidence-based instructional practices

> When staff and faculty operate from routines, change can be challenging. Imagine trying to have STEM faculty move from lectures to active learning. Their underlying belief is that good teaching involves delivery of content. Asking them to move to a mode where they do not deliver content violates their unarticulated beliefs about good teaching. Cultural theories of change emphasize the need to analyze and be cognizant of these underlying systems of meaning, assumptions, and values; while often not directly articulated, they can nonetheless shape institutional operations and prevent or facilitate change (Kezar and Holcombe 2016, p. 38).

For widespread adoption of evidence-based instructional practices (EBIPs) to occur, the complex higher education ecosystem must be altered; it is important for institutional operations and instructional climate to be understood (Association of American Universities 2017; Rankin and Reason 2008). For many faculties in the USA, lecturing remains widespread, with 50.6% of professors indicating a reliance on extensive lecturing (Eagan et al., 2014). This reliance is understandable, as many faculty members teach as they were taught, but this level of reliance is also surprising given the emerging empirical data about the benefits of active learning. Freeman et al., (2014), using a meta-analysis of 225 STEM education research studies, concluded that active learning approaches are robustly superior in regard to reducing course failure rates and increasing student learning in STEM disciplines; Wieman (2014) has referred to lecturing as "…the pedagogical equivalent of bloodletting" (p. 8320). Given the increasing pressures to transform institutions in regard to undergraduate STEM education (Weaver et al., 2016) and the understanding that changing teaching behaviors is personal and difficult to achieve (Andrews and Lemons 2015), those institutions attempting institutional change would benefit

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from measures of instructional climate (Adams Becker et al., 2017) as well as indicators of the adoption of active learning approaches. To fully understand how STEM faculty make changes to their teaching practices, the instructional climate is one of the key indicators; in fact, Kober (2015) concluded that "…a lack of attention to the larger institutional context is one reason why research-based practices in undergraduate science and engineering education have not produced more widespread change, despite evidence of their effectiveness" (p. 177). The ability to assess the current teaching landscape could be an important ally in these efforts. That is, if the subjective norm of the environment becomes teaching via EBIPs rather than lecture, according to Ajzen's (1991) theory of planned behavior faculty members who remain in lecture mode will be more inclined to change given the new environmental conditions. If there is a tipping point (Gladwell 2002) for faculty in STEM departments to be predisposed to transforming their teaching practices toward more active learning approaches, it would be helpful to have measures of instructional climate available so that campus leaders can leverage prevailing trends and ensure adequate support for faculty members in every EBIP adoption stage.

Measuring instructional climate

Measuring the instructional climate of a college or university is important if the desired goal is to create and measure transformational change around teaching and learning. In fact, the values of an organization, including its underlying assumptions, are key drivers of and barriers to change (Kezar and Holcombe 2016). There is no shortage of available instruments for measuring teaching practices, either by using self-report surveys (e.g., Postsecondary Instructional Practices Survey (Walter et al., 2016) and Teaching Practices Inventory (Wieman and Gilbert 2014), or observation protocols (e.g., Reformed Teaching Observation Protocol (Piburn et al., 2000) and Classroom Observation Protocol for Undergraduate STEM (Smith et al., 2013)). However, our interest is in the faculty perceptions of the instructional climate, which includes more than the pedagogies selected for use. Even though there are measures of instructional climate that exist in the literature in various forms (e.g., measuring departmental climate from Walter et al., (2016)), our desire was to create a climate measure that was (a) specific to the instructional climate of a university and (b) designed to measure the climate elicited from a specific organizational change process/theory. Literally, climate is a local phenomenon, and thus, it seemed logical to develop a local instrument, but also to be vigorous in establishing the validity and reliability of its measures.

Many change models exist, such as the Gess-Newsome et al., (2003) model for faculty change and the Henderson et al., (2011) four-quadrant model of strategies for change. We utilized Dormant's (2011) CACAO (Change, Adopters, Change Agent, and Organization) model because of our familiarity with the model and access to local experts in using this model (see also Shadle et al., 2017). The CACAO model is rooted in Rogers (2003) diffusion of innovation theory that specifically outlines a set of actions that can be taken to facilitate change. In the context of the present study, implementation of the CACAO change model is the intervention strategy that has been used to facilitate the adoption of active learning practices by faculty members.

Evidence-based instructional practice adoption stages

With the presumption that there are steps or stages of change through which faculty move as they adopt new teaching approaches, it would be useful to know a faculty member's particular status within the continuum of change; a one-size-fits-all intervention strategy is unlikely to be universally successful when STEM faculty members vary in their readiness to adopt. To this end, we used a Guttman scaling approach to develop our EBIP adoption scale. Well-developed Guttman scales are inherently intuitive because (1) responses are merely yes or no and (2) scoring is easy and obvious by examining when/where the pattern of responses changes. Although there are multiple good survey inventories available in the literature where faculty members describe their usage of pedagogical practices (PIPS, TPI), to our knowledge, there is no existing measure that allows a faculty member to self-identify their level or stage of adoption of evidence-based instructional practices. That is, a teaching practices inventory may help a faculty member report that their predominant teaching pedagogy is lecture, but that same inventory does not yield information about that faculty member's thoughts about alternative EBIP strategies, whether they have imagined using an EBIP in their course, whether they have attended a workshop about adopting a new EBIP, and so on.

In the change model, Dormant (2011) suggested five levels or stages of the potential adopter, described here: (1) Awareness: The potential adopter is passive about the change, has little/no information about the change, and has little/no opinion about the change; (2) Curiosity: The potential adopter wants more information about the change, actively engages in asking questions about the change, and asks questions about personal impact; (3) Mental tryout: The potential adopter is in a pre-commitment stage, imagining how the change would be made, asking job-focused questions (with job-focused concerns) about the impact of the change; (4) Hands-on tryout: The potential adopter has made the commitment to change, wants to learn how to implement the change, has opinions about the change, and asks questions about the change relative to organizational context; and (5) Adoption: The potential adopter has now actively made the change, is able to make suggestions for improvement regarding the change, and may seek out expert opinion for answers to detailed questions about the change. Although not specifically articulated in Dormant's model, if awareness is stage 1, there could essentially be a stage 0, that is, a pre-awareness stage.

Given the goal of changing STEM faculty adoption of active learning, it would be valuable to know the current stage of a faculty member, and perhaps also the cumulative status of a department. A faculty member who is unaware of EBIPs will need a different level of support and training than a faculty member who is a long-time adopter of EBIPs; for faculty developers and campus change agents, one size (intervention) does not fit all. Departmental context is also an important factor to consider when attempting to change faculty teaching behaviors (Lund and Stains 2015; Manduca et al., 2017). The intervention strategies implemented by campus change agents for those in the awareness and curiosity stages should certainly be different compared to those intervention strategies implemented for faculty members in the hands-on tryout or adoption stages (Dormant 2011). An understanding of the adoption stage, paired with current instructional climate data, could provide change agents with useful information about which faculty and departments are most ready for intervention efforts. Given this context, our research questions include: (1) When attempting to measure the construct of instructional climate, what are the reliable and valid components or factors that emerge? (2) Can a straightforward scale be developed that allows STEM faculty to meaningfully self-identify their own adoption stage regarding the usage of evidence-based instructional practices? and (3) How are measures of instructional climate and EBIP adoption stage useful to campus leaders, and how might these measures be related to existing demographic variables that describe the sample?

Method

Participants

In order to understand institutional climate and adoption stages, all Boise State University faculty with teaching responsibilities ($N = 1799$) during the Fall 2015 and Spring 2016 semesters were surveyed in 2016; respondents received $10.00 remuneration placed directly on their campus identification card. To qualify as a faculty member with teaching responsibilities, the following criteria were utilized: (a) the faculty member had to be listed as teaching at least one course in the Registrar's database and (b) the course must have an enrollment greater than one (which allowed ruling out independent study/thesis type courses). This method generated a comprehensive list of instructors, including graduate students, adjunct faculty, tenure and non-tenured full-time faculty, administrators with a teaching appointment, off-campus instructors, and online instructors. With 528 usable responses, the overall response rate was 30.1%.

Materials

Development of the current instructional climate survey

We used Dormant's (2011) change process/protocol in order to engage faculty members in thinking about an end state on our campus (Shadle et al., 2017) that would look like this:

The culture of teaching and learning at Boise State will be characterized by an on-going exploration and adoption of evidence-based instructional practices which includes (a) faculty engaged in continuous improvement of teaching and learning; (b) dialog around teaching supported through a community of practice; and (c) teaching evidenced and informed by meaningful assessment. The fulfillment of this vision will result in increased student achievement of learning outcomes, retention, and degree attainment, especially among underrepresented populations.

Working with two groups of STEM faculty based on convenience sampling, we engaged these faculty members to describe the positive and negative aspects of moving toward the desired end state. Faculty responded on paper surveys in each of the five key characteristic areas in regard to achieving the goal state (relative advantage, simplicity, compatibility, adaptability, and social impact; Dormant 2011). Based on pilot testing from the CACAO-based change adoption process and with the aid of a survey design expert, we organized responses using a modified Q-sorting technique (see Nitzberg (1980) and DeNelsky and McKee (1969) for more Q-sort examples) to identify thematic trends. From these empirically-derived themes, we generated the initial item pool for the current instructional climate survey (CICS). For example, faculty noted "a sense of central administration taking over" as a potential barrier to changing instructional practices. In response, we crafted a semantic differential item with the stem "I believe that the campus culture…" with the anchors ranging from "limits the choice of teaching methods" to "allows for the free choice of teaching methods." Thus, each item in the CICS was based on this analysis of the positive and negative aspects of the potential change (i.e., drivers and barriers) in working toward the desired end state, see Tables 1, 2, and 3 for the CICS items. Items were pilot-tested and re-tested until the resulting pool of 28 items was finalized. It is important to emphasize that all of the items generated for this work originated from STEM faculty members.

The first 24 items of the CICS were answered on a 1 to 7 semantic differential scale, as described above. Another example of this type of scaled item "I believe that the campus culture…" with the low (value = 1) anchor being "connects me with other teachers" and the high (value = 7) anchor being "isolates me from other teachers." The remaining four items of the CICS were answered using a Likert-type agreement scale from 1 = strongly disagree to 5 = strongly agree. After pilot testing, the

Table 1 Campus climate

Item no.	Item	M (SD)	Item
1	is generally supportive of teaching.	2.62 (1.5)	is generally unsupportive of teaching.
2	limits the choice of teaching methods.	5.48 (1.5)	allows for the free choice of teaching methods.
3	promotes faculty-centered teaching.	4.56 (1.5)	promotes student-centered teaching.
4	values research more than teaching.	3.38 (1.7)	values teaching more than research.
5	is student-success oriented.	2.98 (1.5)	is not student-success oriented.
6	connects me with other teachers.	3.50 (1.6)	isolates me from other teachers.
7	does not value teaching ability in hiring decisions.	4.29 (1.7)	does value teaching ability in hiring decisions.
8	discourages me from trying new teaching techniques.	5.48 (1.5)	encourages me to try new teaching techniques.
9	values the assessment of student learning outcomes.	2.92 (1.6)	does not value the assessment of student learning outcomes.
10	values teaching more than research in tenure and promotion decisions.	5.18 (1.5)	values research more than teaching in tenure and promotion decisions.
11	is shaped by leaders who are not supportive of my teaching.	4.83 (1.5)	is shaped by leaders who are supportive of my teaching
12	encourages use of evidence-based instructional practices	2.77 (1.4)	discourages use of evidence-based instructional practices
13	does not value teaching.	5.22 (1.5)	values teaching.
14	does not allow faculty to teach using any method they choose.	5.55 (1.3)	allows faculty to teach using any method they choose.
15	breeds divisiveness in teaching discussions.	5.12 (1.4)	breeds collaborative teaching discussions.
16	is characterized by high faculty-student rapport.	3.12 (1.4)	is characterized by low faculty-student rapport.

For this seven-point semantic differential scale, the left-most response was coded 1 and the right-most response was coded 7. Individual item Ns vary from 516 to 536
Means (M) and standard deviations (SD) for current instructional climate survey (CICS) items
For each item, please select the scale point that best represents your opinion. Each statement begins with "I believe that the campus culture…"

nature of these items appeared to be better answered on an agreement scale rather than a semantic differential scale. Examples from this last section of the CICS include the stem of "I believe that my institution provides…" and items such as "flexible, physical spaces for teaching and learning" and "adequate assessment mechanisms/support."

Development of the evidence-based instructional practices adoption scale

The items in the EBIP adoption scale were developed, a priori, to be used as a Guttman scale with yes/no responses. Our goal was to generate at least one yes/no question for each of the five CACAO adoption stages (Dormant 2011). Members of the research team, working with a survey expert, generated a pool of Guttman scale (yes/no) items that comprised the initial item pool for pilot testing. After pilot testing, one item was selected to map onto each stage of the CACAO change model. One of the objectives of a Guttman scale is unidimensionality, that is, the measure of a singular construct—in the present case, this singular dimension is the faculty members' degree of adoptions of EBIPs.

Table 2 My teaching

Item no.	Item	M (SD)	Item
17	faculty-centered.	5.84 (1.2)	student-centered.
18	unmonitored.	3.78 (1.8)	monitored.
19	a small part of my professional identity.	5.52 (1.5)	a large part of my professional identity.
20	not valued.	5.25 (1.5)	valued.
21	more important than my research.	3.58 (1.8)	less important than my research.
22	not informed by discussions with colleagues.	5.30 (1.5)	informed by discussions with colleagues.
23	less important than my research when I am considered for tenure and promotion.	3.10 (1.5)	more important than my research when I am considered for tenure and promotion.
24	not informed by research about best practices.	5.58 (1.2)	informed by research about best practices.

For this seven-point semantic differential scale, the left-most response was coded 1 and the right-most response was coded 7. Individual item Ns vary from 499 to 532
Means (M) and standard deviations (SD) for current instructional climate survey (CICS) items
For each item, please select the scale point that best represents your opinion. Each statement begins with "I believe that my teaching is…"

Table 3 My institution

Item no.	Item	M (SD)
25	adequate resources to support teaching.	3.82 (1.0)
26	flexible, physical spaces for teaching and learning.	3.38 (1.1)
27	adequate mechanisms for evaluating teaching.	3.10 (1.1)
28	adequate assessment mechanisms/support.	3.32 (1.0)

Individual item Ns vary from 529 to 532
Means (M) and standard deviations (SD) for current instructional climate survey (CICS) items
For each item, please select the scale point that best represents your level of agreement, with 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, and 5 = strongly agree. Each statement begins "I believe that my institution provides…"

Self-scoring of Guttman scales is evident when the pattern of responses changes from yes to no. This goal is operationalized in the calculation of the coefficient of reproducibility (CR); a CR = 1.0 would indicate a perfectly replicable Guttman scale. In practice, a CR > .90 is considered the standard of evidence for unidimensionality (Abdi 2010; Aiken and Groth-Marnat 2006; Guest 2000). However, if extreme patterns of responses to an item emerge or an individual responds with an extreme pattern (e.g., answering all of the items with *yes*), these types of patterns can lead to an artificially high CR (Guest 2000; Menzel 1953). To counteract this, Menzel (1953) developed the coefficient of scalability (CS), "…which measures predictability of the scale relative to the level of prediction afforded by consideration solely of the row and column totals" (p. 351). The recommended standard for a CS is .60 (Guest 2000; Menzel 1953).

Following the formulation and pilot testing of the Guttman scale items, this new instrument was administered to 528 participants at the same time of the CICS item administration, see Additional file 1: Table S1 for the seven EBIP adoption scale items. Following data collection, responses were assembled and ordered from most agreement (highest number of *yes* responses) to least agreement (lowest number of *yes* responses). For each item, scale errors were calculated following Aiken and Groth-Marnat (2006) and Guest (2000) and marginal errors (i.e., non-modal frequencies) were calculated according to the methods suggested by Guest (2000) and Menzel (1953). Any participant who left a Guttman item blank was eliminated from the analysis (N = 14); this resulted in the data from 514 respondents utilized for the Guttman scale analysis.

Similar to the process of eliminating items from a scale to increase inter-item reliability as evidenced by a Cronbach's α, Guttman scale items were systematically tested in order to achieve adequate levels of reproducibility and scalability. Ultimately, the original item #2 was removed from the initial seven items, and this process resulted in a six-item scale (see Additional file 1: Table S1) with a CR = .931 and a CS = .792. This process is similar to using inter-item coefficients when testing the Cronbach's α of Likert-type subscales; removal of the original item #2 allowed for the resulting Guttman item pool to reach acceptable reliability.

Demographics

The demographic questions included faculty rank, total years teaching experience in higher education and at Boise State, the year graduated with their highest academic degree, the highest academic degree in one's primary discipline, the primary academic department or unit, tenure/tenure track or non-tenure track, age, gender, whether or not the faculty member has an office on campus, an approximation of one's normal workload that involves teaching and research, and institutional identification number (this was necessary in order to remunerate participants for survey completion), see Additional file 2: Table S2 for the demographic characteristics of the overall sample.

Procedure

At the end of January 2016, all Boise State faculty with teaching responsibilities were invited via E-mail to complete the current instructional climate survey (CICS), the Postsecondary Instructional Practices Scale (PIPS; Walter et al., 2016), the EBIP Adoption Scale, and demographic questions. The PIPS items are not analyzed as part of the current study. All measures were administered online via Qualtrics. Survey participation closed at the end of February and during the time the survey was available; two follow-up reminders were E-mailed to non-respondents only. Respondents could take as much time as they wanted to reply to survey items. Respondents received $10 placed directly on their university identification card.

Results and discussion

This section is subdivided based on the outcomes of the development of the CICS and the EBIP Adoption Scale, including subsections on descriptive outcomes, CICS factor analysis results, climate and adoption scale results considered together, factor analysis results, and analyses based on select demographic variables. A discussion of each of the outcomes is included here for clarity, followed by a Conclusions section.

Descriptive outcomes for the CICS and EBIP adoption scale

For the overall means and standard deviations for all of the CICS survey items, see Tables 1, 2, and 3. Note that for the first two sections of the CICS, each item was answered on a seven-point semantic differential scale, with the left-most response coded as 1 and the right-most response coded as 7. For example, for the item "I believe that the campus culture ('does not value teaching' to 'values teaching')," a lower score means that faculty responses were closer to the left-most "does not value teaching" anchor (1), and a higher score means that faculty responses were closer to the right-most "values teaching" anchor (7), with an exact midpoint at 4.0. For this particular item, the mean response value was 5.22 (SD = 1.5), meaning that across all faculty respondents, on average, they tend to believe that the campus culture values teaching. With regular and meaningful measurement, answers to particular items can be helpful. For instance, observing relatively high values on the initial measurement can inform researchers that the current campus climate on a particular issue is highly positive; given this observation, efforts to significantly increase perceptions may be difficult due to ceiling effects.

The descriptive outcomes for the EBIP adoption scale responses consist of scale scores and how they map onto Dormant's (2011) CACAO change model adoption stages. For these results, see Additional file 1: Table S1. This type of measure could be particularly valuable over time, as shifts in departmental culture can be tracked based on the distribution of faculty across different stages of EBIP adoption.

Factor analytic outcomes for the CICS

All responses to the 28-item CICS, items were subjected to exploratory factor analysis using a varimax rotation, eigenvalues > 1.25, and factor loadings > 50. A five-factor solution emerges explaining 54.1% of the variance.

The theme that emerges for factor 1 (items 14, 2, 8, and 15; see Tables 1, 2, and 3) is the free choice of teaching methods, which involves the encouragement of using new teaching methods as well as collaborative discussions; inter-item reliability using Cronbach's α = .797. The higher the factor 1 score, the greater the belief that the free choice of teaching methods exists. The theme for factor 2 (items 27, 28, 26, and 25) is institutional support, meaning that there is adequate support for teaching, assessment, evaluation, and the availability of physical, flexible spaces for teaching; inter-item reliability using Cronbach's α = .805. The higher the factor 2 score, the greater agreement that there is institutional support for teaching. The theme for factor 3 (items 10, 4, and 23, with reverse coding for item 10) is teaching-research balance, including the relative valuing of teaching and research in hiring as well as promotion and tenure decisions; inter-item reliability using Cronbach's α = .759. The higher the score for factor 3, the more that teaching is valued over research, including hiring and promotion and tenure decisions. The theme for factor 4 (items 9 and 12, with both items reverse-coded) is the encouragement to use evidence-based instructional practices, especially as related to assessing student learning outcomes; inter-item reliability using Cronbach's α = .619. The higher the score for factor 4, the greater the belief that the campus climate encourages the use of evidence-based instructional practices. Lastly, the theme for factor 5 (items 22 and 6, with item 6 reverse-coded) is teacher connectedness, involving the connections and conversations with teaching colleagues; inter-item reliability using Cronbach's α = .615. The higher the score for factor 5, the greater connectedness with teaching colleagues, especially as related to teaching discussions. Even though the inter-item reliabilities are low for factor 4 and factor 5, they were retained here for explanatory purposes.

Combination of climate and adoption stage: CICS factor scores and EBIP adoption scale outcomes

Scores from the five CICS factor scores were correlated with EBIP adoption scale scores. Due to five correlation coefficients being generated, a Bonferroni correction was employed to minimize family-wise error. The resulting p critical value (p_{crit}) is .01. EBIP adoption scale scores are significantly correlated with (a) factor 1 (the free choice of teaching methods), $r(531) = .13$, $p = .004$; (b) factor 3 (teaching-research balance), $r(531) = -.18$, $p < .001$, (c) factor 4 (encouragement to use evidence-based instructional practices), $r(528) = .14$, $p = .002$, and (d) factor 5 (teacher connectedness), $r(531) = .22$, $p < .001$. What does this mean? The higher the self-reported stage on the EBIP adoption scale (a) the greater the perception of free choice in teaching, (b) the greater the weighting of teaching in considering teaching-research balance, (c) the greater the perceived encouragement on campus to use evidence-based instructional practices, and (d) the more connected the faculty member feels to other teachers on campus.

Select demographic variables as related to CICS scores

For all of the CICS-related analyses in this section, the Bonferroni correction was used for the five comparisons, resulting in $p_{\text{crit}} = .01$.

Age

Answers to the items which comprise factor 1 (the free choice of teaching methods) were significantly correlated with age, $r(493) = .15$, $p = .001$. Younger faculty reports greater freedom to select the teaching method of their choice. Answers to the items which comprise factor 3 (teaching-research balance) were significantly correlated with age, $r(493) = -.12$, $p = .008$. With the negative correlation, younger faculty members report their belief that research is valued over teaching in the teaching-research balance.

Teaching workload

Respondents were asked to report the approximate percentage of their workload that involves teaching. There is a significant correlation between responses to factor 3 (teaching-research balance) and responses to the teaching workload item, $r(526) = .13$, $p = .002$. Faculty members reporting higher workload percentages for teaching perceive teaching is more valued in hiring decisions and promotion and tenure decisions.

Tenure/tenure track vs. non-tenure track

When the responses are compared between tenure/tenure-track faculty and non-tenure-track faculty, significant differences emerge for two CICS factors: (1) tenure/track faculty (mean = 3.18, SD = 0.9) score significantly lower than non-tenure-track faculty (mean = 3.53, SD = 0.8) on factor 2 (institutional support), $t(526) = -4.76$, $p < .001$ and (2) tenure/tenure-track faculty (mean = 2.58, SD = 1.3) score significantly lower than non-tenure-track faculty (mean = 3.45, SD = 1.2) on factor 3 (teaching-research balance), $t(526) = -7.89$, $p < .001$. Tenured/tenure-track faculty believe there is less institutional support for teaching compared to non-tenure-track faculty, and tenured/tenure-track faculty believe that research is more valued over teaching as compared to the balance perceived by non-tenure-track faculty.

Office on campus

For the CICS factor scores, there were three significant differences in answers between those individuals with an office on campus and not having an office on campus: (1) individuals with an office (mean = 3.33, SD = 0.8) scored significantly lower than individuals with no office (mean = 3.68, SD = 0.8) on factor 2 (institutional) support, $t(530) = -4.07$, $p < .001$; (2) individuals with an office (mean = 2.96, SD = 1.3) scored significantly lower than individuals with no office (mean = 3.73, SD = 1.1) on factor 3 (teaching-research balance), $t(530) = -5.91$, $p < .001$; and (3) individuals with an office (mean = 5.00, SD = 1.3) score significantly higher than individuals with no office (mean = 4.52, SD = 1.4) on factor 5 (teacher connectedness), $t(530) = 3.64$, $p < .001$. When answers to the office on campus item are compared to academic status (non-tenure track vs. tenure/tenure track), there is a significant association in the pattern of answering these two items, $X^2(1) = 75.99$, $p < .001$; 98.5% of tenure/tenure-track faculty members have an office on campus compared to 66.1% of non-tenure-track faculty members.

Faculty members with an office on campus actually believe that there are fewer institutional resources for teaching compared to those faculty without offices on campus. Faculty members without an office believe that the institution values teaching over research more than faculty members with an office. Lastly, faculty members with an office report greater connectedness to other teachers on campus compared to those faculty without offices on campus.

Gender

There were no significant differences between male and female responses on each of the five CICS factors.

Demographic variable relationships with EBIP adoption scale scores

EBIP adoption scores were significantly correlated with answers to the item about the percentage of workload involving research, $r(367) = -.12$, $p = .027$; with the negative correlation coefficient, the less workload involving research, the higher the EBIP adoption score. There is a significant difference between tenure/tenure-track faculty (mean = 3.82, SD = 2.0) and non-tenure-track faculty (mean = 3.42, SD = 2.2) on their EBIP adoption scores, $t(526) = 2.08$, $p = .038$; tenure-track faculty report significantly higher EBIP adoption scores. There is a significant difference in answers for those individuals with an office (mean = 3.75, SD = 2.1) and those individuals who do not have an office (mean = 2.91, SD = 2.3) on EBIP adoption scores, $t(530) = 3.84$, $p < .001$; those with an office report higher EBIP adoption score. Also, there is a significant difference between females (mean = 3.86, SD = 2.1) and males (mean = 3.18, SD = 2.1) on EBIP adoption scores, $t(498) = -3.51$, $p < .001$; females report higher EBIP adoption scores than males.

EBIP departmental profiles

With the existence of individually based EBIP adoption scores, departmental profiles can be created to depict the climate or culture within a department concerning the adoption of evidence-based instructional practices. There are strong advocates for changes in STEM education (Freeman et al., 2014; Wieman 2014), and utilizing EBIP departmental profiles for STEM departments could provide a new measure of assessing the landscape. Following the calculation of EBIP scores in the current study, departmental profiles were created for each of the STEM departments under study, see Fig. 1 for examples of STEM department profiles. By reviewing the departmental profiles such as in chemistry or computer science, campus leaders interested in the transformation of both faculty practice and institutional climate may realize that a one-size-fits-all approach in encouraging faculty members to adopt evidence-based instructional practices will likely not work. For instance, multiple strategies for EBIP adoption are needed in Chemistry due to the diversity of scores on the EBIP adoption scale (Fig. 2). However, campus leaders might decide to provide more resource-intensive support to Computer Science since the bulk of respondents are already EBIP adopters.

It is clear to see that different faculty members are aligned at different points on the EBIP Adoption Scale; thus, strategies for those individuals at the awareness stage should be different than the strategies needed for those in the mental tryout or adoption stages. Department profiles could be a powerful source of information for campus leaders in determining tipping points for localized, grassroots efforts to affect teaching practices.

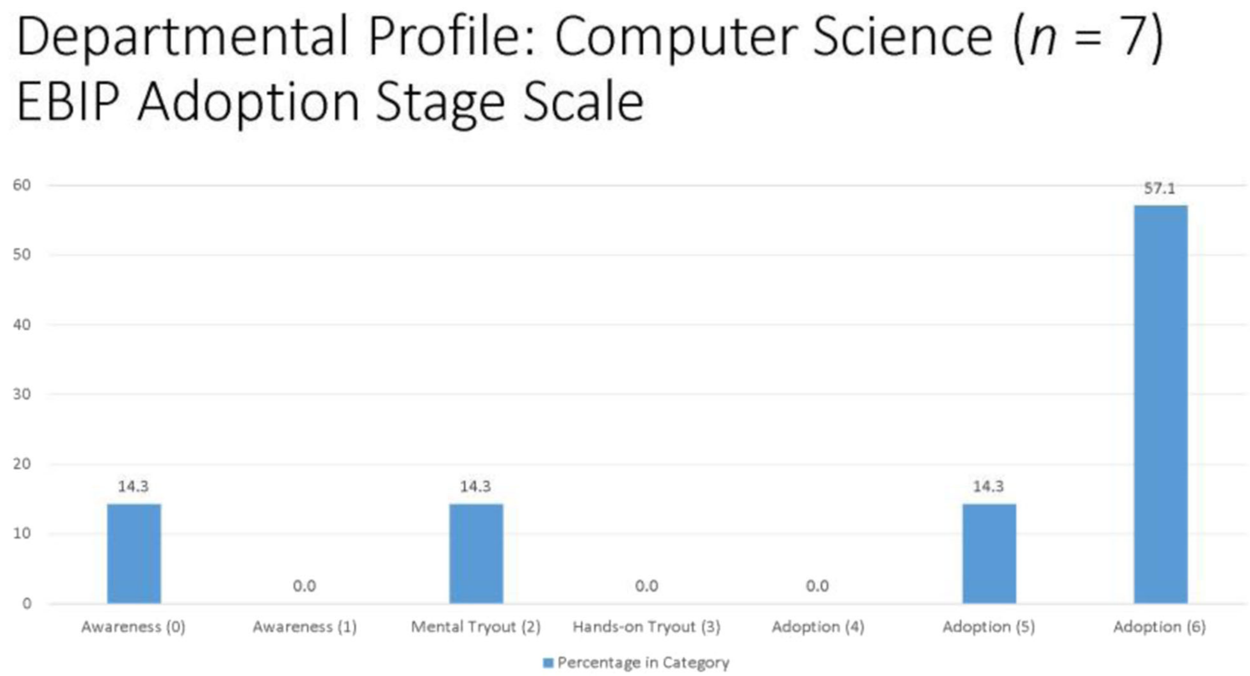

Fig. 1 Departmental Profiles Based on EBIP Adoption Stage Scale Scores

Conclusions

As for limitations, this is a single sample from one institution of higher education; greater use among more and diverse educational institutions would help to re-affirm the reliability of the initial findings presented here. To that end, the specific items that comprise the CICS are shared in Tables 1, 2, and 3, and those of the EBIP adoption scale are in Additional file 1: Table S1, with the goal of facilitating expanded work by other researchers where interested; our team will continue to use this instrument and continue to

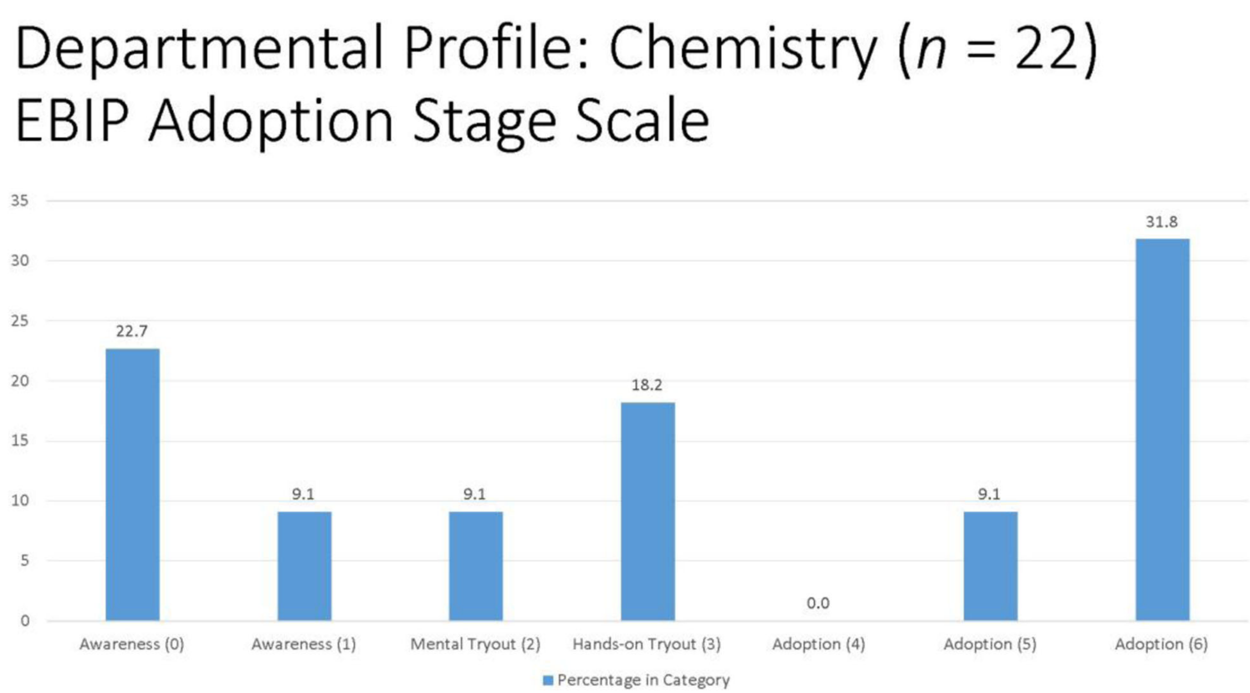

Fig. 2 Departmental Profiles Based on EBIP Adoption Stage Scale Scores

explore the case for its beneficial use. There are also subtle distinctions between measuring the instructional climate of an institution as compared to faculty members' perceptions of the climate. In the present case, perception may be reality; that is, Kober (2015) and Kezar and Holcombe (2016) would argue that understanding the values and the institutional context are vital to the understanding of change and transformation.

The CICS has become a valuable tool in our applied work with STEM departments because it allows for an assessment of the current institutional climate regarding teaching and how it is perceived, valued, and supported on campus. The five-factor structure of this scale makes sense and its use in statistical analyses has already allowed for meaningful insights for our applied work. The overarching goal is for an EBIP adoption scale score to serve as an index of an individual STEM faculty member's placement on the adoption scale as described previously by Dormant (2011) and as adapted here, specifically, for the use of evidence-based instructional practices.

This is a challenging era in higher education; a growing focus on assessment, accountability, student learning and student success is underway. Change will happen, voluntary or otherwise (i.e., innovation or stagnation). Institutions will either effect strategic, planned transformation in alignment with national and regional goals or have it forced upon them. To this end, it would be advantageous to have meaningful measures in place in order to assess the current STEM landscape regarding instructional climate and the adoption of evidence-based instructional practices. The development of such measures is the precise focus of this study, more specifically, to develop a measure of current instructional climate and EBIP adoption stage. Based on our initial findings, the CICS appears to be a useful measure to provide campus leaders with a current "snapshot" of STEM faculty attitudes, beliefs, and behaviors regarding teaching. The EBIP adoption scale allows for the identification of an adoption stage for STEM faculty members, and that information can be useful in designing effective interventions to meet faculty members where they are, and for monitoring changes in faculty EBIP adoption and use over time. We encourage researchers to use these instruments in order to foster a greater understanding of instructional climate as well as EBIP adoption stages for individuals and group from diverse institutional contexts.

Additional files

Additional file 1: Table S1. EBIP adoption scale item development (Groccia and Buskist (2011). (DOCX 15 kb)

Additional file 2: Table S2. Demographic outcomes. (DOCX 14 kb)

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Calculus Reform - Increasing STEM Retention and Post-Requisite Course Success While Closing the Retention Gap for Women and Underrepresented Minority Students

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Calculus Reform – Increasing STEM Retention and Post-Requisite Course Success While Closing the Retention Gap for Women and Underrepresented Minority Students

Abstract

Boise State University (BSU) implemented an across-the-board reform of calculus instruction during the 2014 calendar year. The details of the reform, described elsewhere (Bullock, 2015), (Bullock 2016), involve both pedagogical and curricular reform. Gains from the project have included a jump in Calculus I pass rate, greater student engagement, greater instructor satisfaction, a shift toward active learning pedagogies, and the emergence of a strong collaborative teaching community. This paper examines the effects of the reform on student retention. Since the curricular reform involved pruning some content and altering course outcomes, which could conceivably have negative downstream impacts, we report on student success in post-requisite mathematics and engineering coursework.

To explore the effects of the Calculus reform on retention we focused on whether or not students are retained at the university immediately subsequent to the year in which they encounter Calculus I. We divided 3002 student records into two groups: those who encountered the new version of Calculus and those who had the traditional experience. We then compared retention rates for the two groups. We found that the new Calculus course improved retention (relative to the old) by 3.4 percentage points; a modest, but statistically significant ($p = 0.020$) result. University retention rates for women, under-represented minorities (URM), and Pell-eligible students were also computed. All three subgroups showed gains, with URM leading with 6.3 percentage points of improved retention ($p = 0.107$)

We then considered retention within STEM as a measure of how the Calculus reform influenced students. For the same groups of students, we computed the rate at which STEM majors were retained in STEM. Once again we found a modest overall gain of 3.3 percentage points ($p = .078$). We found strong effects on women and underrepresented minorities (URM). The new Calculus course improved retention for both of these groups by **more than 9 percentage points**, a large effect. At this university, under the old Calculus, women used to lag men in STEM retention by about 8 percentage points. After the Calculus reform this gap nearly vanished, shrinking to 0.5 percentage points. Under the old Calculus, STEM retention of URM students used to lag that of non-URM. After the Calculus reform the gap flipped, so that underrepresented minority students are now retained in STEM at *higher* rates than non-URM.

As a final result we examined student success in courses that typically follow Calculus I. Here the metric is pass rate, and we compared pass rates between the students who took the new Calculus against those who took the old. For additional comparison we also included students who transferred into post-Calculus course work. Once again the reformed Calculus course led to better results.

1.0 Introduction

The department of mathematics recognized a strong need to completely overhaul the instruction of Calculus at Boise State University (BSU). This need resulted from rapid growth in STEM enrollment that occurred which exacerbated underlying weaknesses in the calculus sequence. These weaknesses included first, a lack of alignment of content, despite the presence of a guiding master syllabus and common textbook; second, a lack of alignment concerning assessment, resulting in wide variations in pass rate between instructors of different sections of the same course (Bullock, 2015) and third, very low pass rates – for example, the average pass rate in 2005-6 was 51% (Callahan, 2009).

Transformational curriculum change requires a wide degree of faculty buy-in. The record of how our mathematics faculty engaged in the process is described elsewhere (Bullock, 2015); it was a process that was intrinsically motivated, it had funding that was used to create faculty learning communities that met across a year, and it was phased in mostly across the spring and fall semesters of 2014.

Gains from the project that have previously been reported include pass rate gains that range from 8 to 10%, increased satisfaction by instructors, students and clients, and a shorter prerequisite chain – students may enroll in trigonometry as a co-requisite. While previous work has examined student preparation for Calculus II and shown that the "reformed" Calculus I provides suitable preparation, we have not previously examined student retention. Nor have we examined student performance in post-requisite coursework beyond Calculus II. In this paper we track and report on performance in post-requisite coursework, including post-requisite coursework in Dynamics, Fluids, Calculus III and Differential Equations.

2.0 Background and Experimental Methods

2.1 Pedagogical Approach

The overhauled, or "reformed" Calculus I course (R-Calc) has significant pedagogical differences relative to how it had generally been taught prior to the overhaul (N-Calc). R-Calc devotes a majority of class time to students working in small groups on assignments that were designed along learning cycle principles to target one or two specific learning goals. In-class work is facilitated by the lead instructor and a peer learning assistant. Developing these in-class assignments was facilitated by organizing and holding year-long faculty learning communities (Bullock, 2015).

Whenever possible, students work with data sets and/or continuous models selected from actual physical, biological, financial or other applied models, using notation, language and conventions of the disciplines from which the models are taken. All content is accessible from an intuitive or practical viewpoint, resulting in less abstraction relative to what had been previously taught in N-Calc.

2.2 Experimental Methods

The primary goal of this study/paper is to measure the effect of the Calculus reform on student retention. There is a strong presumption that the math "pipeline" has a negative impact on student retention and especially on student retention in STEM majors. We neither question nor investigate that assumption here. Rather, we seek to measure the retention rates for students in the year during which they encounter Calculus I, with the aim of comparing the effects to two different Calculus experiences that they might have encountered.

> Q1: At what rate are students retained at BSU in the Academic year immediately subsequent to their enrollment in Calculus I?
>
> Q2: What, if any, is the difference in BSU retention rate between students who experience R-Calc versus those who experience N-Calc?
>
> Q3: At what rate are STEM majors retained in STEM in the academic year immediately subsequent to their enrollment in Calculus I?
>
> Q4: What, if any, is the difference in STEM retention rate between students who experience R-Calc versus those who experience N-Calc?
>
> Q5: What, if any, effect does R-Calc have on retention rates for URM, Women, Pell-eligible students?
>
> Q6: What, if any, effect does R-Calc have on pass rates in post-requisite courses?

Questions 1 and 3 are answered with descriptive statistics. The remaining questions ask whether a metric applied to students taking R-Calc differs from the same metric applied to students taking N-Calc. In all cases the metric is a simple proportion (pass rate or retention rate) so all of these questions are answered by testing the following null hypothesis:

> H_0: The [pass/retention] rate of students who took R-Calc is no different than the [pass/retention] rate of students who took N-Calc.

The alternative hypothesis is that the rates are different, either larger or smaller, so we will use 2-tailed *z*-tests. For Question 5 the hypotheses and tests are unchanged; we simply restrict the population.

2.2.1 The Retention Study Population

We gather data on students organized into four cohorts by academic year (AY). Academic years are named for the calendar year containing the spring semester and do not include summer terms. For example, AY 2013 consists of Fall 2012 and Spring 2013 semesters. For each AY, we include in the study all students who:

- Were enrolled for classes in the fall term.
- Were enrolled, as of 10th day, in a section of Calculus I in at least one of the two terms.
- <u>We do not include</u> honors sections and concurrent enrollment (high school AP classes).

Concurrent enrollment students are held out for the obvious reason that their retention is not relevant. Honors students are held out because there are no honors sections in R-Calc.

The most recent year for which data is available is AY 2016. We extend the study back 4 years so that we capture a balanced picture of calculus enrollments before the transition to R-Calc. As indicated in Figure 1, R-Calc was phased in during the study time frame. It was still an experimental course in AY 2013. Scale up began in AY 2014. Since then R-Calc has been the dominant form of Calculus I. The four-year time frame thus includes a reasonable amount of time on each side of the transition, and balances the total number of records as nearly as possible between N-Calc (1560 records) and R-Calc (1442 records).

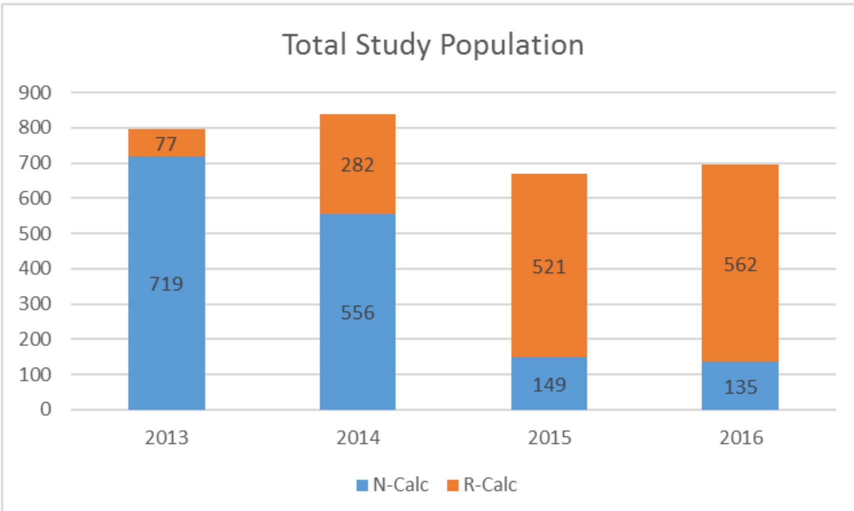

Figure 1: Number of students in N-Calc and R-Calc population

All retention results are presented using this data set aggregated across all four AY's. After checking every data set expanded into a time series we found no confounding trends. Including time series analysis adds little additional information.

2.2.2 Retention Rate Study[1]

Every record in the full data set includes a specified AY. The student in that record is considered "Retained at BSU" if they are enrolled in the fall term of the subsequent AY. However, we do not consider students who graduate during their cohort AY to be either retained or non-retained. Hence

Retention Rate for any subgroup is defined as[2]

= (Number retained at BSU) / (Number of records – Number graduating during cohort AY)

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[1] *Our definitions of the terms "cohort," "retention," and "retention rate" differ from their definitions in State University's official reporting.*

[2] *Note that this differs from traditional definitions of retention used by BSU's official reporting offices. For Federal reporting purposes, retention denominator is the full cohort, but typically only full time, non-transfer students. Further, official reports typically focus on first year retention, which makes graduation effectively impossible. Eliminating these students has a negligible effect on results, since of the 3,002 records in the data set, only 47 represent students who end up in the Graduated category.*

STEM Retention

To study STEM retention, we restrict each cohort or subgroup to those students who have a STEM major declared in the fall term of their cohort AY. For these students, there are four mutually exclusive outcomes that we track in the subsequent AY.

- Graduated = obtained a degree or certificate during the cohort AY.
- Dropped Out = not graduated and not enrolled in subsequent fall term.
- Stem-to-Stem = Retained, not graduated, and has a declared STEM major in the subsequent fall term.
- Stem-to-Non = Retained, not graduated, but does not have a STEM declared major in subsequent fall term.

We then compute three rates for any cohort or subgroup:

STEM Retention Rate

= (Number of Stem-to-Stem) / (Number of STEM Majors – Number Graduated)

Dropout Rate

= (Number Dropped Out) / (Number of STEM Majors – Number Graduated)

Leave STEM Rate

= (Number of Stem-to-Non) / (Number of STEM Majors – Number Graduated)

Students are split into those who encountered R-Calc and those who encountered N-Calc. We consider the former to be a treatment population and the latter a control population. The natural experiment allows us compare their retention rates to determine the effect of calculus transformation.

2.2.3 Course Pairs Study

To examine the effects of the reformed Calculus I curriculum on post-requisite math, physics and engineering courses, we study longitudinally paired courses: the first is always Calculus I and the second is one of:

- Calculus II
- Calculus III
- Differential Equations
- Physics
- Statics
- Dynamics
- Fluids
- Mechanics of Materials

We use a similar population for this study: all students who encountered these courses in the stretch form AY 2013 to AY 2016, but this time we include summer terms. The data set splits into those who used R-Calc as the prerequisite and those who used N-Calc. For these two groups we compare the pass rate in post-requisite courses. Again we have a natural experiment where the comparison of pass rates provides a measure of the impact of the calculus reform.

3.0 Results

The results section is divided into two major categories. First we discuss retention, looking at general retention (retained at BSU), and then we focus on STEM specific retention. In both cases we examine retention for women, URM, and for those who are Pell-eligible. The second major section concerns post-requisite course success, which we examine using course-pair data.

3.1 Retention

3.1.1 General Retention Rates – Retained at BSU

Our first result is a comparison of general retention rates for R-Calc versus N-Calc. The top row of Table 1 shows the retention of all students after R-Calc compared to those who encountered N-Calc. Subsequent rows show the comparative retention rates for women, URM, and Pell-eligible students. Statistical significance (two tailed *z*-test, $p < 0.05$) is highlighted if it occurs.

Table 1: Retention Rate (Retained at BSU)

	N-Calc	R-Calc	Effect Size	*p*-value	*N*
All	78.9%	82.3%	3.4%	0.020	2995
Female	80.8%	84.4%	3.6%	0.181	792
URM	76.6%	82.8%	6.3%	0.107	430
Pell	77.2%	79.6%	2.4%	0.352	1040

The conclusion is that R-Calc shows an improved retention rate (3.4%, $p = 0.020$). The null hypothesis, that R-Calc has no effect on retention is rejected. The effect size is small. This is a modest result and is consistent with the fact that pass rates are better in R-Calc across the same period of time (7.5 points higher for this cohort of students.)

When this picture is restricted to Female students, URM, or Pell-eligible there are similar results. For Female students, the gain is the same as for the full cohort. Pell-eligible students do slightly worse (but still gain in R-Calc). URM gain a decent amount. In all cases the p-values are small but non-significant—we cannot reject the null hypothesis. Nonetheless, the results are still encouraging. In particular, it is clear that retention gains in R-Calc are not obtained as a result of boosting the performance of white males.

3.1.2 STEM Retention Rates – Are STEM majors still in STEM next fall?

While the general retention data is interesting and encouraging, the bigger and perhaps more important story is revealed when examining retention in STEM.

We begin with a look at STEM retention for our full study population, Table 2. The study population drops to 2,352 since we exclude those with no STEM major in their cohort AY and also exclude the tiny number who graduate during their cohort AY. Coincidentally, this results in an almost perfect split into equal numbers for R-Calc and N-Calc. Because it is of some interest where students end up if they are not retained in STEM, we measure three retention outcomes: STEM to STEM (originally STEM major, stayed in STEM); STEM to Non-STEM (originally a STEM major,

Table 2: Retention of STEM Majors -- Headcount

	STEM-to-STEM	STEM-to-Non	Dropped Out	Total
N-Calc	820	111	244	1175
R-Calc	860	108	209	1177
Total	1680	219	453	2352

Table 3: STEM Retention Rates

	N-Calc	R-Calc	Effect Size	p-value
STEM-to-STEM	69.8%	73.1%	3.3%	0.078
STEM-to-Non	9.4%	9.2%	-0.3%	0.821
Dropped Out	20.8%	17.8%	-3.0%	0.064

switched to a Non-STEM major), and Dropped Out (left the university). These are expressed as percentages, along with R-Calc versus N-Calc effects and p-values in Table 3. Things of note in the data from Table 3, depicted in Figure 2 include: The modest retention gain for R-Calc is essentially the same as the general retention gain. However, note that none of the gain comes from keeping students in STEM. All of the gain comes from preventing dropouts. In terms of headcount, it appears that R-Calc keeps about 10 more students per year in STEM, essentially by keeping them in school at all.

3.1.3 STEM Retention Rates – WOMEN and URM

We now consider the retention of women and URM who are STEM majors, where the results begin to show large differences. In Figures 3 and 4 it is clear that the retention gains from R-Calc

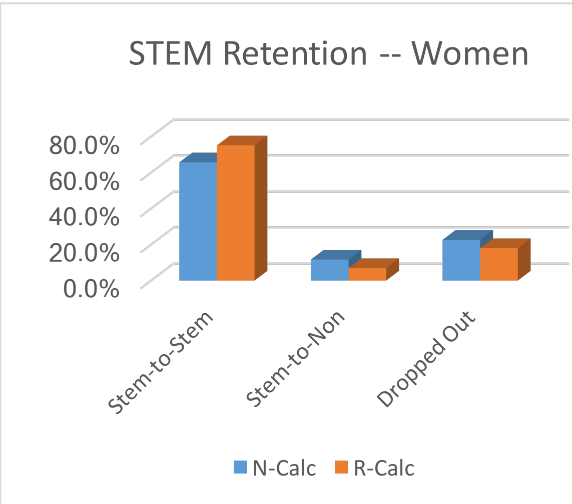

Figure 3: STEM retention - Women

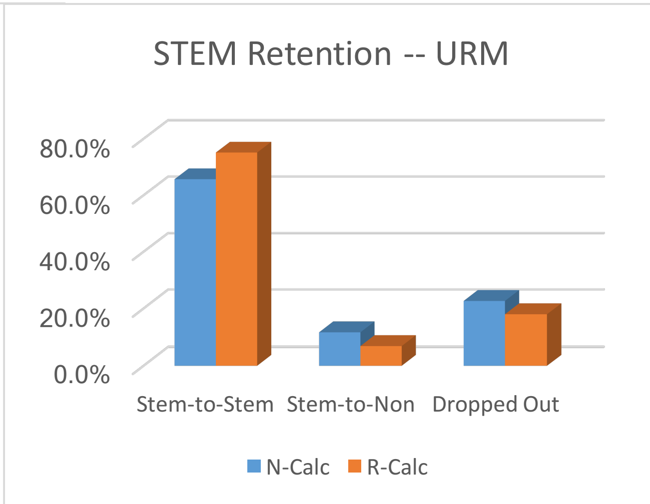

Figure 4: STEM retention - URM

are bigger than for the full cohort. Also, there is a distinct difference in where the students are going. Unlike the general case, there are evident differences in the STEM-to-Non category.

This is more evident in numerical data (Table 4). Both groups show very large gains in retention after R-Calc as compared to N-Calc. For women, the effect size is 9.1% with a p-value of 0.0224. This value is statistically significant; the claim that R-Calc has no effect on retention of Women in STEM at BSU is rejected. For URM, the effect size is 9.4%, with a p-value of 0.0659, which does not quite reach statistical significance.

Table 4: STEM Retention – Women and URM

Women

	N-Calc	R-Calc	Effect Size	p-value
STEM-to-STEM	63.6%	72.7%	9.1%	0.0224
STEM-to-Non	14.6%	10.4%	-4.2%	0.1392
Dropped Out	21.8%	17.0%	-4.9%	0.1482

URM

	N-Calc	R-Calc	Effect Size	p-value
STEM-to-STEM	65.7%	75.1%	9.4%	0.0659
STEM-to-Non	11.7%	6.9%	-4.8%	0.1463
Dropped Out	22.6%	18.0%	-4.6%	0.3066

Moreover, unlike the general case, in both of these groups the retention gain is composed of equal parts "stayed in school", and "stayed in STEM". Given that STEM in particular is prone to retention gaps for these populations, this is an important result.

It is also worth looking at this from the point of view of retention gaps. That is, rather than measuring impact on women, or perhaps comparing to a general baseline, consider the direct comparison of retention rates for women versus men. Table 5 shows our data.

Both men and women benefit from retention gains under R-Calc, but gains for women are so large that an 8% gap is almost entirely eliminated.

A similar picture emerges for URM (Table 5). Here the pre-existing gap was smaller, and there is a complication due to international students. Internationals are retained in STEM at considerably higher rates than any other group. In all prior computations, this has had little to no effect (we checked) because the international students are either not part of any computation, or they are relatively evenly split between N-Calc and R-Calc (163 and 153 respectively, which if anything, gives a boost to N-Calc retention rates.) However, the demographic variable that detects URM takes on three values: URM, Non-URM, and International. So, for measuring gaps between URM and other student groups, it matters whether or not International students are included in the non-URM control group. Table 5 shows the results for both cases. As usual, all groups gain. Here, the gains for URM versus either alternative group are so large that a retention gap actually flips.

Table 5: STEM Retention Gap – Women, URM w/o Intn'l, URM w/Intn'l		
Women v Men		
	N-Calc	**R-Calc**
Female	63.6%	72.7%
Male	71.5%	73.2%
Gap	-7.9%	-0.5%
URM v Non-URM, no International		
	N-Calc	**R-Calc**
URM	65.7%	75.1%
Non-URM, no Intn'l	67.9%	70.5%
Gap	-2.2%	4.6%
URM v Non-URM plusInternational		
	N-Calc	**R-Calc**
URM	65.7%	75.1%
Non-URM and Intn'l	70.3%	72.7%
Gap	-4.6%	2.5%

After R-Calc, URM are retained at *higher rates* than either comparison group.

3.1.4 STEM Retention Rates – Pell-Eligible

Unfortunately, there is no such good news for Pell-eligible students, see Table 6. This group still gains from R-Calc, but the gain is slightly less than the gain for all R-Calc students. Also, like the general case, the gain is entirely from fewer dropouts. There is no additional capture of students departing for other majors.

Table 6: STEM Retention – Pell-Eligible				
	N-Calc	**R-Calc**	**Effect Size**	**p-value**
STEM-to-STEM	68.4%	71.2%	2.8%	0.3885
STEM-to-Non	8.0%	8.1%	0.1%	0.9590
Dropped Out	23.6%	20.8%	-2.9%	0.3236

3.2 Success in post-requisite coursework: Course Pair Data

In this section, we revisit an analytic device (Bullock, 2016). We track student performance in courses that are typically taken subsequent to Calculus I. We then compare their success in these post-requisite courses' follow-on courses, depending on which version of Calculus I they used as the prerequisite. In our previous work we studied only the performance in Calculus II, comparing students who took R-Calc against those who took N-Calc. In this paper, we significantly expand the analysis to include a large set of Math, Engineering and Physics post-courses (section 2.2.3) We also include, for additional comparison, performance in the post-course for the cohort of student who did not take Calculus I at BSU. This is intended as primarily descriptive statistical evidence. However, we include significance testing of the difference in post-course pass rates for the R-Calc and N-Calc groups.

The Effect Size and *p*-value are only for the comparison of R-Calc to N-Calc. We do not analyze transfer students; they are provided only for descriptive comparison.

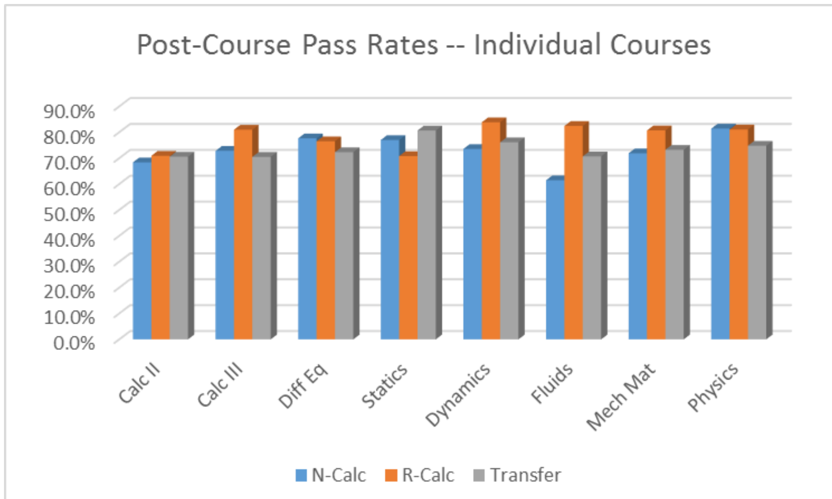

Figure 5: Post-Course Pass Rates

3.2.1 Course-pair data – success in post-requisite courses

<u>Overall</u>

Table 7 and Figure 5 show the pass rate data for the full range of post courses, with students split into N-Calc, R-Calc, and Transfer.[3] Notice that in most courses, R-Calc students outperform N-Calc. Positive effect sizes tend to be larger than negative effects, and have greater statistical significance as denoted by smaller p-values. The only negative effect

Table 7: Post-Course Pass Rates -- Individual Courses

	N-Calc	R-Calc	Transfer	Effect Size	*p*-value	*N*
Calc II	68.4%	70.9%	70.6%	2.5%	0.247	1,983
Calc III	72.8%	81.0%	70.5%	8.1%	0.005	1,055
Diff Eq	77.7%	76.5%	72.3%	-1.2%	0.719	953
Statics	77.0%	70.8%	80.7%	-6.2%	0.104	693
Dynamics	73.5%	83.9%	76.1%	10.3%	0.038	391
Fluids	61.5%	82.5%	70.7%	21.0%	0.001	**352**
Mech Mat	71.9%	80.7%	73.2%	8.8%	0.098	**420**
Physics	81.4%	81.1%	74.8%	-0.3%	0.883	1,462

[3] Footnote: There is a small chance that students in "Transfer" did not actually transfer the prerequisite Calculus I course to BSU, but the number of such instances would be very small.

that calls for attention is perhaps the effect in Statics. This is unsurprising, since statics relies very heavily on good preparation in trigonometry and vector analysis, which are not treated in Calculus I.

The post-course analysis technique was created to study the effect of R-Calc on subsequent Math courses, since there were concerns that content changes in R-Calc could have negative effects on later Math courses. Table 8 presents the result of aggregating all the Math post-courses, and some other aggregates.

There are positive effects in Math and Engineering, in the aggregate. While these are not quite statistically significant, they are still an encouraging result. When all courses are aggregated, the positive effect is statistically significant: We may conclude that R-Calc does a better job of preparing students for subsequent course work, although the effect is fairly small.

Table 8: Post-Course Pass Rates -- Course Types

	N-Calc	R-Calc	Transfer	Effect	*p*-value	*N*
Math	71.8%	74.4%	71.2%	2.6%	0.094	3,991
Engr	72.0%	76.7%	75.5%	4.7%	0.058	1,856
Phys	81.4%	81.1%	74.8%	-0.3%	0.883	1,462
ALL	76.6%	78.7%	75.9%	2.1%	0.040	8,765

3.2.2 Course-Pair Data – Success in post-requisite courses, Female and URM

Figure 6 and Table 9 show the effects on subgroups of female, URM, and Pell-eligible students, with all students included for comparison.

All three groups experience a boost from R-Calc. Consistent with the findings on retention, we see that the gains for URM and women are visibly larger than the general gain for all students. Pell-eligible students, however, actually get less value (but still gain) from R-Calc. Also, interestingly, all three groups perform

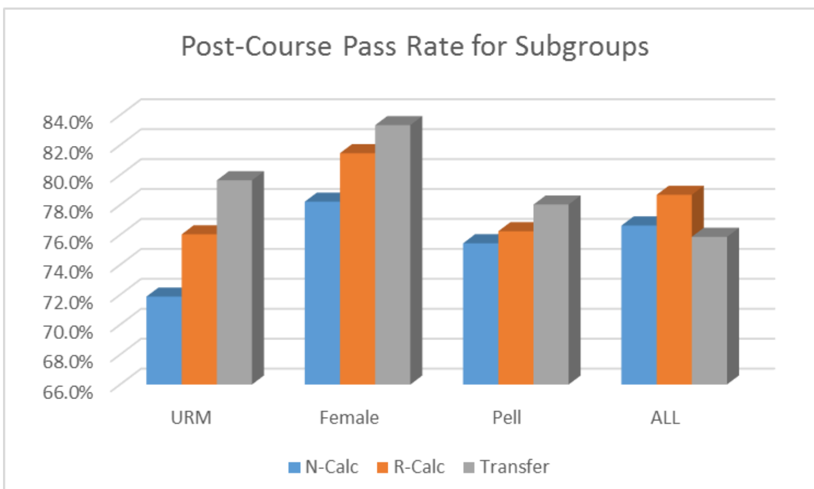

Figure 6: Post-Course Pass Rates for URM, Female, Pell

Table 9: Post-Course Pass Rate for Subgroups

	N-Calc	**R-Calc**	**Transfer**	**Effect**	**p-value**	**N**
URM	71.9%	76.0%	79.6%	4.2%	0.158	1,108
Female	78.2%	81.5%	83.3%	3.2%	0.162	1,496
Pell	75.4%	76.2%	78.0%	0.8%	0.671	2,709
ALL	76.6%	78.7%	75.9%	2.1%	0.040	8,765

much better than transfer students. Both this and the Pell data are revealing and must be considered important targets for future reforms.

4.0 Discussion

The improved retention and performance in certain post-requisite courses that we have seen as a result of R-Calc is now discussed, and is likely influenced by (1) improved grades in the course, (2) increased relevancy of content, (3) active learning, (4) increased self-efficacy, and (5) increased sense of belonging. Other factors may also be relevant.

Improved Grades: The literature on first-year academic success as measured by grade point average shows a clear association with retention; for example, see Whalen (2010) and Herzog (2005). Herzog (2005) also found that after GPA, the strongest predictor of retention was performance in first-year mathematics courses. The role of first course grade in mathematics was also studied by Callahan (2017), who showed that earning a grade of "A" or "B" in mathematics doubled the likelihood of persistence, and that grades earned are more important than the actual level of mathematics course (whether Calculus, Precalculus or College Algebra) taken in the students' first year. Thus, improved retention is expected simply based on the fact that student grades are higher in the R-Calc relative to N-Calc courses. The underlying rationale behind why improved grades increase retention is that students who earn higher grades have a higher sense of self-efficacy. Students with a higher math self-efficacy are more likely to view difficult tasks as something to be mastered than something to be avoided (Bandura, 1977).

Increased Relevancy of Content: The strong focus in R-Calc on providing actual examples from physics, biology, finance or other applied models for students to work with to solve calculus problems is likely to have contributed to increasing student engagement with the mathematics they were learning. Students who don't find value in mathematics learning are likely to disengage; for example, see Allexsaht-Snider and Hart (2001). Ames (1992) reviews key characteristics of tasks that are likely to foster a willingness in students to put forth effort and become actively engaged in learning. These characteristics include tasks that involve variety and diversity, tasks that provide meaningfulness of content, tasks that students perceive they can accomplish with reasonable effort, and tasks that structure student engagement. The pedagogical approaches used in R-Calc very much align with these underlying theoretical principles. Other researchers have also focused effort on improving student learning by means of adding an applications-focus into calculus. For example, Young et al. (2011) developed two, one-credit applications "add-on" courses for students to take alongside "normal" calculus. Their work showed that while the first course, taken with Calculus I, did not have a statistically significant effect, the second 1-credit course, taken with Calculus II, did. Relative to the improved results seen in post-requisite coursework, it is natural that students might more easily recognize when to use calculus in post-requisite courses if they have already seen such examples when they took R-Calc.

Active Learning: It is well known that active learning increases student performance in science, engineering and mathematics. A metaanalysis by Freeman, et al. (2013) of 225 studies, of which 29 were focused on mathematics shows that active learning improves average examination scores by 6%, and students in classes with traditional lecturing are 1.5 times more likely to fail than students in classes with active learning. The metaanalysis shows, based on 15 different independent studies, that a shift to active learning shows an average of approximately 8% decrease in failure rate in the discipline of Mathematics. In Freeman's PNAS report, active learning was defined as learning that "engages students in the process of learning through activities and/or discussion in class, as opposed to passively listening to an expert. The pedagogical shifts in this work (R-Calc) were all shifts away from "exposition-centered methods" (lecturing) toward constructivist approaches (active learning).

Self-efficacy: Self-efficacy is a critical element that is strongly associated with the literature on retention in STEM disciplines. Bandura's research has shown that high perceived self-efficacy leads students to view difficult tasks as something to be mastered, rather than something to be avoided. (Bandura, 1977). A recent article by Ellis, et al. (2016) shows that women are 1.5 times more likely to leave the STEM pipeline after calculus compared to men and identifies lack of mathematical confidence as a potential culprit. Their paper shows that women start and end the term with significantly lower confidence than men. The approaches taken in our work, which closed the gap in persistence between men and women in STEM, thus may have improved student self-efficacy, although this was not measured.

Belongingness: Dasgupta (2011) describes the importance of the need to belong, and its influence on self-concept. In this work, Dasgupta summarizes how people's behavior and choices are driven by the need to belong and be accepted by others within a community of peers. The need to belong is particularly strong under adversity or stress – and thus "is likely to play an important role in the lives of individuals who belong to historically disadvantaged groups and find themselves in adverse situations where their group is numerically scarce and their abilities cast in doubt, such as high-stakes academic" environments. Dasgupta reviews relevant literature about the imposter syndrome, and more, and goes on to suggest that collectively, "*the experience of being in a numeric minority in high-stakes academic environments where stereotypes are in the air may reduce individuals' self-efficacy or confidence in their own ability, especially in the face of difficulty, even if their actual performance is objectively the same as majority-group members*."

The focus of Dasgupta's 2011 article is to highlight two factors that are likely to contribute to increasing social belonging and to build resilience against stereotypes. These factors are, (1) exposure to ingroup experts, and (2) exposure to peers in high-achievement contexts. In the context of the increased retention of women and URM as a result of R-Calc, "ingroup experts" might refer to a woman enrolled in R-Calc being exposed to other women in R-Calc during the course of the semester in group work. Dasgupta's "stereotype inoculation" model proposes that exposure to ingroup experts and peers in high-stakes achievement contexts functions as a social vaccine that helps inoculate individuals against self-doubt. In this work, we have not focused any effort to date on analyzing the group compositions, which are not regulated, but which rather self-aggregate according to where students place themselves in the classroom. Future work could examine the degree to which students align themselves with ingroup peers.

Belongingness – feeling as though one belongs – cannot be emphasized enough relative to the results we have seen. As summarized by Herzig, 2005, building a sense of "belongingness in mathematics" has been proposed as a critical feature of an equitable K-12 education, where "belongingness refers to the extent to which each student senses that she or he belongs as an important and active participant in all aspects of the learning process." Allexsaht-Snider and Hart (2001) also discuss belongingness – the extent to which each student senses that she or he belongs. The sense that each student feels as though she or he belongs in calculus is critical relative to future decisions made by the student to remain in STEM.

5.0 Summary

Our reform of Calculus has positively affected retention, at least in the year that students encounter Calculus I. Overall retention improved by about 4 percentage points, with gains for women, URM and Pell-eligible students all similar to the general case. Retention in STEM was improved, in general, by about the same amount. We noted especially large gains in STEM retention for women and URM (exceeding 9%). These increases closed the gap in retention of men versus women at this university and resulted in a retention rate for URM that exceeded non-URM students by 4.6%. We attribute these results primarily to the pedagogical shifts that have taken place relative to how the course is taught. These shifts include (1) collaborative work that occurs each day in class, and (2) a strategy of being explicit about the relevancy of calculus by using actual physical situations, data and units in homework problems, in-class work and exams.

Relative to post-requisite coursework, students who experience R-Calc versus N-Calc as the prerequisite to later Math and Engineering course work receive a small, but statistically significant boost in pass rate. The effect is larger in engineering courses, as would be expected given the curriculum in R-Calc. As we have seen elsewhere in the Calculus Transformation project, the gains are even larger for women and URM, but Pell-eligible students are not as well served.

6.0 Acknowledgments

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Enhancing STEM Majors' College Trigonometry Learning through Building Mobile Apps

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Abstract

This study aimed to strengthen students' trigonometry knowledge and skills by providing authentic contexts for knowledge application. An innovative approach was applied to guide students to integrate trigonometry in programming mobile apps and in developing learning content. Three research questions guided this study: 1) How do students apply trigonometry concepts in developing their mobile apps? 2) How do students perceive the experiences of applying trigonometry concepts in developing their mobile apps? 3) What motivates students in a Trig-APPS course? We found students were overwhelmingly positive about their experiences of reviewing, revisiting, and utilizing trigonometry through programming mobile apps. The innovative approach is promising in motivating students to learn foundational mathematics while solving design problems.

Objectives

Many students enter colleges with a need to strengthen their foundational mathematics, such as trigonometry. In addition, college students in STEM majors (science, technology, engineering, and mathematics) often struggle to apply trigonometry concepts in post-requisite courses. For example, in statics, a sophomore engineering course, even though many students are able to find the sine or cosine of a right triangle oriented in any of the four quadrants as taught in mathematics, they struggle with correctly applying the sine function if the triangle is presented in a different orientation. Similarly, students have little number sense when applying the sine, cosine, or tangent functions, and many cannot recognize an obviously wrong result generated by a calculator. Further, students have very little sense of how to resolve a vector into components. In this study, we aimed to strengthen students' knowledge and skill development in trigonometry by providing authentic contexts for knowledge application with mobile app development.

Creating mobile apps can help motivate learners because they can create their own applications that work on the mobile devices that are important in their lives (Morelli et al., 2011). It also can help build their confidence in programming and creative problem solving, especially when a good visual programming tool (e.g., App Inventor) is used (Hsu, Rice, & Dawley, 2012; Hsu & Ching, 2013; Wolber, 2011). Hsu and Ching (2013) found students had strong feelings of empowerment and success when making mobile apps because they could unleash their creativity and turn their ideas into something real and tangible. Students also enjoyed testing peers' apps—this process helped others and also provided inspiration for their own app development.

Our study piloted an innovative approach to mathematics learning. It engaged college STEM majors studying trigonometry by asking them to develop mobile apps for learning/reviewing trigonometry. These apps were expected to help in two ways by: (1) meeting immediate trigonometry course needs. In this constructionist approach, students who simultaneously act as both learners as well as app developers will be more engaged and gain improved learning outcomes from trigonometry instruction; (2) serving future mathematics and engineering course needs. Students who developed the apps can leverage them as interactive study aids to refresh their knowledge, should they advance to the calculus and engineering statics courses that follow for STEM majors one to two semesters later.

Two potential student benefits were expected to result from this approach. First, to develop apps, trigonometry students need to revisit and apply the knowledge they have learned in class. Doing so provides mathematic content practice and review opportunities. Second, developing their own mobile apps empowers and motivates students to take ownership of their learning. Producing thoughtful app designs also helps them see that mathematics is relevant to authentic projects and can have real-world impact. These benefits are important for students and their ability to transfer their understanding of trigonometry to other STEM contexts later in their academic programs and careers.

The following research questions (RQ's) guide this study:

1. How do students apply trigonometry concepts in developing their mobile apps?
2. How do student perceive the experiences of applying trigonometry concepts in developing their mobile apps?
3. What motivates students in a Trig-APPS course?

Theoretical Framework

Artifact Construction

Artifact construction is a well-known learning approach; it engages students in their learning by having them create tangible artifacts. Students can apply content knowledge and skills through artifact construction and collaboration (Harel & Papert, 1991). When students construct artifacts, they also construct ideas simultaneously (Noss & Holyes, 2006). During the construction process, students can and need to iteratively refine artifacts and ideas to achieve the learning goals and solve design problems. Artifacts can be physical (e.g., furniture, robots, clothes) or digital (e.g., graphics, computer programs, or mobile applications). A study found that after a semester in mobile app development, students showed significant improvement in their ability to design comprehensive solutions to a given problem (Dekhane, Xu, & Tsoi, 2013).

Collaboration

Collaboration initiates more complex and iterative refinement of ideas and mental models (Harel & Papert, 1991) than artifact construction pursued alone. Collaboration in small teams or a large community leads to feedback that requires learners who build artifacts to critically examine their working products and ideas. Learning through collaboration includes a wide spectrum of methods that can take many different forms, such as cooperative learning, collaborative learning, and collective learning. Each emphasizes different levels and ways of learning by the group and community (Dillenbourg, 1999), which all lead to collaborative knowledge construction (Barab, Hay, Barnett, & Squire, 2001) and varying ways for participants to interact during the process (Hsu, Ching, & Grabowski, 2014). Noss and Hoyles (2006) discussed how students can explore mathematics through construction and collaboration. In one of the projects, students worked together to apply their mathematics knowledge to program animated robots to achieve desired sequence and actions. In a subsequent project, Noss and Hoyles developed a system and specifically built in a mechanism (asynchronous discussion) for learners to communicate their emerging understanding of mathematics and share their developing mental models regarding mathematics knowledge they have learned. Considering the class size, benefits and efficiency of collaboration, and multiple knowledge/skill sets required in the Trig-APPS course (discussed below), the students in our study were asked to work collaboratively in pairs on app development.

Methods

A two-credit co-curricular course was created for students who enrolled in college level trigonometry courses in a northwestern public university in the United States in spring 2016. Twelve students from six STEM majors enrolled in the class participated in this study. Among them, 10 were males and two were female, with an average of age at 22.8 years old. Five were first-year students and seven were in Computer Science.

This course requires 2-hour meeting time across 15 weeks. Students learned about the foundations of App Inventor for mobile app programming in the first 4 weeks by creating individual apps while working through assigned tutorials. In the following three weeks, students were introduced to three example apps incorporating or demonstrating trigonometry concepts. Students worked in pairs on debugging, customizing, and improving the provided source codes of these apps. For the rest of the semester, students worked in pairs to conceptualize mobile apps that applied trigonometry concepts, created app proposals, and built the actual apps they proposed. Students were provided with online discussion forums to communicate and collaborate throughout the semester, in addition to interacting face-to-face.

Data Sources

Content analysis on the app development process and projects was conducted to examine students' application of trigonometry concepts in designing and developing mobile apps. Data sources included students' reflective journals and developed mobile apps, and the codes of student-developed apps. Student perception of the learning experience and group process of designing apps was investigated through interviews with 10 students. The innovative Trig-APPS curriculum aims to excite and motivate students in learning trigonometry through authentic app development activities. We slightly modified a validated survey, Instructional Materials Motivational Survey (Keller, 2010) to measure the motivational characteristics of the instructional materials through 36 Likert-scale items. We also added several open-ended questions to obtain input from students to help improve the design of the curriculum.

Results

RQ1: Students' application of trigonometry concepts in developing their mobile apps

A total of 5 complete team apps were developed by 10 students. One of the teams did not complete a final project app due to one student's attendance and participation issues. The types of mobile apps the students developed included some combination of the following: quiz, game, and review guide.

Trigonometry was applied in coding the mobile apps and also in the apps' content. The concepts included degrees and radians conversion, unit circle, and trigonometric functions, which were covered in the two apps we discuss below.

For example, one team created a quiz+review app that allows users to take a quiz regarding different angles on a unit circle. The trigonometric functions were used in the codes to make the app draw different graphs based on the pre-assigned angles in the question bank. The app presented, along with the angle, the three representations (coordinate, degree, and radian) of the tested angle for review purpose, after the app users have made their selection. The team also developed a review guide that provided clear and concise text and graphics related to major concepts of a unit circle.

Another app took advantage of game mechanics and the orientation sensor in mobile devices to create a quiz/game. It required users to tilt their devices left or right for a spinning boomerang image from top to collide with one of two choices at the bottom. It helped provide a sense of excitement that encouraged game players to quickly make a correct choice on degree-radian conversion. The team also incorporated random assignment in their codes to display various lengths of triangle sides to generate a series of questions on trigonometry functions.

RQ2: Student perception of the experience of applying trigonometry concepts in developing their mobile apps

Student responses were overwhelmingly positive. Many students commented on how app making fostered their learning of trigonometry. A sample comment reads, "The app we created made me think about the trig conversion from radians to degrees so much while testing it, I'll probably never forget it." Another student stated "The fact that I had to research concepts within trigonometry and create games/quizzes for this material allowed me to understand trigonometry at a more conceptual level. This made going to math class much more enjoyable and relevant to the real world."

Some students commented on their problem solving skills and practice through the app creation process. A sample comment reads, "I think that the process of identifying a problem and creating a solution is an important lesson to learn." Some students commented on the overall course design and instruction. One student indicated that "…the instructor is amazing at keeping the attention of people in a subject like CS. Some of the people in the class weren't even CS students, but everyone seemed motivated and looked like they were having a blast every class."

RQ3: Student motivation in the Trig-APPS course

100% of the students rated "very true" or "mostly true" for the following statements:

- The content of this course will be useful to me.
- I really enjoyed working on projects for this course.
- I enjoyed this course so much that I would like to know more about this topic.
- As I worked on this course, I was confident that I could learn the content.
- Completing the exercises in this course gave me a satisfying feeling of accomplishment.

90% of the students rated "very true" or "mostly true" for the following statements:

- I could relate to the content of this course to things I have seen, done or thought about in my own life.
- This course has things that stimulated my curiosity.
- The content of this course is relevant to my interests.

Scientific/Scholarly Significance

Based on our initial findings, we found this innovative approach of creating and using a co-curricular mobile app development course to enhance trigonometry learning promising in motivating and engaging students. The students were overwhelmingly positive about their experiences of reviewing, revisiting, and utilizing trigonometry through programming mobile apps. Through the curriculum, they were able to witness and experience a real-world implementation of their trigonometry knowledge. They also created tangible products that they can relate to and that have potential impact on learning foundational subjects in STEM. The mobile apps also created a need for the students to enhance and solidify their learning of trigonometry because the integration of their knowledge needs to be 100% accurate for the apps to function.

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The Crux: Promoting Success in Calculus II

Abstract

In the 2013-14 school year, Boise State University (BSU) launched a major overhaul of Calculus I. The details of the reform, described elsewhere, involved both pedagogical and curricular changes. In subsequent years, we developed several assessment tools to measure the effects of the project on students' grades and retention. The toolkit includes: (1) pass rate and GPA in Calculus I, (2) longitudinal analysis of pass rates and GPA in subsequent courses, (3) impact of Calculus I on retention in STEM and retention at BSU, (4) all of the above comparing students in reformed Calculus vs traditional Calculus, (5) all of the above for underrepresented minorities, women, or other demographic subsets. While these tools were originally developed to study the Calculus I project, they are available for studying the effects of other courses on student academic performance and retention.

In this paper, we briefly describe a rebuild of Calculus II, overhauled in the 2015-16 school year following the same general plan as was used for Calculus I. We then present the results of applying the full toolkit to the new Calculus II course. Pass rate and GPA improvements in Calculus II were evident immediately after scale up in the spring of 2016. Sufficient time has now passed so that we can apply the full set of assessment tools built for Calculus I to measure the effectiveness of the Calculus II transformation on academic performance in post-requisite coursework and on student retention in STEM.

1.0 Introduction

The grade earned in mathematics courses is critical when considering student retention in engineering and in STEM majors. For example, the work by Budny, et al. (1998) shows that the grade earned in the first semester course of mathematics (whether Precalculus, Calculus I, or Calculus II) is a strong predictor of retention in engineering. Callahan & Belcheir (2017) showed that of the two – level of first semester mathematics, or grade earned – that the grade earned in the mathematics class is a better predictor of student retention in STEM one year later than the level of mathematics course taken. Success in the first year of mathematics in engineering is paramount.

Because of this, Boise State University is five years into an overhaul of the entry-year calculus sequence. Implementation of the initial, Calculus I, phase and early results were reported in Bullock, Callahan, Shadle (2015). This included pass rate gains that range from 8 to 10%, increased satisfaction by instructors, students and clients, and more. An examination of how students who have taken the overhauled Calculus I have fared in post-requisite coursework has been investigated in Bullock, Johnson, Callahan (2016) and Bullock, Callahan, Cullers (2017). The latter paper (2017) also presented the effects of the Calculus I project on retention.

As a natural next step in continuous improvement, the mathematics department turned to Calculus II as their next focal area for reform. In this paper, we report on what this reform of Calculus II consists of, and also track and report on student grade performance in the course as well as in post-requisite coursework including Dynamics, Fluids, Calculus III, and Differential Equations.

2.0 Background and Methods

2.1 Calculus II Redesign

The redesign of Calculus II followed the general plan that was used to redesign Calculus I (Bullock, et. al. 2015), with four major components of change:

1. Substantial changes to content, seeking to maximize relevance to future coursework.
2. Voluntary opt-in to a “master course” model.
3. Redesign of each daily lesson to support active learning pedagogies.
4. Formation of a community of practice to deliver the course.

The content of a typical second semester Calculus course usually includes: techniques of integration (symbolic with no machine assistance, plus some numerical integration), applications of integration (physical applications and solids of revolution), sequences and series (emphasizing proofs of convergence and culminating in Taylor series), and a smorgasbord of parametric functions, polar coordinates, conics, and differential equations. We rebuilt the content to focus tightly on four units:

- 4 weeks of symbolic integration. Restricted to a minimal list of types vetted by stakeholders.
- 4 weeks of sequences and series. No proofs. Qualitative understanding of convergence. Quantitative speed of convergence. Taylor polynomials as applied approximations.

- 4 weeks of applications of integration. Heavy emphasis on student understanding and communication of the underlying geometry (vs. formalism). Applications to loads, forces, moments, centroids, work, and energy.
- 3 weeks of 2-D parametric and vector valued functions. Mimicking the notation and language of the 3-D material that begins Calculus III at Boise State.

Previously, redesigned Calculus II was delivered as a collection of independent single sections with little to no governance beyond a common text and a suggestion of content coverage (the traditional list above). We replaced this with a master course specifying all homework, quizzes, exams, daily lesson order and content, and overall grade weighting. The master course was copied to each individual section, with the understanding that no instructor would be coerced by the department. Voluntary opt-in meant adopting the master course structure. We have had approximately 95% opt-in since the launch of the project.

Opt-in does not require any particular pedagogical approach. However, each homework set in the master course is designed to be best delivered in an active learning style, with most class time devoted to students progressing through carefully scaffolded exercises with guidance from the instructor and a learning assistant. All instructors who have opted in have also adopted some form of active learning.

The group of instructors in any given semester works as a team to deliver the course – collaborating on quizzes and exams, meeting regularly to discuss classroom practice and course delivery logistics. They are supported by a team of more senior instructors dedicated to the continued operation of the restructured Calculus I and II courses. The result is a strong community of practice.

Consensus and buy-in was developed over the 2015-16 scale up period by forming a Faculty Learning Community (FLC) that met for a full year (e.g. see Cox and Richlin, 2004). In the fall term of 2015, instructors debated and agreed upon lesson objectives and content. During the spring of 2016, all FLC members who had been assigned Calculus II taught their sections using the agreed upon curriculum and content. Weekly meetings during the spring semester served to further build out content, to discuss real-time issues in course delivery, and to agree on common weekly quizzes and midterm exams. These weekly meetings formed the basis for the ongoing community of practice that has continued the project. The result is a closely coordinated, multi-section Calculus II course with common content, assessments, and exams.

2.2 Methods

The toolkit developed to assess the effects of Calculus I transformation includes descriptive statistics:

- Time series of aggregate pass rate across all of Calculus I. (Bullock, et. al. 2015, 2016)
- Before/After comparisons of pass rates for individual instructors who taught both the old Calculus I and the reformed Calculus I. (Bullock, et. al. 2015)
- Pass rates in courses subsequent to Calculus I, with comparisons between students who reached the subsequent course via old Calculus I, reformed Calculus I, or by transfer credit. (Bullock, et. al. 2017)

- Retention of students, both in the sense of “retained at Boise State University” and “retained in STEM”, across the year in which they first encounter Calculus I, again with comparisons between old and reformed Calculus I. Effects on retention were studied for subpopulations of female, Pell eligible, and underrepresented minority students. Bullock, et. al. 2017).

Features of the course transformation process allowed us to identify a treatment group (those who took reformed Calculus I) and a control group (those who took the old Calculus I). The two groups co-existed across a time span that extended to either side of the year of course development and implementation. Before implementation, some students took the reformed curriculum as it was in preliminary development and testing, and after implementation, some instructors opted out of the project. The result is a natural experiment with two roughly equal sized study populations taking different versions of Calculus I in the same time frame. We used this opportunity to conduct the following statistically rigorous assessments:

- Comparison of Calculus I pass rates for treatment vs. control. Significance tests were applied to the research question: “Does treatment improve Calculus I pass rate?” Control variables were used to test whether the two groups had different levels of academic preparation or ability. (Bullock, et. al. 2016)
- Comparison of Calculus I average GPA for treatment vs. control. Significance tests were applied to the research question: “Does treatment improve Calculus I GPA?” Control variables were used to test whether the two groups had different levels of academic preparation or ability. (Bullock, et. al. 2016)
- Comparison of Calculus II pass rates and GPA for treatment vs. control. Note that Calculus II in this context is not the treatment course. It is a testing ground for the results of reforming Calculus I. Significance tests were applied to the research question: “Does the treatment (reformed Calculus I) have any detrimental effects on Calculus II?” Control variables were used to test whether the two groups had different levels of academic preparation or ability. (Bullock, et. al. 2016)
- Comparison of some (not all) of the various retention metrics. Significance tests were applied to the research question: “Does treatment improve retention?” Control variables were not used, so this is less rigorous. The significance testing here is perhaps best thought of as a refinement of the descriptive statistics on retention. (Bullock, et. al. 2017)

In the subsequent sections of this paper we will, for each assessment instrument or group above, present the results of applying the same tools or tests to measure the effects of transforming Calculus II. In each case, we will compare or contrast the findings with what we learned about Calculus I across the last three years.

3.0 Results – Descriptive Statistics

3.1 Aggregate Pass Rate

Figure 1 shows the pass rate for all of Calculus II in each non-summer term for the last decade (line graph). The bars graph shows total enrollment. Color coding indicates students in old Calculus II (blue) versus new Calculus II (orange). The implementation term is visible in the shift from mostly blue to mostly orange bars. Orange before implementation is due to small development and testing sections. Blue after implementation is due to instructors opting out of the coordinated course design. Despite some volatility and a potential trend leading up to transition, there is a fairly clear jump in pass rate.

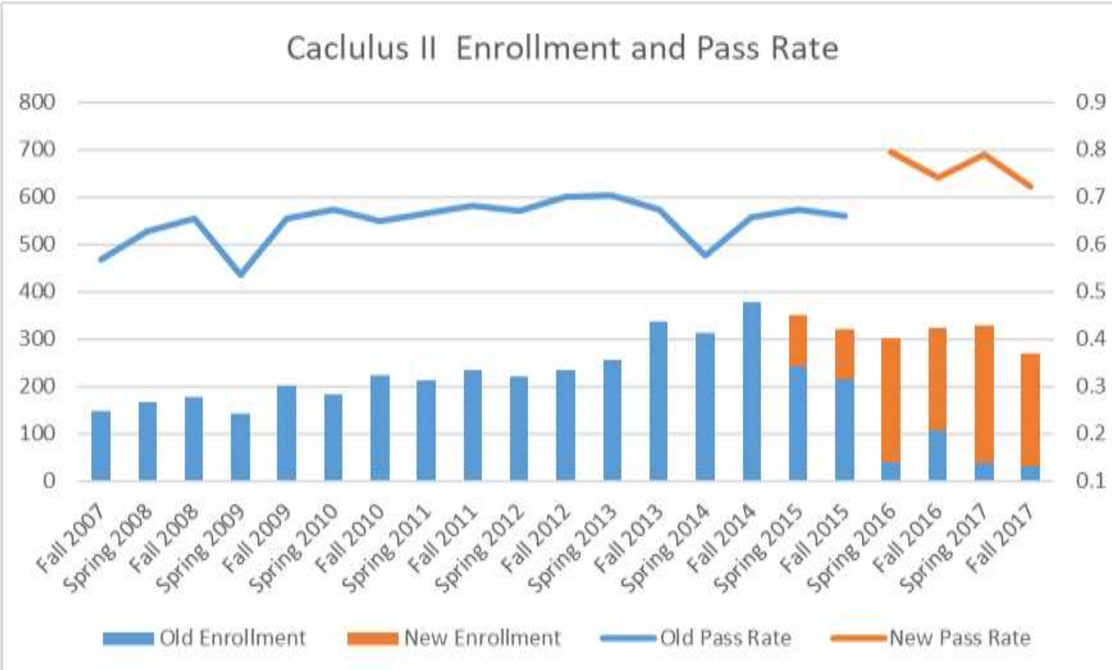


Figure 1: Calculus II enrollment and pass rate

For comparison, Figure 2 shows the corresponding decade of Calculus I.

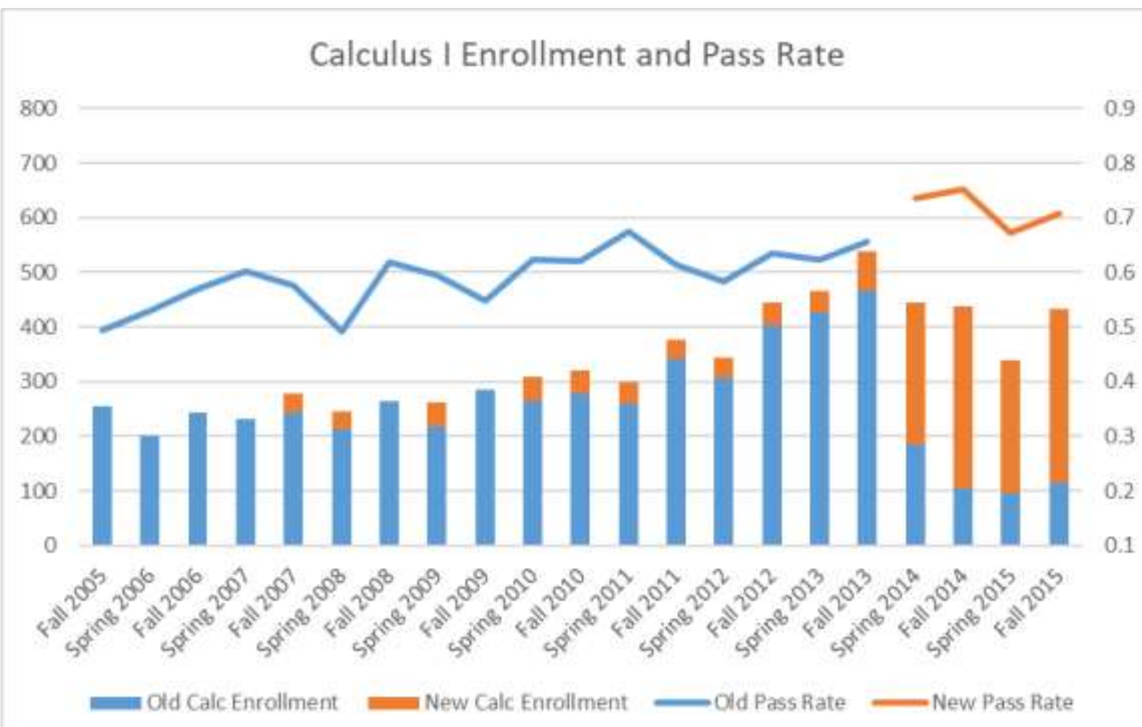


Figure 2: Calculus I enrollment and pass rate

In both graphs we have chosen to present one decade of data, with the cut off exactly 2 years after the course was transformed. Both graphs show that after transition, the bulk of calculus was taught using the reformed curriculum, and pass rates increased.

3.2 Before and After Pass Rate

In the Calculus I transition, fortuitously, we had a group of six instructors who taught Calculus I both before and after the reform, allowing us to compare pass rates while keeping instructors constant. In the Calculus II transition we, coincidentally, ended up with six instructors who had taught both the old and new Calculus II. Figures 3 and 4 show the individual pass rates, per instructor, for both Calculus I (Bullock, et. al., 2015) and Calculus II.

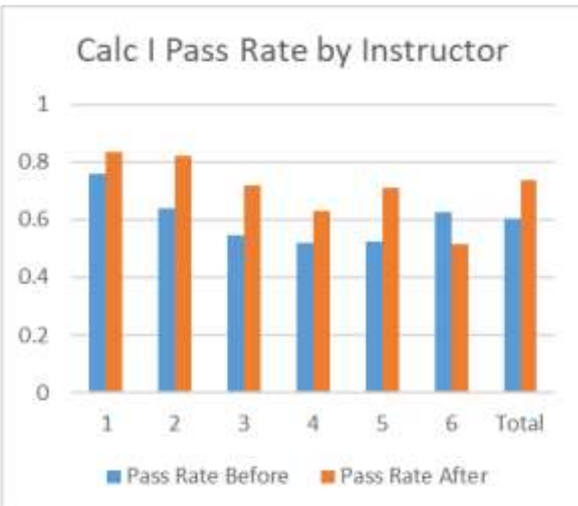


Figure 3: Calculus I pass rate by instructor

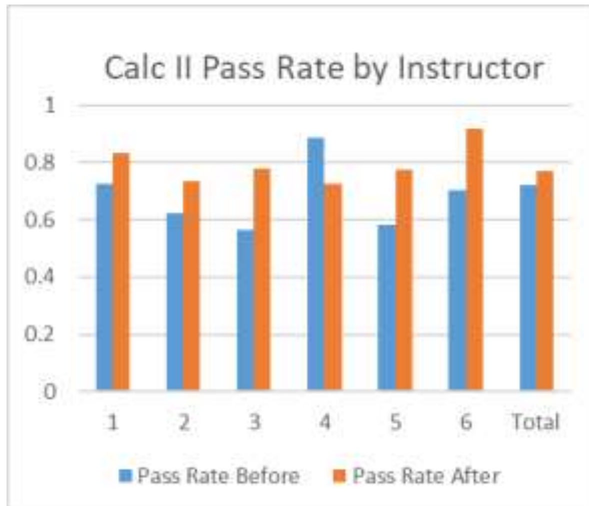


Figure 4: Calculus II pass rate by instructor

For both Calculus I and II, five of the six instructors saw jumps in pass rate. However, this data is highly volatile, with small population sizes. The rightmost bar aggregates the pass rate across the six instructors, giving a decent comparison of before/after pass rates while holding the instructor corps constant.

3.3 Subsequent Course Work

As an assessment of the efficacy of Calculus II, we monitor pass rates in courses that carry Calculus II as a prerequisite or for which Calculus II knowledge could be meaningful even if not a prerequisite. We consider all students who took and passed Calculus II between Spring 2015 and Summer 2017. This range is chosen to include a full calendar year before the implementation term (Spring 2016) for transforming Calculus II, and to end at the last point when a student could pass Calculus II and subsequently attempt another course. In this time frame, there is one data record for each pair of events of the form:

(Student Passed Calculus II, Same student subsequently attempted a target course)

A student can appear more than once in the data set, if they have attempted more than one of the subsequent target courses. All students attempting any given target course are sorted by whether they passed new Calculus II or old Calculus II. We compute the pass rate for each group in each course. For comparison, we also include the pass rate for students who transferred the prerequisite. These students have no record of a Boise State Calculus II course prior to the target course, so they are not affected by our course redesign. Results are shown in Table 1.

Table 1: Post Calculus II pass rates -- individual courses

Post Calculus II Pass Rates -- Individual Courses						
Course	Old Calculus II	New Calculus II	Transfer	Effect Size	*p*-value	*N*
Calculus III	84.2%	84.2%	80.6%	0.0%	0.988	811
Circuits I	80.0%	81.3%	71.4%	1.3%	0.876	98
Circuits II	65.0%	83.9%	100.0%	18.9%	0.133	51
Diff Eq	80.9%	76.0%	70.7%	-4.9%	0.154	582
Dynamics	84.0%	77.7%	84.2%	-6.3%	0.240	212
E and M	72.7%	75.0%	100.0%	2.3%	0.901	23
Fluids	84.4%	89.5%	61.9%	5.1%	0.403	121
Heat	92.9%	88.2%	50.0%	-4.6%	0.616	45
Mech Mat	81.2%	77.2%	87.0%	-4.0%	0.532	164
Phys I	89.0%	89.7%	88.7%	0.8%	0.850	235
Phys II	88.7%	91.5%	81.3%	2.8%	0.278	519
Statics	75.8%	78.9%	64.4%	3.1%	0.477	360
ALL COURSES	83.3%	82.8%	77.9%	-0.5%	0.728	3221

For those who prefer a graphical description, see Figure 5.

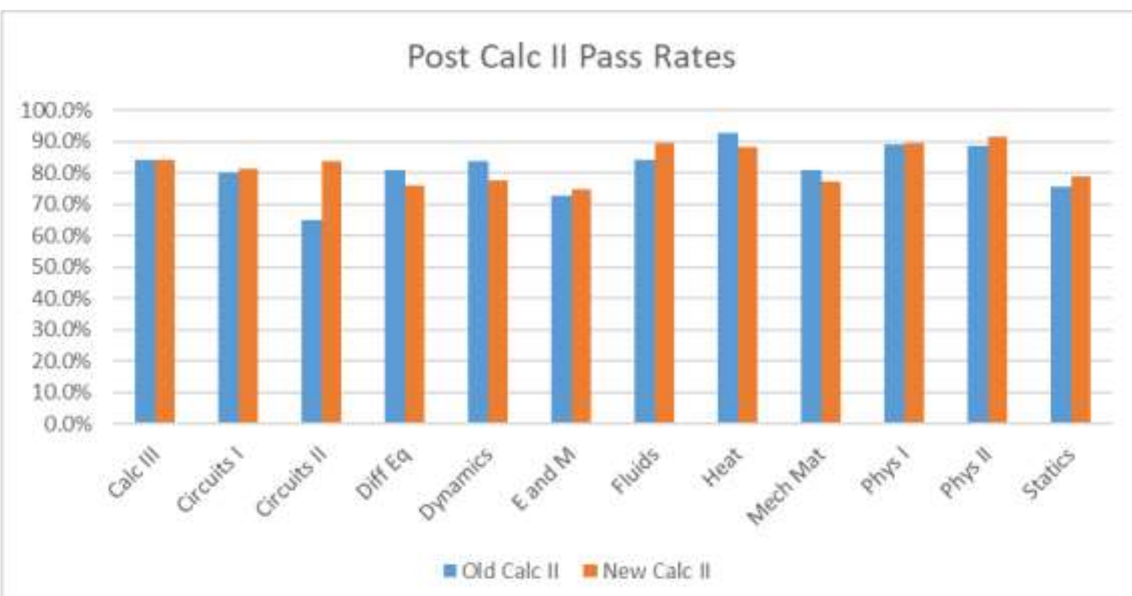


Figure 5: Post Calculus II pass rates

In Table 1, effect size is the difference between the pass rates of students originating in new Calculus II compared to old Calculus II. Positive effects mean that the new Calculus II students perform better. *N* and *p*-value are included to help judge significance. However, since none of the effects are significant, this is simply additional descriptive statistics. For example, if there is a negative effect with a small *p*-value, even if not meeting the 0.05 significance threshold, this is a potential cause for concern. Details on the computational methods are available in Bullock, et. al. (2017).

The purpose of this computation is to give a sense of whether the changes to curriculum and content in the new Calculus are creating any problems in downstream courses. Since the content changes have made Calculus II more accessible, there is some possibility that subsequent coursework would expose students' weaknesses. Since we see a scattering of positive and negative effects, but none statistically significant, this descriptive report suggests that there are no ill effects.

This tool allows for easy aggregation of post Calculus II courses by discipline, which is of interest to specific course owners. It also includes demographic slicers. The discipline aggregates are Math, Physics, and Engineering (Table 2).

Table 2: Post Calculus II pass rates -- by discipline

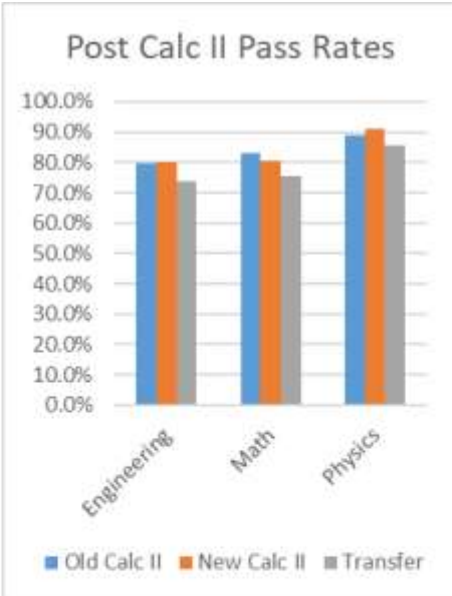
Post Calculus II Pass Rates – By Discipline						
Discipline	Old Calculus II	New Calculus II	Transfer	Effect Size	*p*-value	*N*
Engineering	79.9%	80.3%	73.8%	0.1%	0.952	1189
Math	82.9%	80.6%	75.3%	-2.7%	0.195	1530
Physics	88.8%	90.9%	85.7%	2.0%	0.373	866

The subpopulations of most interest to us are women, underrepresented minorities (URM), and Pell eligible students. For this, we aggregate post Calculus II courses (Table 3).

Table 3: Post Calculus II pass rates -- by demographic

Post Calculus II Pass Rates -- By Demographic						
Demographic	Old Calculus II	New Calculus II	Transfer	Effect Size	*p*-value	*N*
URM	79.8%	80.4%	66.7%	0.6%	0.879	410
Female	88.7%	88.1%	84.7%	-0.6%	0.803	699
Pell	81.3%	78.3%	80.5%	-3.0%	0.254	915

As always, these are descriptive statistics, with *N* and *p*-value included to provide suggestions of which numbers might be of most interest.



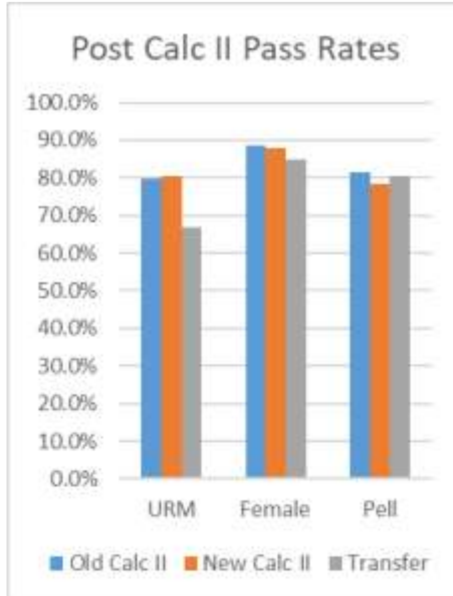


Figure 6: Post Calculus II pass rates - by discipline and by demographic

4.0 Results – Rigorous Hypothesis Tests

4.1 Outcomes in the Transformed Course

Our first use of statistical testing of hypotheses to address a research question in the Calculus I project was a comparison of outcomes in Calculus I for treatment (new Calculus I) vs control (old Calculus I). Details of the methodology are in Bullock, et. al. (2016), where we found large and significant gains in pass rate and GPA in Calculus I. For this paper, we applied the same methodology to treatment and control populations of Calculus II students. The study population was all students in Calculus II from Spring 2013 through Fall 2017, a four-year span straddling the implementation term, Spring 2016. There were 2845 data records, split into 1307 treated students and 1538 in the control group. The research question was:

> "Does treatment (reformed Calculus II) improve results in Calculus II?"

We tested two null hypotheses. Regarding pass rates:

H_0: Students in treatment and control are equally likely to pass Calculus II.

Regarding grades:

H_0: Treatment and control groups will have the same average grades in Calculus II.

The experimental variables we measured were Pass Rate and Average Grade Points (GPA) for each group in Calculus II. We also sought to control for the possibility that the treatment and control groups had different levels of academic preparation or aptitude. For each group, we measured four additional variables: High School GPA, College GPA (in the term they took Calculus II), Admission Index (computed by our admissions office from HS GPA and composite SAT and/or ACT scores), and ACT Math score, using concordances if a student has an SAT Math score instead. The results are shown in Table 4.

Table 4: Calculus II pass rate and GPA, treatment vs. control

Calculus II Pass Rate and GPA: Treatment vs. Control					
Variable	Variable Type	Control	Treatment	Effect Size	*p*-value
Calculus II Pass Rate	Study Variable	63.7%	77.4%	13.6%	0.0000
Calculus II GPA	Study Variable	1.90	2.38	0.48	0.0000
College GPA	Control Variable	3.01	3.09	0.08	0.0002
Admission Index	Control Variable	62.42	63.98	1.56	0.0708
High School GPA	Control Variable	3.35	3.40	0.06	0.0061
Concordant ACT Math	Control Variable	25.14	25.59	0.45	0.0160

It is immediately evident that there are massive gains in pass rate (13.6%) and GPA (an increase of half a letter grade) for the treatment group. However, it is also clear that the treatment group in this natural experiment is stronger in academic preparation. We have used this "academic preparation control" process in all of the previous Calculus I papers – and in each case, we found that treatment and control groups were not academically different, so we were satisfied with this form of control. However, the results in Calculus II make it clear that better tools are needed – either a multivariable regression to determine what portion of the gains are due to treatment instead of incoming academic ability, or perhaps non-parametric methods. Unfortunately, this will have to wait for a subsequent study. For now, we can report enormous gains with statistical significance on the study variables. These are more than twice as large as the gains shown in Calculus I at the equivalent stage of that project. If even half of the Calculus II gains are due to the treatment, this is still an excellent outcome.

4.2 Outcomes in Subsequent Courses

Section 3.3 provided descriptive statistics on pass rates in courses subsequent to the transformed Calculus II course. We can also use the tool to address the research question:

> "Does treatment (reformed Calculus II) have any negative effect on subsequent courses?"

Essentially, this is a test of "do no harm." Early in the Calculus I project, there was some fear that pass rate gains in Calculus I might be coming at the expense of success in subsequent courses, so we built and applied this tool as a rigorous test to check if there was any harm. We found none for the Calculus I reform. Similarly, for Calculus II, we test the null hypothesis:

H_0: Treatment and control groups (in Calculus II) are equally likely to pass subsequent courses.

Here, we hope to find no evidence that causes us to reject the null hypothesis. We set up a natural experiment in Calculus II following exactly the protocol we used for Calculus I (Bullock, et. al. 2016). In that paper, we tested only the pass rate in one critical course subsequent to Calculus I – namely Calculus II. However, with Calculus II as the treatment focus, there is less clarity as to what subsequent course is the most important test of treatment effects. We chose two: Calculus III and Differential Equations. Both courses are part of the standard STEM track; either course may be taken immediately after Calculus II. Which comes first is typically a matter of advising within various STEM disciplines. There are additional technical details of how we restricted the study population to most effectively test our hypothesis, which we will not restate here (see Bullock, et. al. 2016). Tables 5 and 6 present the results for the two subsequent courses.

Table 5: Post Calculus II results in Calculus III

Post Calculus II results in Calculus III					
Variable	Variable Type	Control	Treatment	Effect Size	*p*-value
Calculus III Pass Rate	Study Variable	84.3%	83.8%	-0.4%	0.884
Calculus IIII GPA	Study Variable	2.64	2.59	-0.05	0.620
College GPA	Control Variable	3.19	3.19	0.00	0.937
Admission Index	Control Variable	66.95	67.34	0.39	0.802
High School GPA	Control Variable	3.48	3.47	-0.01	0.781
Concordant ACT Math	Control Variable	25.45	25.89	0.43	0.236

In Calculus III, both study variables show a small negative effect of treatment, but very large *p*-values mean this is insignificant, so the null hypothesis of "did no harm" is retained. This is what we found when we studied the effect of Calculus I on subsequent Calculus II. Also, similarly, the treatment and control groups display no significant differences in academic ability or preparation.

The picture for Differential Equations, however, is less appealing.

Table 6: Post Calculus II results in Differential Equations

Post Calculus II results in Differential Equations					
Variable	Variable Type	Control	Treatment	Effect Size	*p*-value
Diff Eq Pass Rate	Study Variable	80.3%	70.4%	-9.9%	0.068
Diff Eq GPA	Study Variable	2.46	2.16	-0.30	0.078
College GPA	Control Variable	3.06	3.11	0.06	0.378
Admission Index	Control Variable	63.27	64.00	0.73	0.796
High School GPA	Control Variable	3.38	3.41	0.03	0.638
Concordant ACT Math	Control Variable	24.67	26.53	1.86	0.008

Here, we see very large negative effects on the treatment population. While the *p*-value is just above the threshold at which one would typically reject the null hypothesis, it would not be safe to comfortably conclude that the treatment of reforming Calculus II has done no harm in Differential Equations. Also, since there is evidence in the control variables that indicates the treatment group was academically stronger than the control group, it puts the negative treatment effects in an even worse light. Again, it is clear that a more robust statistical model is necessary. But this data is sufficient to require immediate engagement with the Calculus II project team and possible intervention to ameliorate potential trouble in Differential Equations. It is unclear what causal mechanism (if any) may be at work.

4.3 Retention

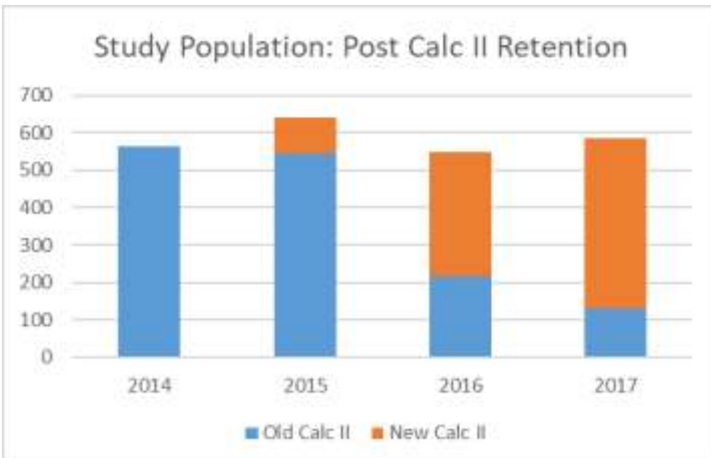
Here again we developed a natural experiment as the Calculus I project evolved (Bullock, et. al. 2017). We used the experiment to study the effect of reforming Calculus I on the retention of students in the year that they encountered Calculus I. For this paper, we apply an identical protocol to Calculus II students, addressing the research question:

"What effect does treatment (reformed Calculus II) have on retention of students in the year that they encounter Calculus II?"

There are actually two research questions: one in which retention is "retained at the university," regardless of major, and one in which retention is "retained in STEM," and applies only to students who were STEM majors in the year they encountered Calculus II[1]. We answer each question for the general study population and then again for demographics of women, underrepresented minorities, and Pell eligible students. In all cases, we test the null hypothesis:

H_0: Students in treatment and control are equally likely to be retained.

We do not, however, include the additional variables for academic preparation and ability.



Details on the protocol for forming the study population, technical definitions of variables, and other elements of the experimental design can be found in Bullock, et. al. (2017). Figure 7 presents a snapshot of the size of the study population (2340 records), distributed across 4 academic years and broken out as treatment (new Calculus II) or control (old Calculus II).

Figure 7: Study population – post Calculus II retention

4.3.1 Retained at the University

Treatment delivers a bit more than four percentage points of additional retention at the university in the year that students encounter Calculus II (Table 7 and Figure 8). The result is statistically significant.

Table 7: Post Calculus II retention rates

Post Calculus II Retention Rates					
Demographic	Control	Treatment	Effect Size	*p*-value	*N*
ALL	81.0%	85.2%	4.2%	0.008	2340
Female	83.1%	88.2%	5.1%	0.104	496
URM	81.5%	85.3%	3.7%	0.372	324
Pell	82.6%	84.9%	2.3%	0.400	789

When sliced by demographics, we see that there are slightly larger retention gains for women. URM and Pell eligible students also gain, but not as much as the full study population. None of

[1] Our definitions of the terms "retention" and "retention rate" differ from the definitions used in Boise State University's official reporting offices. Details available in Bullock, et. al. (2017).

the demographically specific gains are statistically significant, since these are much smaller populations compared to the full study population.

Figure 8: Retention rate gains, Calculus II

4.3.2 Retained in STEM

We restrict the study population to students who were STEM majors in the year they encountered Calculus II. There are now three possible outcomes: Retained in STEM, switched to non-STEM, and left school. Treatment delivers a similar gain in STEM-to-STEM retention.

Table 8: Post Calculus II STEM Retention

Post Calculus II STEM Retention					
Result	Control	Treatment	Effect Size	*p*-value	*N*
STEM-to-STEM	75.5%	79.8%	4.3%	0.040	1659
STEM-to-Non	5.9%	6.0%	0.1%		128
Dropped Out	18.5%	14.2%	-4.3%		363

This is very much like what we saw for Calculus I (Bullock, et. al. 2017) in two ways. One is that the size of the gain is what one would expect as a simple consequence of the pass rate gains, and two is that the entire gain in STEM-to-STEM retention is caused by preventing dropouts. Both observations suggest that all of this is directly attributable to pass rate.

When we drill down to demographics (Table 9) we see similar results, albeit none that are statistically significant. There is one notable difference involving underrepresented minority students.

For female students, treatment may confer a gain in STEM retention that is, again, entirely the result of preventing dropouts. The STEM-to-STEM retention gain for women is not as large as the gain in retention at the university, which is a stark contrast to the result from transforming Calculus I (Bullock, et. al. 2017). In that paper we found a much larger benefit to women retained in STEM as compared to women retained at the university. Also, note that the starting point for female retention in STEM, about 75%, is much lower than the starting point for female retention in college

Table 9: STEM retention by Demographic

STEM Retention by Demographic						
Demographic	Result	Control	Treatment	Effect Size	*p*-value	*N*
Female	STEM-to-STEM	74.8%	78.5%	3.7%	0.435	328
	STEM-to-Non	8.9%	9.3%	0.4%		39
	Dropped Out	16.3%	12.2%	-4.1%		63
URM	STEM-to-STEM	77.2%	76.9%	-0.3%	0.953	225
	STEM-to-Non	5.3%	9.9%	4.7%		21
	Dropped Out	17.5%	13.2%	-4.3%		46
Pell Eligible	STEM-to-STEM	76.6%	79.5%	3.0%	0.403	576
	STEM-to-Non	6.1%	6.1%	0.0%		45
	Dropped Out	17.4%	14.4%	-3.0%		121

For Pell eligible students, there is the same story: small gains that are due to preventing dropouts.

There is an oddity for URM. Here, the treatment effect on STEM retention is negative. Reforming Calculus II could have cost some URM retention in STEM. As with other groups, we have obtained a nice reduction in the dropout rate, but here all of the non-dropouts seem to have departed for non-STEM fields.

While informative, none of the demographically specific results are statistically significant.

4.3.3 STEM Retention Gaps

The previous subsection details STEM retention rates for demographic subgroups, which can be compared to STEM retention for the full study population.

Table 10: STEM retention gaps

STEM Retention Gaps		
Demographic	Control	Treatment
Female	74.8%	78.5%
Male	75.7%	80.2%
Gender Gap	**0.9%**	**1.7%**
URM	77.2%	76.9%
non-URM	77.8%	82.0%
URM Gap	**0.6%**	**5.1%**
Pell	76.6%	79.5%
non-Pell	78.5%	82.0%
Pell Gap	**2.0%**	**2.5%**

Where retention gaps are concerned, what is more appropriate is a head-to-head comparison. These are displayed for treatment and control in Table 10. Here, we show only STEM-to-STEM retention. It is evident that the treatment seems to confer STEM retention gains for all groups. However, because the gains for men are highest, the pre-existing gaps for women, underrepresented minorities, and Pell eligible students widened after treatment.

5.0 Summary

The transformation of Calculus II has achieved very large gains in Calculus II pass rates and grades, which translate into reasonably large gains in retention, both at the overall university level and specifically for STEM majors. All of these results, when studied via natural experiment, are statistically significant. None are restricted to *a priori* advantaged demographic groups. The gains in pass rate, grades, and retention are similar to those achieved by the earlier transformation of Calculus I at Boise State University. The Calculus II gains are even larger.

Descriptive statistics on performance in courses beyond Calculus II suggest that there is no negative effect from altering the Calculus II content and curriculum. However, when statistical tools are carefully applied to test this hypothesis on immediately subsequent math courses, there is one important and actionable exception; although Calculus II transformation seems to have no effect on Calculus III, there is a sizable and significant negative impact on Differential Equations. It is, at least, a positive outcome of this study to have caught this effect and to have data to support and guide an intervention to address it.

Retention effects are smaller and less statistically robust than the pass rate gains in Calculus II. They also did not display STEM specific impacts that were as profound as those observed after Calculus I transformation. However, this does not mean that the Calculus II reform is failing female, URM, or Pell eligible students. It simply means that issues with retention will need to be kept in view.

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