Instructions
Read today's Notes and Learning Goals.

1. Question Details
Finish the Warmup Worksheet. Then write the 6-th degree Taylor polynomial for \(e^x\) centered at \(x = 0\).

\[ e^x \approx \text{ } \]

From the pattern in your worksheet table, guess the 7-th degree coefficient.

\[ a_7 = \text{ } \]

Guess the 25-th degree coefficient. Write your answer using factorial notation.

\[ a_{25} = \text{ } \]

Write the Taylor polynomial in sigma notation. Start by entering the lower and upper bounds.

Lower bound: \( \text{ } \)
Upper bound: \( \text{ } \)

\[ e^x \approx \sum_{n=j}^{k} \text{ } \]

2. Question Details
Finish the Warmup Worksheet. Then write the 8-th degree Taylor polynomial for \(f(x) = \cos(x)\) centered at \(x = 0\).

\[ \cos(x) \approx \text{ } \]

From the pattern in your worksheet table, guess the 10-th derivative of \(\cos(x)\), evaluated at \(x = 0\).

\[ f^{(10)}(0) = \text{ } \]

What is the 10-th degree coefficient? Use factorial notation.

\[ a_{10} = \text{ } \]

How about the 20-th degree coefficient?

\[ a_{20} = \text{ } \]
3. Question Details

For each expression below, decide if it is a finite sum, a series, a polynomial, or a power series. If you are unsure of the differences, consult today’s Notes and Learning Goals as well as those from Infinite Series.

- \[ \sum_{n=1}^{\infty} \frac{1}{2^n} \]  
  - Finite sum  
  - Series  
  - Polynomial  
  - Power series

- \[ 1 + x + \frac{1}{3}x^2 + \frac{1}{9}x^3 \]  
  - Finite sum  
  - Series  
  - Polynomial  
  - Power series

- \[ \sum_{n=1}^{\infty} \frac{x^n}{2^n} \]  
  - Finite sum  
  - Series  
  - Polynomial  
  - Power series

- \[ \sum_{n=0}^{10} \frac{(-1)^n x^{2n}}{(2n)!} \]  
  - Finite sum  
  - Series  
  - Polynomial  
  - Power series

- \[ 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots \]  
  - Finite sum  
  - Series  
  - Polynomial  
  - Power series

4. Question Details

For each expression below, decide if it is a Taylor polynomial or a Taylor series. Then determine the center point.

- \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \]  
  - Taylor polynomial  
  - Taylor series

  Centered at \( x = \) 

- \[ 1 + \frac{1}{2}(x - 2) + \frac{1}{3}(x - 2)^2 + \frac{1}{4}(x - 2)^3 + \cdots \]  
  - Taylor polynomial  
  - Taylor series

  Centered at \( x = \) 

- \[ \sum_{n=0}^{8} \frac{1}{2^n} (x + 3)^n \]  
  - Taylor polynomial  
  - Taylor series

  Centered at \( x = \)
5. In the Taylor series for \( \sin(x) \) centered at \( x = 0 \), what is the 5-th degree coefficient?

\[ a_5 = \]

How about the 10-th degree coefficient?

\[ a_{10} = \]

The 15-th degree coefficient?

\[ a_{15} = \]

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6. Find the 4-th degree Taylor polynomial for \( f(x) = \frac{1}{x} \) centered at \( x = 1 \). Write your answer in standard form.

\[ \frac{1}{x} \approx \]

From the pattern in your work, guess the 7-th degree coefficient.

\[ a_7 = \]

Guess the 20-th degree coefficient.

\[ a_{20} = \]

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7. Suppose that you are given the sigma notation for a Taylor series of an unknown function

\[ f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n \]

Find the 4-th degree Taylor polynomial approximation for \( f(x) \) centered at zero.

\[ f(x) = \]

If you expanded further, what would be the 10-th degree coefficient?

\[ a_{10} = \]

How about the 15-th?

\[ a_{15} = \]
8. Suppose that you are given the following Taylor series for an unknown function

\[ f(x) = 1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \frac{x^4}{5} + \cdots \]

Write the Taylor series in sigma notation. Start by entering the lower and upper bounds.

**Note:** It's a series, not a polynomial.

Lower bound: 
Upper bound: 

\[ f(x) = \sum_{n=j}^{\infty} \]
12. **Question Details**

Find the 5-th order Taylor polynomial, centered at zero, for $-3\sin(2x)$. You can do this by distributing $-3$ across the result after you substitute $2x$.

$-3\sin(2x) \approx \ldots$

13. **Question Details**

Find the 4-th order Taylor polynomial, centered at zero, for $\frac{1}{2} \cdot \frac{1}{1 - (-x/2)} \approx \ldots$

14. **Question Details**

Find the 5-th order Taylor polynomial, centered at zero, for $x - \sin x \approx \ldots$

15. **Question Details**

Find the 5-th order Taylor polynomial, centered at zero, for $\frac{\cos(x) - 1}{x} \approx \ldots$

There is a famous limit problem from Calculus I (below). It is considered difficult in Calculus I because you clearly cannot plug in $x = 0$. What happens if you use the Taylor polynomial instead?

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x} = \ldots$$

**Discussion Question:** Taylor polynomials are only approximations. What happens to this limit problem if you use higher degree approximations? What is the exact answer?