Consider the following sequence

\[ a_n = \frac{n(n + 1)}{2} \]

a. Calculate the first four terms of this sequence, starting with \( n = 1 \).

\[
\begin{align*}
    a_1 &= \quad \\
    a_2 &= \\
    a_3 &= \\
    a_4 &= 
\end{align*}
\]

b. Which of the following is the graph of \( a_n \)?

![Graphs of \( a_n \) and \( a_{n1} \)]

\[
\arg\lim_{n \to \infty} \frac{n(n + 1)}{2} = 
\]

C. What is the limit of this sequence? Use \( \infty \) as needed.
Consider the following sequence

\[ a_n = \frac{n}{n + 1} \]

a. Calculate the first four terms of this sequence, starting with \( n = 0 \).

\[ a_0 = \]

\[ a_1 = \]

\[ a_2 = \]

\[ a_3 = \]

b. Which of the following is the graph of \( a_n \)?

![Graph options]

c. What is the limit of this sequence? Use \( \infty \) as needed.

\[ \lim_{n \to \infty} \frac{n}{n + 1} = \]
Consider the following sequence
\( a_n = n! \) (n factorial)

a. Calculate the first four terms of this sequence, starting with \( n = 1 \).

\[
\begin{align*}
    a_1 &= \frac{(n - 1)!}{(n + 1)!} \\
    a_2 &= \frac{(n - 1)!}{(n + 1)!} \\
    a_3 &= \frac{(n - 1)!}{(n + 1)!} \\
    a_4 &= \frac{(n - 1)!}{(n + 1)!}
\end{align*}
\]

b. What is the limit of this sequence? Use \( \infty \) as needed.

\[
\lim_{n \to \infty} \frac{n!}{(n - 1)!} = \frac{n!}{(n + 1)!}
\]
Consider the following two sequences that both go to infinity:

\[ a_n = n^3 \]
\[ b_n = n^2 \]

a. Find the limit of the ratio. Use \( \infty \) as needed.

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \]

b. Which sequence goes to infinity faster?

- \( n^3 \) goes to infinity faster than \( n^2 \).
- \( n^3 \) goes to infinity slower than \( n^2 \).
- \( n^3 \) goes to infinity at about the same speed as \( n^2 \).

---

Consider the following two sequences that both go to infinity:

\[ a_n = 5n^3 \]
\[ b_n = 7n^3 \]

a. Find the limit of the ratio. Use \( \infty \) as needed.

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \]

b. Which sequence goes to infinity faster?

- \( 5n^3 \) goes to infinity faster than \( 7n^3 \).
- \( 5n^3 \) goes to infinity slower than \( 7n^3 \).
- \( 5n^3 \) goes to infinity at about the same speed as \( 7n^3 \).

---

Consider the following two sequences that both go to infinity:

\[ a_n = n! \]
\[ b_n = 2^n \]

a. Find the limit of the ratio. Use \( \infty \) as needed.

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \]

b. Which sequence goes to infinity faster?

- \( n! \) goes to infinity faster than \( 2^n \).
- \( n! \) goes to infinity slower than \( 2^n \).
- \( n! \) goes to infinity at about the same speed as \( 2^n \).
Consider the following two sequences that both go to infinity:
\[ a_n = 2^n \]
\[ b_n = n^8 2^n \]

a. Find the limit of the ratio. Use \( \infty \) as needed.
\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \]

b. Which sequence goes to infinity faster?
\[ 2^n \text{ goes to infinity faster than } n^8 2^n. \]
\[ 2^n \text{ goes to infinity slower than } n^8 2^n. \]
\[ 2^n \text{ goes to infinity at about the same speed as } n^8 2^n. \]
Consider the following two sequences that both go to infinity:

\[ a_n = n - \ln(n) \]
\[ b_n = n \]

a. Find the limit of the ratio. Use \( \infty \) as needed.

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \]

b. Which sequence goes to infinity faster?

- \( a_n \) goes to infinity faster than \( b_n \).
- \( a_n \) goes to infinity slower than \( b_n \).
- \( a_n \) goes to infinity at about the same speed \( b_n \).

12. All the following families of sequences go to infinity, meaning \( \lim_{n \to \infty} a_n = \infty \).

Order these families by the speed at which they go to infinity (1 = slowest to 4 = fastest).

- Powers: \( a_n = n^r \) where \( r > 0 \)
- Logarithms: \( a_n = \ln(n) \)
- Factorial: \( a_n = n! \)
- Exponential: \( a_n = b^r \) where \( b > 1 \)

13. All of the following sequences go to infinity, meaning \( \lim_{n \to \infty} a_n = \infty \).

Order them by the speed at which they go to infinity (1 = slowest to 10 = fastest).

- \( n! \)
- \( n \)
- \( n \ln(n) \)
- \( n^2 \)
- \( 2^n \)
- \( 3^n \)
- \( \ln(n) \)
- \( n^3 \)
- \( n^{1/3} \)
- \( n^{1/2} \)
Consider the following two sequences that both have horizontal asymptotes at zero,

\[
\lim_{n \to \infty} a_n = 0 \quad \text{and} \quad \lim_{n \to \infty} b_n = 0:
\]

\[
a_n = \frac{1}{n^{1/3}}
\]

\[
b_n = \frac{1}{n^{1/2}}
\]

a. Find the limit of the ratio. Use \(\infty\) as needed.

\[
\lim_{n \to \infty} \frac{a_n}{b_n} = \_\_\_\_
\]

b. Which sequence has the thicker tail? That is, which goes to zero more slowly?

- \(a_n\) has a thicker tail than \(b_n\). \((a_n \gg b_n)\)
- \(a_n\) has a thinner tail than \(b_n\). \((a_n \ll b_n)\)
- \(a_n\) and \(b_n\) have about the same tail thickness.

Consider the following two sequences that both have horizontal asymptotes at zero,

\[
\lim_{n \to \infty} a_n = 0 \quad \text{and} \quad \lim_{n \to \infty} b_n = 0:
\]

\[
a_n = \frac{1}{n!}
\]

\[
b_n = \frac{1}{e^n}
\]

a. Find the limit of the ratio. Use \(\infty\) as needed.

\[
\lim_{n \to \infty} \frac{a_n}{b_n} = \_\_\_\_
\]

b. Which sequence has the thicker tail? That is, which goes to zero more slowly?

- \(a_n\) has a thicker tail than \(b_n\). \((a_n \gg b_n)\)
- \(a_n\) has a thinner tail than \(b_n\). \((a_n \ll b_n)\)
- \(a_n\) and \(b_n\) have about the same tail thickness.
Consider the following two sequences that both have horizontal asymptotes at zero, \( \lim_{n \to \infty} a_n = 0 \) and \( \lim_{n \to \infty} b_n = 0 \):

\[
\begin{align*}
    a_n &= \frac{1}{\log_2(n)} \\
b_n &= \frac{1}{\log_3(n)}
\end{align*}
\]

a. Find the limit of the ratio. Use \( \infty \) as needed.

\[
\lim_{n \to \infty} \frac{a_n}{b_n} = \quad \square
\]

Hint: The change of base formula for logarithms is \( \log_b(n) = \frac{\ln(n)}{\ln(b)} \).

b. Which sequence has the thicker tail? That is, which goes to zero more slowly?

- \( a_n \) has a thicker tail than \( b_n \) \( (a_n \gg b_n) \)
- \( a_n \) has a thinner tail than \( b_n \) \( (a_n \ll b_n) \)
- \( a_n \) and \( b_n \) have about the same tail thickness.
All of the following sequences have horizontal asymptotes at zero, meaning \( \lim_{n \to \infty} a_n = 0 \).

Order them by the thickness of their tails, 1 = thickest (goes to zero slowest) to 8 = thinnest (goes to zero fastest).

- \(2^{-n}\)
- \(\frac{1}{n^2}\)
- \(\frac{1}{n}\)
- \(\frac{1}{n!}\)
- \(\left(\frac{1}{3}\right)^n\)
- \(\frac{1}{\ln(n)}\)
- \(\frac{1}{n^{1/2}}\)
- \(\frac{1}{n\ln(n)}\)
Consider the following sequence
\[ a_n = \frac{2n^2 - 5n^3 + 6}{2n^3 + n + 8} \]

Find the limit of this sequence as follows

1. For large values of \( n \) this fraction is approximately the ratio of the dominant terms (the ones that go to infinity the fastest).
   Select the correct ratio of dominant terms.
   - \[ \frac{2n^2}{2n^3 + n + 8} \approx \frac{-5n^3}{n} \]
   - \[ \frac{2n^2}{2n^3 + n + 8} \approx \frac{2n^2}{2n^3} \]
   - \[ \frac{2n^2}{2n^3 + n + 8} \approx \frac{-5n^3}{2n^3} \]
   - \[ \frac{2n^2}{2n^3 + n + 8} \approx \frac{6}{8} \]

2. Use the above approximation to find the limit of this sequence.
\[ \lim_{n \to \infty} a_n = \]

Consider the following sequence
\[ a_n = \frac{n^2 + 2n + 3}{\sqrt{4n^4 + 2n^2 + 1}} \]

Find the limit of this sequence as follows

1. For large values of \( n \) this fraction is approximately the ratio of the dominant terms (the ones that go to infinity the fastest).
\[ a_n \approx \]

2. Use the above approximation to find the limit of this sequence.
\[ \lim_{n \to \infty} a_n = \]
20. Consider the following sequence
\[ a_n = \frac{2^n + 3^{-n}}{5^{-n} + 3^n} \]

Find the limit of this sequence as follows

1. For large values of \( n \) this fraction is approximately the ratio of the dominant terms.
   \[ a_n \approx \]

2. Use the above approximation to find the limit of this sequence.
   \[ \lim_{n \to \infty} a_n = \]

21. Find the limit of the following sequence
   \[ \lim_{n \to \infty} \frac{\sqrt{n^2 + 3}}{1 - 3n} = \]

22. Find the limit of the following sequence
   \[ \lim_{n \to \infty} \frac{1 + 2^{n+1}}{1 + 2^n} = \]