Instructions

- Read today’s Notes and Learning Goals
- In this assignment you are **NOT ALLOWED** to use the exponential function on your calculator or on any equivalent device
- Many questions in this assignment ask for decimal answers. All such answers must be rounded to 4 decimal places. Anything less accurate or more accurate will be marked incorrect.

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**1.** True or False? For this assignment you are **NOT ALLOWED** to use the $e^x$ button on your calculator (or any equivalent device). If you are unsure, read the assignment instructions.

- [ ] False
- [ ] True

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**2.** True or False? All the numerical non-integer calculations and/or estimates on these homework problems must be rounded to four (4) decimal places.

- [ ] True
- [ ] False
3. **Question Details**

Find the 2\(^{nd}\) degree Taylor Polynomial for \(e^x\) centered at \(x = 0\).

\[ e^x \approx \]

Use exact coefficients. Decimals will not be accepted.

Use the Taylor polynomial to estimate \(e^1\). That is, plug in \(x = 1\).

\[ e^1 \approx \]

Find the 5\(^{th}\) degree Taylor Polynomial for \(e^x\) centered at \(x = 0\). All coefficients must be exact.

\[ e^x \approx \]

Use this Taylor polynomial to estimate \(e^1\). Round to 4 decimal places.

\[ e^1 \approx \]

**Note:** WebAssign tolerances are set very tightly. You will probably have to enter the full expression into your calculator so that you do not accumulate roundoff error.

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4. **Question Details**

The exercise you just completed, using two different degrees of Taylor polynomial to estimate \(e^1\), can be repeated for every degree. The table below shows the results for degrees 0 through 10. Complete the table by entering your answers from above in the correct cells.

<table>
<thead>
<tr>
<th>degree</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e\approx)</td>
<td>1.0000</td>
<td>2.0000</td>
<td></td>
<td>2.6667</td>
<td>2.7083</td>
<td></td>
<td>2.7181</td>
<td>2.7183</td>
<td>2.7183</td>
<td>2.7183</td>
<td>2.7183</td>
</tr>
</tbody>
</table>

Higher degrees give better approximations, but eventually they stop getting any better, at least when rounded to 4 decimal places.

To 4 decimal places, what is the best possible approximation?

\[ e \approx \]

What is the minimum degree that achieves this approximation?

degree =

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5. **Question Details**

Using Taylor polynomials of degree 0 through 6, centered at 0, to estimate \(e^{0.5}\). Enter your answers in the table below. Round to 4 decimal places.

<table>
<thead>
<tr>
<th>degree</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^{0.5})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To 4 decimal places, what is the best possible approximation?

\[ e^{0.5} \approx \]

What is the minimum degree that achieves this approximation?

degree =
6. **Question Details**  

The 8\textsuperscript{th} degree Taylor polynomial approximation for the function $\cos(x)$ centered at 0 is  

$$\cos(x) \approx 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8$$

Make a table like the one in the previous problem. Use it to find the best possible approximation of $\cos(1)$ to 4 decimal places. Do it without touching the "cos" button on your calculator.

What is the minimum degree that achieves this approximation?

$$\text{degree} = \phantom{00}$$

7. **Question Details**

Download and print **Worksheet #1**. If you are working in class, get a copy from your instructor. As you fill out the table in the worksheet, you will be asked to return here and answer the questions below.

In the row for $e^{0.5}$ ...

a. What degree resulted in "No Change"?

b. How many cells did you leave blank?

c. What is the stepsize?

In the row for $e^{0.2}$ ...

d. What degree resulted in "No Change"?

e. What is the best approximation for $e^{0.2}$?

f. What degree resulted in the best approximation?

In the full table...

g. How many blank cells are there in the degree 7 column?

h. What is the best approximation for $e^{0.1}$?

i. What is the quadratic approximation $e^{0.05}$?

j. What is your table entry for stepsize $= 0.2$ and degree $= 3$?
8. Question Details

All questions refer to your clean and complete Worksheet #1. If you don’t have one, complete the worksheet before you start this problem.

a. Suppose that you are required to use degree 3 polynomials. What is the largest allowable stepsize if you need 4 decimal places of accuracy? Choose from this list:
   - 1.0
   - 0.5
   - 0.2
   - 0.1
   - 0.05

b. Suppose that you are required to use stepsize = 0.2. What is the smallest degree that will give you 4 decimal places of accuracy?

c. Suppose that you are required to use stepsize = 0.5, but you are allowed to select any degree ≤ 4. What's the best approximation you can get?

d. What's the error in that approximation? That is, what is the difference between your answer above and the best possible 4 decimal place approximation of $\sqrt{0.5}$?

e. You are required to use cubic polynomials and stepsize = 0.5. What is the error in your approximation?