The graph of the functions $y = x^2$ and $y = \frac{1}{x^2}$ are shown below.

a. Which of the following limit expressions best describes the graph of $y = x^2$ as $x \to \infty$?

- $\lim_{x \to \infty} x^2 = 2x$
- $\lim_{x \to \infty} x^2 = 0$
- $\lim_{x \to \infty} x^2 = 2$
- $\lim_{x \to \infty} x^2 = \infty$

b. Which of the following limit expressions best describes the graph of $y = \frac{1}{x^2}$ as $x \to \infty$?

- $\lim_{x \to \infty} \frac{1}{x^2} = \frac{-2}{x^3}$
- $\lim_{x \to \infty} \frac{1}{x^2} = 0$
- $\lim_{x \to \infty} \frac{1}{x^2} = 2$
- $\lim_{x \to \infty} \frac{1}{x^2} = \infty$
2. Question Details

The graph of the functions \( y = e^x \) and \( y = e^{-x} = \frac{1}{e^x} \) are shown below.

Find the following limits

a. \( \lim_{x \to \infty} e^{-x} = \)

b. \( \lim_{x \to \infty} e^x = \)

3. Question Details

The graph of the functions \( y = \sqrt{x} \) and \( y = \ln(x) \) are shown below.

Find the following limits

a. \( \lim_{x \to \infty} \sqrt{x} = \)

b. \( \lim_{x \to \infty} \ln(x) = \)

4. Question Details

Suppose \( f(x) \) is a function such that \( \lim_{x \to \infty} f(x) = \infty \). Find the following limit

\[ \lim_{x \to \infty} \frac{1}{f(x)} = \]
5. Question Details

a. Find the following limit

\[ \lim_{x \to \infty} \frac{x^2}{x^3} = \]

**Hint:** Simplify the fraction first.

b. Which of the following statements best describes the relationship the above limit gives between \( x^2 \) and \( x^3 \) as \( x \to \infty \)?

- \( x^2 \) goes to infinity slower than \( x^3 \).
- \( x^2 \) goes to infinity about the same speed as \( x^3 \).
- \( x^2 \) goes to infinity faster than \( x^3 \).

6. Question Details

a. Find the following limit

\[ \lim_{x \to \infty} \frac{e^x}{x^2} = \]

**Hint:** Use L'Hopital's Rule or think about which function is dominant as \( x \to \infty \).

b. Which of the following statements best describes the relationship the above limit gives between \( e^x \) and \( x^2 \) as \( x \to \infty \)?

- \( e^x \) goes to infinity slower than \( x^2 \).
- \( e^x \) goes to infinity faster than \( x^2 \).
- \( e^x \) goes to infinity about the same speed as \( x^2 \).

7. Question Details

a. Find the following limit

\[ \lim_{x \to \infty} \frac{5x^2}{3x^2} = \]

**Hint:** Simplify the fraction first.

b. Which of the following statements best describes the relationship the above limit gives between \( 5x^2 \) and \( 3x^2 \) as \( x \to \infty \)?

- \( 5x^2 \) goes to infinity faster than \( 3x^2 \).
- \( 5x^2 \) goes to infinity at about the same speed as \( 3x^2 \).
- \( 5x^2 \) goes to infinity slower than \( 3x^2 \).
All of the following functions go to infinity as $x \to \infty$, $\lim_{x \to \infty} f(x) = \infty$.

Order these by the speed at which they go to infinity. Enter the functions in the boxes below using the notation "Slower Function $\ll$ Faster Function". **Only 3 submits per answer box.**

8. Question Details

All of the following functions go to infinity as $x \to \infty$, $\lim_{x \to \infty} f(x) = \infty$.

Order these by the speed at which they go to infinity. Enter the functions in the boxes below using the notation "Slower Function $\ll$ Faster Function". **Only 3 submits per answer box.**

9. Question Details
10. All the following families of functions go to infinity as $x \to \infty$, $\lim_{x \to \infty} f(x) = \infty$.

- **A**: Exponential: $a^x$ where $a > 1$
- **B**: Powers: $x^n$ where $n > 1$
- **C**: Logarithms: $\log_a(x)$ where $a > 1$
- **D**: Roots: $\sqrt[n]{x} = x^{1/n}$ where $n > 1$

Order these families by the speed at which they go to infinity.

---Select--- $<$ $<$ ---Select--- $<$ $<$ ---Select--- $<$ $<$ ---Select---

11. All of the following functions go to infinity as $x \to \infty$, $\lim_{x \to \infty} f(x) = \infty$.

Order these by the speed at which they go to infinity. 1 is the slowest, 8 is the fastest.

- $x^5$ ---Select---
- $x^{1/5}$ ---Select---
- $e^x$ ---Select---
- $5^x$ ---Select---
- $x^2$ ---Select---
- $x^{1/2}$ ---Select---
- $\ln(x)$ ---Select---
- $2^x$ ---Select---

12. Consider the limit

$$\lim_{x \to \infty} \frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8}$$

a. For large values of $x$ this fraction is approximately the ratio of the dominant terms (the ones that go to infinity the fastest). Select which is the correct approximation for large values of $x$.

- $2x^2 - 5x^3 + 6 \approx \frac{6}{8}$
- $2x^2 - 5x^3 + 6 \approx \frac{2x^2}{2x^3}$
- $2x^2 - 5x^3 + 6 \approx \frac{-5x^3}{x}$
- $2x^2 - 5x^3 + 6 \approx \frac{-5x^3}{2x^3}$

b. Use the above approximation to find the limit.

$$\lim_{x \to \infty} \frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8} =$$
Consider the limit
\[
\lim_{{x \to \infty}} \frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5}
\]
a. For large values of \(x\), the above ratio is approximately:

- \[\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{-5}{3}\]
- \[\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{-4x^{5/2}}{3x^2}\]
- \[\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{5x^{3/2}}{3x^2}\]
- \[\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{12}{5}\]

b. Use the above approximation to find the limit.
\[
\lim_{{x \to \infty}} \frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} = \]

Consider the limit
\[
\lim_{{x \to \infty}} \frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}}
\]
a. For large values of \(x\), the above fraction is approximately:

- \[\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{2x^2}{9x^4}\]
- \[\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{5}{2}\]
- \[\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{2x^2}{3x^2}\]
- \[\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{5}{2}\]

b. Use the above approximation to find the limit.
\[
\lim_{{x \to \infty}} \frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} = \]

15. Consider the limit
\[ \lim_{x \to \infty} \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \]

a. For large values of \( x \), the above fraction is approximately:
   - \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx \frac{3}{5} \)
   - \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx -\frac{6 \cdot (2^x)}{5 \cdot (3^x)} \)
   - \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx -\frac{6}{5} \)
   - \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx \frac{3}{x} \)

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} = \]

16. Consider the limit
\[ \lim_{x \to \infty} \frac{\sqrt{25x^4 + 15x^2 + 6}}{7x^2 + 9x + 10} \]

a. For large values of \( x \), the above fraction is approximately:
\[ \frac{\sqrt{25x^4 + 15x^2 + 6}}{7x^2 + 9x + 10} \approx \]

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{\sqrt{25x^4 + 15x^2 + 6}}{7x^2 + 9x + 10} = \]

17. Consider the limit
\[ \lim_{x \to \infty} \frac{3x^3 - 5x^5 + 1}{5x^5 - 7 \cdot (2^x)} \]

a. For large values of \( x \), the above fraction is approximately:
\[ \frac{3x^3 - 5x^5 + 1}{5x^5 - 7 \cdot (2^x)} \approx \]

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{3x^3 - 5x^5 + 1}{5x^5 - 7 \cdot (2^x)} = \]
Consider the limit
\[ \lim_{x \to \infty} \frac{3x^5 + 9x^4 - 31x}{2x^4 - 31x^2 + 12} \]

a. For large values of \( x \), the above fraction is approximately:
\[ \frac{3x^5 + 9x^4 - 31x}{2x^4 - 31x^2 + 12} \approx \]

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{3x^5 + 9x^4 - 31x}{2x^4 - 31x^2 + 12} = \]

---

Evaluate the following limit
\[ \lim_{x \to \infty} \frac{\sqrt{9x^4 - 3x^2 + 5x + 1}}{5x^2 - 6x + 1} = \]

---

Evaluate the following limit
\[ \lim_{x \to \infty} \frac{3x^2 + 5x}{4x^2 + 6^{-x} + 3^x} = \]
Answer the following questions. **You only get two submits per answer box.**

a. Order the functions, $x^2$, $x^3$ and $\sqrt{x}$, by the speed they go to infinity.

b. Order the functions, $\frac{1}{x^2}$, $\frac{1}{x^3}$ and $\frac{1}{\sqrt{x}}$, by the their tail thickness, as shown in the graph below. Use the notation "Thinner Tail $\ll$ Thicker Tail".

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22. **Question Details**

The following functions, $\frac{1}{x}$, $\frac{1}{\ln(x)}$ and $\frac{1}{2^x}$, all have a horizontal asymptote at 0, $\lim_{x \to \infty} f(x) = 0$.

Order these functions by their tail thickness. **You only get two submits per answer box.**
All of the following functions have a horizontal asymptote at 0, \( \lim_{x \to \infty} f(x) = 0 \).

Order these by the thickness of their tail. 1 is the thinnest, 6 is the thickest.

\[
\begin{align*}
\frac{1}{x^{1/5}} & \quad \text{---Select---} \\
\frac{1}{e^x} & \quad \text{---Select---} \\
\frac{1}{x^{3/5}} & \quad \text{---Select---} \\
\frac{1}{\ln(x)} & \quad \text{---Select---} \\
\frac{1}{3^x} & \quad \text{---Select---} \\
\frac{1}{x^5} & \quad \text{---Select---}
\end{align*}
\]