Instructions

- Be sure you know the hierarchies listed in the Convergence Notes
- Order things on paper before you enter answers.
- Very Limited Submissions!

---

1. Question Details

All of these sequences have end behavior \( \lim_{n \to \infty} a_n = 0 \).

Get out a clean sheet of paper. Write down all eight sequences, ordered by tail thickness.

After you are done ordering them on paper, order them in WebAssign below. Select 1 for the thinnest tail and 8 for the thickest.

\[
\begin{align*}
\frac{1}{n!} & \quad - \\
\frac{1}{n} & \quad - \\
\frac{1}{\ln(n)} & \quad - \\
\frac{1}{2^n} & \quad - \\
\frac{1}{n \ln(n)} & \quad - \\
\frac{1}{2^n n!} & \quad - \\
\frac{1}{n^2} & \quad - \\
\frac{1}{\sqrt{n}} & \quad - 
\end{align*}
\]
All of the following sequences have end behavior \( \lim_{n \to \infty} a_n = \infty \).

Get out a clean sheet of paper. Write down all eight sequences, ordered by the speed at which they go to infinity.

After you are done ordering them on paper, order them in WebAssign below. Select 1 for the slowest and 8 for the fastest.

\[
\begin{align*}
\sqrt{n} & \quad - \quad \diamond \\
10n^{1/4} & \quad - \quad \diamond \\
n \ln(n) & \quad - \quad \diamond \\
n & \quad - \quad \diamond \\
n2^n & \quad - \quad \diamond \\
n! & \quad - \quad \diamond \\
10^n & \quad - \quad \diamond \\
en & \quad - \quad \diamond
\end{align*}
\]

Consider the two sequences

\[
A_n = \frac{n}{\sqrt{n^3 + n + 1}} \quad \text{and} \quad B_n = \frac{1}{n^{1/2}}
\]

1. Both sequences have end behavior zero. Which of the following is a true statement about their tail thicknesses?
   - \( A_n \) has a thicker tail than \( B_n \).
   - \( A_n \) has a thinner tail than \( B_n \).
   - Both tails are the same thickness

2. If possible, determine whether the series \( \sum_{n=1}^{\infty} A_n \) converges or diverges.
   - \( \sum_{n=1}^{\infty} A_n \) converges
   - \( \sum_{n=1}^{\infty} A_n \) diverges
   - There is not enough information to decide.
Consider the two sequences
\[ A_n = \frac{5}{n^{3/2} \ln(n)} \quad \text{and} \quad B_n = \frac{1}{n^{3/2}} \]

1. Both sequences have end behavior zero. Which of the following is a true statement about their tail thicknesses?
   - \( A_n \) has a thicker tail than \( B_n \).
   - \( A_n \) has a thinner tail than \( B_n \).
   - Both tails are the same thickness

2. If possible, determine whether the series \( \sum_{n=1}^{\infty} A_n \) converges or diverges.
   - \( \sum_{n=1}^{\infty} A_n \) converges
   - \( \sum_{n=1}^{\infty} A_n \) diverges
   - There is not enough information to decide.

Consider the sequence
\[ a_n = \frac{n^2 - n}{n^5 + n} \]

Write down the simplest possible sequence that has the same tail thickness.
\[ b_n = \]

Which is true?
- \( \sum_{n=1}^{\infty} a_n \) converges
- \( \sum_{n=1}^{\infty} a_n \) diverges
- There is not enough information to decide.
6. Consider the sequence
\[ a_n = \frac{4^n - n^4}{5^n + n^5} \]
Write down the simplest possible sequence that has the same tail thickness.
\[ b_n = \]
Which is true?
- \[ \sum_{n=1}^{\infty} a_n \text{ converges} \]
- \[ \sum_{n=1}^{\infty} a_n \text{ diverges} \]
- There is not enough information to decide.

7. Consider the following hierarchy, in which the boxes represent spaces to put other series.

\[
\begin{align*}
& A \quad B \quad C \quad D \quad E \\
& \frac{1}{\sqrt{n}} \quad \frac{1}{n} \quad \sqrt[n]{x} \quad \frac{1}{5^n} \quad \frac{1}{n^2} \quad \frac{1}{5^n}
\end{align*}
\]
For each series below, select the letter of the box it belongs in. Select 'None of these' if this series has the same tail thickness as one of the four series shown above.

- Select \[ \sum 2^n \]
- Select \[ \sum (0.2)^n \]
- Select \[ \sum \frac{1}{2^n} \]
- Select \[ \sum \frac{1}{n^2} \]
- Select \[ \sum \frac{n^2}{5^n} \]
Consider the following hierarchy, in which the boxes represent spaces to put other series.

\[ A \Rightarrow \sum 2^n \Rightarrow B \Rightarrow \sum 1^n \Rightarrow C \Rightarrow \sum \left( \frac{1}{2} \right)^n \Rightarrow D \Rightarrow \sum \left( \frac{1}{10} \right)^n \Rightarrow E \]

For each series below, select the letter of the box it belongs in. Select 'None of these' if this series has the same tail thickness or speed to infinity as one of the four series shown above.

---Select--- \( \sum \frac{1}{n} \)

---Select--- \( \sum \frac{1}{\sqrt{n}} \)

---Select--- \( \sum \frac{1}{5^n} \)

---Select--- \( \sum \frac{1}{n^5} \)

9. **Question Details**

**sp16 give any series thinner 2** [3573937]

Find a reasonably simple series whose tail is thinner than the tail of the given series.

\[ \sum \frac{1}{10^n} \lll \sum \underline{} \]

10. **Question Details**

**sp16 give any series thicker 2** [3573941]

Find a reasonably simple series with a tail that is thicker than the tail of the given series.

\[ \sum \underline{} \lll \sum \left( \frac{1}{2} \right)^n \]

11. **Question Details**

**sp16 give any series between 2** [3573935]

Find a reasonably simple series whose tail thickness is between the tail thicknesses of two given series.

\[ \sum \frac{n^4}{n^5 + 16n} \lll \sum \underline{} \lll \sum \frac{n^5 + n}{2^n} \]

12. **Question Details**

**sp16 give any series thinner 3** [3573938]

Find a reasonably simple series whose tail is thinner than the tail of the given series.

\[ \sum \frac{1}{n!} \lll \sum \underline{} \]
Find a reasonably simple series with a tail that is thicker than the tail of the given series.

\[ \sum \quad \quad \quad \quad \quad \quad \quad \quad > > \sum \frac{1}{\ln n} \]