Partial Fractions Overview

- The *partial fractions decomposition* is the result of turning a single fraction into the sum of simpler fractions.
- The process starts by stating the **form** of the decomposition, in terms of unknown constants, $A$, $B$, $C$, $D$, etc. Then uses the form to setup a system of equations to find the constants.
- The form of the decomposition is determined by factors of the denominator of the original fraction.
  - Non-repeated linear factors.
  - Irreducible quadratic factors.
  - Repeated factors.
- When adding multiple fractions together, the *least common denominator* is the simplest denominator. The *partial fractions form* represents the most general sum that could produce the given denominator.
Non-repeated linear factors: \((ax + b)\)

- Non-repeated linear factors produce a single term: \(\frac{A}{ax + b}\)
- There is one term per factor, using different constants.
- Example:

\[
\frac{5x + 7}{(x + 2)(2x + 5)(x - 3)} = \frac{A}{x + 2} + \frac{B}{2x + 5} + \frac{C}{x - 3}
\]

- Factor before finding form:

\[
\frac{5x + 7}{x^2 - x - 6} = \frac{5x + 7}{(x + 2)(x - 3)} = \frac{A}{x + 2} + \frac{B}{x - 3}
\]
Irreducible quadratic factors: \((ax^2 + bx + c)\)

- If a quadratic can be factored into two linear factors, use the previous case to find the form.
- An irreducible quadratic is a quadratic that cannot be factored in the real numbers (or has non-real zeros).
- Irreducible quadratic factor term: \(\frac{Ax + B}{ax^2 + bx + c}\)
- Example:

\[
\frac{x^2}{(x^2 - 4)(x^2 + 4)} = \frac{x^2}{(x - 2)(x + 2)(x^2 + 4)} = \frac{A}{x - 2} + \frac{B}{x + 2} + \frac{Cx + D}{x^2 + 4}
\]
Repeated Factors: \((ax + b)^k\) or \((ax^2 + bx + c)^k\)

- Repeated factors can produce multiple terms.
- Example:

\[
\frac{x^2 + x + 1}{(x + 2)^3} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 2)^3}
\]

- The general form has multiple terms because they have the desired least common denominator.
- More Examples:

\[
\frac{x^2 + 1}{x^2(x + 4)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 4} + \frac{D}{(x + 4)^2}
\]

\[
\frac{x^2 + 1}{x(x^2 + 4)^2} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4} + \frac{Dx + F}{(x^2 + 4)^2}
\]
Work Expectations

For partial fractions you must clearly show:

- Correct and clearly stated *partial fractions form*.
- All algebraic work to determine the coefficients:
  - Work to setup a system of equations on the constants.
  - Clear statement of the system of equations.
  - Work to solve the system of equations.
- A rewritten integral using the partial fractions decomposition.
- Find the antiderivative of the rewritten integral then state the final answer.