Partial Fractions I
Warmup: Answers

\[ \int \frac{1}{x} \, dx = \ln(|x|) + C \]

\[ \int \frac{1}{x^2} \, dx = -\frac{1}{x} + C \]

\[ \int \frac{1}{5x + 2} \, dx = \frac{1}{5} \ln(|5x + 2|) + C \]

\[ \int \frac{1}{(5x + 2)^2} \, dx = -\frac{1}{5(5x + 2)} + C \]
Which of the following two antiderivatives can be computed using elementary antiderivatives?

\[ \int \frac{13x + 2}{(x - 2)(3x + 1)} \, dx = \text{Not Elementary} \]

\[ \int \left( \frac{4}{x - 2} + \frac{1}{3x + 1} \right) \, dx = 4 \ln \left( |x - 2| \right) + \frac{1}{3} \ln \left( |3x + 1| \right) + C \]
Algebra Review: Combine the following two fractions. Find the *least common denominator*. Simplify the numerator. Combine two fractions into one.

\[
\frac{4}{x - 2} + \frac{1}{3x + 1} = \frac{4(3x + 1) + (x - 2)}{(x - 2)(3x + 1)} = \frac{12x + 4 + x - 2}{(x - 2)(3x + 1)} = \frac{13x + 2}{(x - 2)(3x + 1)}
\]
Example: Rewrite the single fraction as two. Find elementary antiderivatives.

Use \( \frac{4}{x - 2} + \frac{1}{3x + 1} = \frac{13x + 2}{(x - 2)(3x + 1)} \) to compute

\[
\int \frac{13x + 2}{(x - 2)(3x + 1)} \, dx = \\
\int \left( \frac{4}{x - 2} + \frac{1}{3x + 1} \right) \, dx = \\
4 \ln (|x - 2|) + \frac{1}{3} \ln (|3x + 1|) + C
\]
Partial Fractions

- Partial fractions transforms a single fraction, \( \frac{13x + 2}{(x - 2)(3x + 1)} \), into the sum of fractions, \( \frac{4}{x - 2} + \frac{1}{3x + 1} \).

- **Step 1:** Assume the partial fraction form: \( \frac{A}{x - 2} + \frac{B}{3x + 1} \).

- **Step 2:** Combine the partial fraction form together:

  \[ \frac{A}{x - 2} + \frac{B}{3x + 1} = \cdots = \frac{(3A + B)x + (A - 2B)}{(x - 2)(3x + 1)} \]

- **Step 3:** Compare to original fraction:

  \[ 3A + B = 13 \quad \text{and} \quad A - 2B = 2 \]

- **Step 4:** Solve system of equations to find \( A = 4 \) and \( B = 1 \).

- **Step 5:** Use result to compute original antiderivative.