A limit describes behavior of a function close to a point.

\[ \lim_{x \to a} f(x) \]

A limit exists (and equals \( L \)) if output values are arbitrary close to \( L \), for all input values sufficiently close to \( a \).

Limits can describe the behavior at a division by zero point:

**Derivative Definition:**

\[ \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \]

Limits can describe the behavior as a value becomes infinite:

**Riemann Integral:**

\[ \int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^n f(x_i^*) \Delta x \]
Limits at Infinity: \( \lim_{x \to \infty} f(x) \)

- Limits at infinity describe the behavior as the input value grows larger and larger without bound (approaches infinity).
- Some example behaviors as \( x \to \infty \)
  - Limit is \( L \).
  - Grows to \( \infty \).
  - Does not exist.

\[
\begin{align*}
\lim_{x \to \infty} f(x) & = L \\
\lim_{x \to \infty} f(x) & = \infty \\
\lim_{x \to \infty} f(x) & \text{ DNE}
\end{align*}
\]
Comparing speeds functions go to infinity.

Suppose $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to \infty} g(x) = \infty$.

- $f(x) \to \infty$ faster than $g(x) \to \infty$: $f(x) \gg g(x)$
  \[
  \lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty
  \]

- $f(x) \to \infty$ about the same speed as $g(x) \to \infty$: $f(x) \sim g(x)$
  \[
  \lim_{x \to \infty} \frac{f(x)}{g(x)} = L \neq 0
  \]

- $f(x) \to \infty$ slower than $g(x) \to \infty$: $f(x) \ll g(x)$
  \[
  \lim_{x \to \infty} \frac{f(x)}{g(x)} = 0
  \]

- Use this to order functions by their speed to infinity.