Instructions

- Read today's Notes and Learning Goals.
- Each problem will have a computational component and a graphing component.
- The graphing component is at least as important as the computational component.
- The graphing component will involve limited submission questions, sometimes also penalty scored.
- Make sure you complete all graphing instructions before you attempt these questions.
- Most problems that ask you for a graph will allow you to compare your graph to a solution key that appears after you submit one answer for each answer box.

1. Question Details

The position of a train from its home station, \( s(t) \), is a function of time where \( s \) is measured in feet and \( t \) in seconds. The graph of the velocity of this train is

![Graph of velocity](image)

a. Find the area of the shaded region. Give an exact answer with correct units.

b. Compute the change in the train's position on the time interval \( 2 \leq t \leq 4 \) seconds. Give an exact answer with correct units.

\[ \Delta s = \quad \]  

c. Discuss the above two results with your study group.
The position of a train from its home station, \( s(t) \), is a function of time where \( s \) is measured in feet and \( t \) in seconds. The graph of the velocity of this train is

a. Find the area of the shaded region. Give an exact answer with correct units.

b. Compute the change in the train’s position on the time interval 1 \( \leq t \leq 3.5 \) seconds. Give an exact answer with correct units.

\[ \Delta s = \]

c. Discuss the above two results with your study group.
The position of a train from its home station, \( s(t) \), is a function of time where \( s \) is measured in feet and \( t \) in seconds. The graph of the velocity of this train is

![Graph of velocity vs time](image)

a. Find the area of the shaded rectangle. Give an exact answer with correct units.

b. Estimate the change in the train's position on the time interval \([2,6]\) by using the left-hand endpoint. Give the estimate with correct units.

\[
\Delta s = \boxed{} 
\]

c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{ds}{dt} \).

- The rectangle leaves space under the graph.
- The rectangle sticks out over graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (b) is

- Less than the actual change in the train's position.
- More than the actual change in the train's position.
- Equal to the actual change in the train's position.
The position of a train from its home station, \( s(t) \), is a function of time where \( s \) is measured in feet and \( t \) in seconds. The graph of the \textit{velocity} of this train is

\[
\begin{array}{c}
\text{\( \frac{ds}{dt} \) (ft/s)} \\
\end{array}
\]

\[
\begin{array}{c}
t (\text{sec})
\end{array}
\]

a. Find the area of the shaded rectangle. Give an exact answer with correct units.

b. Estimate the change in the train's position on the time interval \([2, 6]\) by using the \textit{right-hand endpoint}. Give the estimate with correct units.

\[ \Delta s \approx \]

c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{ds}{dt} \).

- The rectangle leaves space under the graph.
- The rectangle sticks out over graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (b) is

- Less than the actual change in the train's position.
- More than the actual change in the train's position.
- Equal to the actual change in the train's position.
The position of a train from its home station, \( s(t) \), is a function of time where \( s \) is measured in feet and \( t \) in seconds. The graph of the velocity of this train is

![Graph of velocity vs time](image)

\( t \) (sec)

\( \frac{ds}{dt} \) (ft/s)

a. Find the area of the shaded rectangle. Give an exact answer with correct units.

b. Estimate the change in the train's position on the time interval [2,6] by using the left-hand endpoint. Give the estimate with correct units.

\[ \Delta s \approx \]

c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{ds}{dt} \).

- The rectangle leaves space under the graph.
- The rectangle sticks out over graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (b) is

- Less than the actual change in the train's position.
- More than the actual change in the train's position.
- Equal to the actual change in the train's position.
The position of a train from its home station, \( s(t) \), is a function of time where \( s \) is measured in feet and \( t \) in seconds. The graph of the velocity of this train is shown.

a. Find the area of the shaded rectangle. Give an exact answer with correct units.

b. Estimate the change in the train's position on the time interval \([2, 6]\) by using the right-hand endpoint. Give the estimate with correct units.

\[ \Delta s = \Box \]

c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{ds}{dt} \)?

- The rectangle leaves space under the graph.
- The rectangle sticks out over graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (b) is

- Less than the actual change in the train's position.
- More than the actual change in the train's position.
- Equal to the actual change in the train's position.

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The height of a moving object is a function of time, \( h(t) \), with \( h \) in meters and \( t \) in seconds. The velocity of the object is graphed below.
a. Use the velocity at the left-hand endpoint to estimate the change in height of this object on the interval \([0.5, 1]\).

\[ \Delta h \approx \text{[ ]} \]

b. Select which of the following shaded rectangles represents the estimate found in part (a).
c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{dh}{dt} \)?

- The rectangle leaves space under the graph.
- The rectangle sticks out above the graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (a) is

- Less than the actual change in height.
- More than the actual change in height.
- Equal to the actual change in height.

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8. Question Details

The graph of the velocity, \( \frac{dh}{dt} \), of an object is

a. Use the velocity at the right-hand endpoint to estimate the change in height of this object on the interval \([0.5, 1]\).

\[ \Delta h = \text{ } \]

b. Select which of the following shaded rectangles represents the estimate found in part (a).
c. Which sentence best describes how well the shaded rectangle fits the graph of $\frac{dh}{dt}$?

- The rectangle leaves space under the graph.
- The rectangle sticks out above the graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (a) is

- Less than the actual change in height.
- More than the actual change in height.
- Equal to the actual change in height.

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9. The height of a moving object is a function of time, $h(t)$, with $h$ in meters and $t$ in seconds. The velocity of the object is graphed below.
a. Use the velocity at the left-hand endpoint to estimate the change in height of this object on the interval \([0.5, 1]\).

\[ \Delta h = \text{[Blank]} \]

b. Select which of the following shaded rectangles represents the estimate found in part (a).
c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{dh}{dt} \)?

- The rectangle leaves space under the graph.
- The rectangle sticks out above the graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (a) is

- Less than the actual change in height.
- More than the actual change in height.
- Equal to the actual change in height.

10. Question Details

The height of a moving object is a function of time, \( h(t) \), with \( h \) in meters and \( t \) in seconds. The velocity of the object is graphed below.

a. Use the velocity at the right-hand endpoint to estimate the change in height of this object on the interval \([0.5, 1]\).

\[ \Delta h = \]

b. Select which of the following shaded rectangles represents the estimate found in part (a).
c. Which sentence best describes how well the shaded rectangle fits the graph of \( \frac{dh}{dt} \)?

- The rectangle leaves space under the graph.
- The rectangle sticks out above the graph.
- The rectangle matches the graph perfectly.

d. The estimate in part (a) is

- Less than the actual change in height.
- More than the actual change in height.
- Equal to the actual change in height.
11. An object moves along the y-axis. Its velocity is always 4 m/s.

a. Graph the velocity data. Be sure to correctly label your graph. Use $t$ in seconds for the input axis.

b. Compute the change in the object’s position during the time interval $3 \leq t \leq 7$ seconds. Give an exact answer with correct units.

\[ \Delta y = \]

c. Shade an area in your graph that matches your computation of $\Delta y$.

d. After you are confident that you have a correct graph with a correct shaded region, answer these questions. Warning! You only get one try on each question. After you answer the questions (and the one above) a solution will appear.

The correct shaded region has the shape of a

- Triangle
- Rectangle

The correct shaded region is located

- Below the $t$-axis
- Above the $t$-axis

12. A falling object has constant velocity $-125$ m/s.

a. Graph the velocity data. Be sure to correctly label your graph. Use $t$ in seconds for the input axis.

b. Compute the change in height during the time interval $4 \leq t \leq 10$ seconds. Give an exact answer with correct units.

\[ \Delta h = \]

c. Shade an area in your graph that matches your computation of $\Delta h$.

d. After you are confident that you have a correct graph with a correct shaded region, answer this questions. Warning! You only get one try on each question. After you answer the questions (and the one above) a solution will appear.

The correct shaded region is located

- Above the $t$-axis
- Below the $t$-axis

13. Suppose that the rate of change of the volume of water in a tank is

\[ \frac{dV}{dt} = 3.2 - 1.3\sin(2.5t + 2.2) \text{ in}^3/\text{sec} \]

Where $t$ is in seconds.

a. Estimate $\Delta V$ on $[1, 1.5]$. Use 5 intervals, each having width 0.1, and use rate of change data from the left side of each interval. Round your answer to 3 decimal places.

\[ \Delta V \approx \]

b. From the graphs below, select the one in which the shaded region best matches the answer in part (a).
c. Which sentence best describes how well the shaded rectangles fit the graph of \( \frac{dV}{dt} \)?

- The rectangles leave space under the graph.
- The rectangles stick out above the graph.
- The rectangles match the graph perfectly.

d. The estimate in part (a) is

- Less than the actual change in volume.
- More than the actual change in volume.
- Equal to the actual change in volume.
An object is launched straight upward. Its height, \( h \), is measured in feet, and time, \( t \), is in seconds. Its velocity is shown in the table below.

<table>
<thead>
<tr>
<th>( \frac{dh}{dt} ) (ft/s)</th>
<th>96</th>
<th>80</th>
<th>64</th>
<th>48</th>
<th>32</th>
<th>16</th>
<th>0</th>
<th>-16</th>
<th>-32</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t ) (s)</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

a. Graph the velocity data. Be sure to correctly label your graph.

b. Estimate \( \Delta h \) on \([0.0, 0.5]\), assuming a constant velocity of 96 ft/s. Shade a rectangle in your graph that matches your computation. Enter an exact answer with correct units.

\[ \Delta h \text{ on } [0.0, 0.5] = \] ____________

c. Estimate \( \Delta h \) on \([0.5, 1.0]\), assuming a constant velocity of 80 ft/s. Shade a rectangle in your graph that matches this computation. Enter an exact answer with correct units.

\[ \Delta h \text{ on } [0.5, 1.0] = \] ____________

d. Estimate \( \Delta h \) on the time intervals shown below. On each interval, assume constant velocity measured at the first instant in the interval. Also, shade rectangles that match each computation.

\[ \Delta h \text{ on } [1.0, 1.5] = \] ____________
\[ \Delta h \text{ on } [1.5, 2.0] = \] ____________
\[ \Delta h \text{ on } [2.0, 2.5] = \] ____________
\[ \Delta h \text{ on } [2.5, 3.0] = \] ____________

e. Estimate \( \Delta h \) on \([0, 3]\) by adding up your last 6 answers.

\[ \Delta h \text{ on } [0, 3] \approx \] ____________

**Warning!** The next two questions are **penalty scored**. Half credit after one wrong answer. Zero points after two wrong answers.

**Shade your rectangles first!** Get feedback. After one submission for every answer box (below and above) you can see a solution.

f. In a correct graph, with a correct shaded region, how many rectangles are shaded?

__________

g. Which sentence best describes how well the shaded rectangles fit the graph of \( \frac{dh}{dt} \)?

- The rectangles stick out above the graph.
- The rectangles match the graph perfectly.
- The rectangles sit under the graph.
An object moves along the y-axis. Its velocity at various times is:

<table>
<thead>
<tr>
<th>$\frac{dy}{dt}$ (m/s)</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>t (s)</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
<td>5.0</td>
<td>5.5</td>
<td>6.0</td>
<td>6.5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

a. Graph the velocity data. Be sure to correctly label your graph.

b. Estimate $\Delta y$ on $[3, 7]$. Use 8 time intervals, each lasting 0.5 seconds. In each interval assume constant velocity, using data from the left side of each interval.

$$\Delta y \approx \text{Rectangles}$$

c. In your graph of the velocity data, shade a collection of rectangles that matches your computation.

**Warning!** The next two questions are **penalty scored**. Half credit after one wrong answer. Zero points after two wrong answers.

**Shade your rectangles first!** Get feedback. After one submission for every answer box (below and above) you can see a solution.

e. Which sentence best describes how well the shaded rectangles fit the graph of $\frac{dy}{dt}$?

- The rectangles leave space under the graph.
- The rectangles stick out above the graph.
- The rectangles match the graph perfectly.

f. Which sentence best describes the accuracy of your estimate?

- Your estimate is larger than the true value of $\Delta y$.
- Your estimate is the correct value of $\Delta y$.
- Your estimate is smaller than the true value of $\Delta y$. 
Suppose that the rate of change of the volume of water in a tank is
\[
\frac{dV}{dt} = 3.2 - 1.3\sin(2.5t + 2.2) \text{ in}^3/\text{sec}
\]
Where \( t \) is in seconds.

a. Graph \( \frac{dV}{dt} \) on the domain \([0, 2]\).

b. Estimate \( \Delta V \) on \([0, 1]\). Use 10 intervals, each having width 0.1, and use rate of change data from the left side of each interval. Round your answer to 3 decimal places.
\[
\Delta V \approx \underline{\text{ }}
\]

c. In your graph of \( \frac{dV}{dt} \), shade a collection of rectangles that matches your computation.

**Warning!** The next two questions are **penalty scored**. Half credit after one wrong answer. Zero points after two wrong answers.

**Shade your rectangles first!** Get feedback. After one submission for every answer box (below and above) you can see a solution.

d. Which sentence best describes how well the shaded rectangles fit the graph of \( \frac{dV}{dt} \)?

- The rectangles leave space under the graph.
- The rectangles stick out above the graph.
- The rectangles match the graph perfectly.

e. Which sentence best describes the accuracy of your estimate?

- Your estimate is smaller than the true value of \( \Delta V \).
- Your estimate is the correct value of \( \Delta V \).
- Your estimate is larger than the true value of \( \Delta V \).
The population of fish, \( P \), in a lake is a function of time, \( t \), measured in years. The rate of change of \( P \) is given by
\[
\frac{dP}{dt} = \frac{7600e^{0.4t}}{(19 + e^{0.4t})^2} \text{ fish/year.}
\]

a. Graph \( \frac{dP}{dt} \) on the domain \([0, 20]\). Make sure your graph is properly labeled.

b. Estimate the change in population on the time interval \( 0 \leq t \leq 20 \) years. Use 10 intervals, each lasting two years. Use rate of change data from the left side of each interval. Round your answer to the nearest fish. (Units are provided with the answer box.)
\[
\Delta P \approx \text{fish}.
\]

c. In your graph of \( \frac{dP}{dt} \), shade a collection of rectangles that matches your computation of \( \Delta P \).

**Warning!** The next two questions are penalty scored. Half credit after one wrong answer. Zero points after two wrong answers.

Shade your rectangles first! Get feedback. After one submission for every answer box (below and above) you can see a solution.

d. In a correct graph, with a correct shaded region, how many rectangles are shaded?

\[\boxed{\text{rectangles}}\]

e. Which sentence best describes how well the shaded rectangles fit the graph of \( \frac{dP}{dt} \)?

- The rectangles stick out above the graph.
- The rectangles leave space below the graph.
- The rectangles match the graph perfectly.
- Some stick out above; some leave space below.
The population of fish, \( P \), in a lake is a function of time, \( t \), measured in years. The rate of change of \( P \) is given by
\[
\frac{dP}{dt} = \frac{7600e^{0.4t}}{(19 + e^{0.4t})^2} \text{ fish/year.}
\]

A student estimates \( \Delta P \) on \([0, 20]\). The area corresponding to his estimate is shown below. What was his estimate? Do not include units.
\[
\Delta P \approx \boxed{\text{fish.}}
\]