1. (10 pts.) The graph below shows the velocity, \( v = f(t) \), of a particle moving along a coordinate line. Explain your reasoning for each question below.

![Velocity Graph](image)

(a) When does the particle move forward? Backward?

- **Forward** when \( 0 \leq t \leq 2 \), and when \( 9 \leq t \leq 11 \). Because velocity is positive.
- **Backward** when \( 4 \leq t \leq 9 \). Because velocity is negative.

(b) When does the particle speed up? Slow down?

- **Speeds up** when \( 8 \leq t \leq 10 \). Because velocity is increasing.
- **Slows down** when \( 0 \leq t \leq 2, 4 \leq t \leq 6, \) and \( 10 \leq t \leq 11 \). Because vel. is decreasing.

(c) When does the particle move at its greatest speed?

- \( t = 10 \). Because here velocity is longest, even compared to negative velocities.

(d) When does the particle stand still for more than an instant?

- **Stands Still** when \( 2 \leq t \leq 4 \), because velocity is 0.
2. (10 pts.) A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometers to the north of P and traveling at 80 km/hr, while car B is 15 kilometers to the east of P and traveling at 10 km/hr. How fast is the distance between the two cars changing at that time?

\[ x^2 + y^2 = z^2 \]
\[ 2xx' + 2yy' = 2zz' \]
\[ xx' + yy' = zz' \]
\[ z' = \frac{xx' + yy'}{z} \]
\[ = \frac{15 \cdot 10 + 10 \cdot 80}{\sqrt{10^2 + 15^2}} \]
\[ = \frac{190}{\sqrt{13}} \text{ km/hr.} \]

3. (15 pts.) Find where the curve \( y = \cot(x) + 2x \) has a horizontal tangent in the interval \((0, \pi)\).

\[ y' = -\csc^2 x + 2 = 0 \]
\[ \csc^2 x = 2 \]
\[ \csc x = \pm \sqrt{2} \]
\[ \sin x = \pm \frac{1}{\sqrt{2}} \]

\[ \chi = \frac{\pi}{4}, \frac{3\pi}{4} \]
4. (15 pts.) The graph of the equation $x^2 - xy + y^2 = 9$ is an ellipse. Find the lines tangent to this curve at the two points where it intersects the x-axis. Show that these lines are parallel.

(1) Find where it intersects: set $y = 0$.  
\[ x^2 = 9 \]
\[ x = \pm 3 \]

(2) Differentiate: 
\[ 2x - xy' - y + 2yy' = 0 \]

(3) Plug in locations:
At $(3,0)$: 
\[ 2(3) - 3y' + 2yy' = 0 \]
\[ 6 + 3y' = 0 \Rightarrow y' = -2 \]

At $(-3,0)$:
\[ 2(-3) + 3y' + 0 + 2yy' = 0 \]
\[ -6 + 3y' = 0 \Rightarrow y' = 2 \]

Equations of lines:
\[ y = 2(x-3) \]
\[ y = 2(x+3) \]

Parallel because slopes are equal.

5. (5 pts.) Let $f(x) = \sin(2x)$. Find $dy$ if $a = \pi$ and $dx = \frac{\pi}{100}$.

Raphase: 
$y = \sin(2x)$. Find $\Delta y$ at location $x = \pi$ with $\Delta x = \frac{\pi}{100}$

Solution: 
$y' = \cos(2x) \cdot 2$

\[ \Delta y = y'(\pi) \Delta x \]
\[ = \cos(2\pi) \cdot 2 \cdot \frac{\pi}{100} = \frac{2\pi}{50} \]
6. (5 pts.) Given \( f(0) = 3 \), and \( f'(0) = 4 \) find the derivative of \( g(x) = \frac{\sin^{-1}(x) + 2x}{f(x)} \) at \( x = 0 \).

\[
g'(x) = \frac{f(x) \left[ \frac{1}{\sqrt{1-x^2}} + 2 \right] - \left[ \sin^{-1}(x) + 2x \right] f'(x)}{[f(x)]^2}
\]

\[
g'(0) = 3 \cdot \left[ \frac{1}{\sqrt{1-0}} + 2 \right] - \left[ \sin^{-1}(0) + 0 \right] \cdot 4 \cdot 3^2
\]

\[
= 1
\]

7. (5 pts. each) Find \( y' \). Show your work and only do obvious simplifications.

(a) \( y = (x^2 + 5x)^4 \)

\[
y' = 4 (x^2 + 5x)^3 (2x + 5)
\]

(b) \( y = xe^{\sin(x)} \)

\[
y' = 1 \cdot e^{\sin x} + x \cdot e^{\sin x} \cdot \cos x
\]

(c) \( y = (\ln(\sqrt{x}))^3 \)

\[
y' = 3 \left( \frac{1}{2x} \right)^2 \cdot \frac{1}{x^{\frac{1}{2}}} - \frac{1}{2} \cdot \frac{1}{x^{\frac{3}{2}}}
\]
8. (5 pts. each) Find \( \frac{dy}{dx} \) in each of the following equations.

(a) \( \ln(xy) + y^3 = 4x \)

\[
\frac{1}{xy} \cdot (xy' + y) + 3y^2 y' = 4 \quad \Rightarrow \quad \frac{y'}{y} + \frac{1}{x} + 3y^2 y' = 4 - \frac{1}{x}
\]

\[
\int \frac{y'}{y} + \frac{1}{x} + 3y^2 y' = 4 - \frac{1}{x} \quad \Rightarrow \quad y' = \frac{4 - \frac{1}{x}}{\frac{1}{y} + 3y^2}
\]

(b) \( y = (\sin(x))^{\cos(x)} \)

\[
\ln y = \cos x \ln(\sin x)
\]

\[
\frac{1}{y} y' = -\sin x \ln(\sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x
\]

\[
y' = (\sin x)^{\cos x} \left[ -\sin x \ln(\sin x) + \frac{\cos^2 x}{\sin x} \right]
\]

9. (5 points) Let \( f(x) = \log_4(x^2) \). Find \( f''(x) \).

\[
f = \frac{\ln x^2}{\ln 4} = \frac{2}{\ln 4} \ln x
\]

\[
f' = \frac{2}{\ln 4} \cdot \frac{1}{x} = \frac{2}{\ln 4} \cdot x^{-1}
\]

\[
f'' = -\frac{2}{\ln 4} \cdot x^{-2}
\]
10. (10 points) Consider \( y = (x^2 - 3)^{4x} \).

(a) Explain why \( y' \neq 4x(x^2 - 3)^{4x-1}(2x) \).

Variable base and variable exponent.
Must use logarithmic differentiation.

(b) Find \( y' \)

\[
\ln y = 4x \ln (x^2 - 3)
\]

\[
\frac{y'}{y} = 4 \ln (x^2 - 3) + 4x \cdot \frac{1}{x^2 - 3} \cdot 2x
\]

\[
y' = (x^2 - 3)^{4x} \left[ 4 \ln (x^2 - 3) + \frac{8x^2}{x^2 - 3} \right]
\]