1. (10 pts.) The graph of a function \( g(x) \) is shown at right. Approximate the rate of change of \( g \) at the point \( x = 0.5 \). Your solution must include at least three reasonably accurate secant slopes. Be sure to show all calculations.

<table>
<thead>
<tr>
<th>( \Delta x )</th>
<th>( \frac{\Delta g}{\Delta x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.3</td>
<td>.67</td>
</tr>
<tr>
<td>.4</td>
<td>1</td>
</tr>
<tr>
<td>.6</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ \frac{\Delta g}{\Delta x} \text{ at } (3, 5): \]
\[ \frac{\Delta g}{\Delta x} = \frac{5 - 5}{3 - 5} = -0.5 \]

\[ \frac{\Delta g}{\Delta x} \text{ at } (4, 5): \]
\[ \frac{\Delta g}{\Delta x} = \frac{5 - 5}{4 - 5} = 0 \]

\[ \frac{\Delta g}{\Delta x} \text{ at } (5, 6): \]
\[ \frac{\Delta g}{\Delta x} = \frac{6 - 5}{5 - 5} = \text{undefined} \]

Guess the slope between 4 and 6:
Say \( 1.5 \)
2. (10 pts.) Use the graph of $f$ shown at right to answer each of the following questions. Read carefully; there will be no partial credit.

(a) $\lim_{x \to -1^+} f(x)$ exists.  **TRUE** or **FALSE**

(b) $\lim_{x \to -1} f(x)$ exists.  **TRUE** or **FALSE**

(c) $\lim_{x \to -3} f(x) = \frac{1}{3}$

(d) $f(x)$ is differentiable at $x = 3$.  **TRUE** or **FALSE**

(e) On what intervals is $f$ continuous?

$[-5, -3), (-3, -1], (-1, 5]$  

3. (5 pts.) Compute $\lim_{t \to 0} \frac{\tan(3t)}{t}$. Decimal approximation will not suffice for full credit nor will graphs.

\[
\lim_{t \to 0} \frac{\sin(3t)}{t \cos(3t)} = \lim_{t \to 0} \frac{3}{3} \cdot \frac{1}{\cos(3t)} = 1 \cdot 3 \cdot \frac{1}{\cos 0} = 3
\]
4. (15 pts) Determine each of the following limits. If a limit does not exist, explain why. Remember to show your work. Decimal approximations will not suffice for full credit nor will graphs.

(a) \[ \lim_{x \to \sqrt{2}} \frac{x^2 - 2}{x - \sqrt{2}} \cdot \frac{(x + \sqrt{2})}{(x + \sqrt{2})} = \lim_{x \to \sqrt{2}} \frac{(x^2 - 2) \cdot (x + \sqrt{2})}{(x - \sqrt{2})} = \sqrt{2} + \sqrt{2} = 2\sqrt{2} \]

OR
\[ \frac{(x - \sqrt{2}) \cdot (x + \sqrt{2})}{(x - \sqrt{2})} \to \sqrt{2} + \sqrt{2} \]

(b) \[ \lim_{x \to 1} \frac{x^2 - 1}{|x - 1|} = \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)(x + 1)}{x - 1} = 2 \]
\[ \lim_{x \to 1^-} \frac{x^2 - 1}{-(x - 1)} = \lim_{x \to 1^-} \frac{(x - 1)(x + 1)}{-(x - 1)} = -2 \]

Therefore, 2-sided limit does not exist.

(c) \[ \lim_{x \to \infty} \frac{4x^3 - x^2}{5x^3 + 1} = \lim_{x \to \infty} \frac{\frac{1}{x^3} \cdot 4 - \frac{1}{x}}{\frac{1}{x^3} \cdot 5 - \frac{1}{x}} = \frac{4}{5} \]
5. (10 pts.) Sketch the graph of a function, \( f \), that has all of the following properties:

- \( f(0) = 0, f(-3) = 4, f(3) = 4 \)
- \( \lim_{x \to 2^-} f(x) = \infty, \lim_{x \to 2^+} f(x) = -\infty, \lim_{x \to \infty} f(x) = 1 \),
- \( \lim_{x \to -\infty} f(x) = 1, \lim_{x \to 2^-} f(x) = -\infty, \lim_{x \to 2^+} = \infty \).
6. (10 pts.) USE THE DEFINITION of the derivative to compute \( \frac{dy}{dx} \) for \( y = \frac{4}{x-1} \)

\[
\frac{\Delta y}{\Delta x} = \frac{\frac{y}{x-1} - \frac{y}{t-1}}{x-t} = \frac{4(t-1) - 4(x-1)}{(x-t)(x-1)(t-1)}
\]

\[
= \frac{4t - 4 - 4x + 4}{(x-t)(x-1)(t-1)} = \frac{-4(x-t)}{(x-t)(x-1)(t-1)}
\]

\[
\lim_{x \to 1} \frac{\Delta y}{\Delta x} = \frac{-4}{(x-1)(x-1)} = \frac{-4}{(x-1)^2}
\]

7. (5 pts.) Find the value of \( a \) that will make \( f(x) \) continuous at \( x = 2 \) where

\[
f(x) = \begin{cases} 
  ax + 3, & x > 2 \\
  ax^2, & x \leq 2 
\end{cases}
\]

\[
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax + 3 = 2a + 3
\]

\[
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} ax^2 = 4a
\]

\[
2a + 3 = 4a
\]

\[
3 = 2a
\]

\[
a = \frac{3}{2}
\]
Note: For the remainder of the test you may use derivative methods from section 3.2.

8. (20 pts.) Compute $y'$ for each of the following. Assume that $f$ and $g$ are unknown differentiable functions of $x$.

(a) $y = 4x^3e^x$

$$y' = 12x^2e^x + 4x^3e^x$$

(b) $y = \frac{3x^5 - x^2}{x^2 + 3}$

$$y' = \frac{(x^2 + 3)(24x^7 - 7x) - (3x^8 - x^2)(2x)}{(x^2 + 3)^2}$$

(c) $y = x^7 + \sqrt{x} - \frac{1}{\pi + 1}$

$$y' = 7x^6 + \frac{1}{2\sqrt{x}} - 0$$

(d) $y = 3e^x f(x) + g(x)$

$$y' = 3e^x f(x) + 3e^x f'(x) + g'(x)$$
9. (10 pts.) Find an equation of the line tangent to \( y = 3x + \sqrt{x} \) at the point \( x = 4 \).

\[
y = 3x + x^{\frac{1}{2}}
\]

\[
y' = 3 + \frac{1}{2} x^{-\frac{1}{2}}
\]

\[
= 3 + \frac{1}{2\sqrt{x}}
\]

\[
\text{at } x = 4,
\]

\[
\text{slope} = 3 + \frac{1}{2\sqrt{4}} = 3.25
\]

\[
y \text{-value} = 3(4) + \sqrt{4} = 14
\]

\[
\text{line} \quad y - 14 = 3.25(x - 4)
\]

\[
\text{or} \quad y = 3.25x + 1
\]

10. (5 pts.) Draw the graph of a function that is defined for all values on the closed interval \([-3, 3]\) and is continuous for all values except at \( x = 1 \).