This homework introduces a new technique for computing limits called L’Hôpital’s rule. You will need to recall the notation and meaning of \( \lim_{x \to a} h(x) \), from Lesson 2-2.

1. Know that you always start a limit computation by attempting to plug in \( x = a \).

2. Know the three most common results and what each one tells you about the limit.
   - You get a number. The formal math vocabulary is, “\( h \) is defined at \( x = a \).” The limit problem is done. The answer is your number.
   - You get \( \frac{\text{non-zero}}{0} \). The limit problem is done. The answer is “Does Not Exist”.
   - You get \( \frac{0}{0} \). The limit is indeterminate. You have to do more work.

3. If your plug-in gives \( \frac{0}{0} \) you are allowed to use L’Hôpital’s rule. Here’s how it works.
   - The original problem must have had a top and bottom
     \[ \lim_{x \to a} \frac{f(x)}{g(x)} \]
   - Differentiate top and bottom separately. (No quotient rule.)
     \[ \lim_{x \to a} \frac{f'(x)}{g'(x)} \]
   - Simplify anything obvious, and then plug in again. Usually this resolves the limit.
   - If it’s still unresolved, repeat the process.

You can find examples in Section 4.5, at Khan Academy or Patrick JMT.

4. After you resolve the limit, know how to describe the graph of \( f \) near \( x = a \). There are three common situations.
   - If \( f \) is defined at \( x = a \), you say the graph is continuous.
   - If initial plug-in or later plug-in gives \( \frac{\text{non-zero}}{0} \), the graph has a vertical asymptote.
   - If the limit was indeterminate, but eventually you got a number, the graph has a hole.
Here’s a recap of the common situations.

<table>
<thead>
<tr>
<th>Plug-in gives . . .</th>
<th>Limit equals . . .</th>
<th>Graph . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(a) ) is defined</td>
<td>( f(a) )</td>
<td>is continuous at ( x = a )</td>
</tr>
<tr>
<td>non-zero ( \frac{0}{0} )</td>
<td>DNE</td>
<td>has asymptote at ( x = a )</td>
</tr>
<tr>
<td>( \frac{0}{0} )</td>
<td>(don’t know yet)</td>
<td>(usually has a hole) (could have an asymptote)</td>
</tr>
</tbody>
</table>

5. Know that many other initial situations can occur. Here are two. Others may appear (with hints) in your homework.

- L’Hôpital’s rule applies if the initial plug-in gives

\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}
\]

- L’Hôpital’s rule applies if the limit location is infinity:

\[
\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}
\]

6. **Remember!** Always start with a plug-in to see what happens.

You CANNOT use L’Hôpital’s rule unless you have \( \frac{0}{0} \) or \( \frac{\infty}{\infty} \).