Goals:

- Compute quick derivatives.
- Compute higher derivatives.
- Know what to do when you encounter an inverse trig function or a non-base-$e$ exponential.

Exercises:

1. Find the second derivative, or $y''$, for each of the following. Assume that $a$, $b$, $c$ and $n$ are constants. Assume that $f$ and $g$ are unknown functions of $x$.

   (a) $y = c$
   (b) $y = x$
   (c) $y = x^n$
   (d) $y = e^x$
   (e) $y = \ln x$
   (f) $y = \sin x$
   (g) $y = \cos x$
   (h) $y = \tan x$
   (i) $y = \cot x$
   (j) $y = \sec x$
   (k) $y = \csc x$
   (l) $y = af + bg$

2. Find $f''$ for each of the following. Assume that $A$, $k$ and $\omega$ are constants.

   (a) $f(t) = \sin 4t$
   (b) $f(t) = \cos 0.031$
   (c) $f(t) = e^{-0.02t}$
   (d) $f(t) = e^{-t} \cos 2t$
   (e) $f(t) = A \cos \omega t$
   (f) $f(t) = e^{kt} \sin \omega t$
3. Find $y'$ for each of the following. You do not need to completely simplify. You can leave your answers as $y' = \text{stuff with } y$'s in it.

(a) $y = \arccsc x$
(b) $y = (x - 1)^{(x+1)}$

Challenges: Find $y''$ for each of the following:

1. $y = \arccsc x$
2. $y = (x - 1)^{(x+1)}$
Hints and Answers

Exercises:

1. (a) \( y'' = 0 \)
   (b) \( y'' = 0 \)
   (c) \( y'' = n(n - 1)x^{n-2} \)
   (d) \( y'' = e^x \)
   (e) \( y'' = -x^{-2} \)
   (f) \( y'' = -\sin x \)
   (g) \( y'' = -\cos x \)
   (h) \( y'' = 2 \sec x \sec x \tan x \)
   (i) \( y'' = 2 \csc x \csc x \cot x \)
   (j) \( y'' = \sec x \sec^2 + \sec x \tan x \tan x \)
   (k) \( y'' = \csc x \csc^2 + \csc x \cot x \cot x \)
   (l) \( y'' = af'' + bg'' \)

2. (a) \( f'' = -16 \sin 4t \)
   (b) \( f'' = -0.000961 \cos 0.031 \)
   (c) \( f'' = 0.004e^{-0.02t} \)
   (d) \( f'' = 4e^{-t} \sin 2t - 3e^{-t} \cos 2t \)
   (e) \( f'' = -A \omega^2 \cos \omega t \)
   (f) \( f'' = (k^2 - \omega^2)e^{kt} \sin \omega t + 2k\omega e^{kt} \cos \omega t \)

3. (a) \( y' = -\frac{1}{\csc y \cot y} = -\sin y \tan y = -\frac{1}{x \sqrt{x^2 - 1}} \)
   (b) \( y' = y \left[ \frac{x + 1}{x - 1} + \ln(x - 1) \right] = (x - 1)^{(x+1)} \left[ \frac{x + 1}{x - 1} + \ln(x - 1) \right] \)

Challenges:

1. \( y' = \frac{x \cdot \frac{1}{2}(x^2 - 1)^{-1/2} \cdot 2x + (x^2 - 1)^{1/2}}{x^2(x^2 - 1)} \)

2. \( y' = (x - 1)^{(x+1)} \left[ \frac{x + 1}{x - 1} + \ln(x - 1) \right] \left[ \frac{x + 1}{x - 1} + \ln(x - 1) \right] + (x - 1)^{(x+1)} \left[ -\frac{2}{(x - 1)^2} + \frac{1}{x - 1} \right] \)

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