A long tapered pole is anchored to a wall, as shown below. Cross-sections of the pole perpendicular to the x-axis are circles of radius
\[ r(x) = 0.8 - 0.04x \]

Here \( x \) is the distance in feet from the wall and \( r \) is measured in feet. The pole is 10 feet long.

Use slices perpendicular to the x-axis to answer the following questions. Express all symbolic answers in terms of \( x \). Express all numerical answers with units.

1. **a.** What is the volume of a typical slice?
   \[ dV = \]

2. **b.** What is the volume of the pole? Be accurate to one decimal place.
   \[ V = \]

3. **c.** The pole is made of aluminum, which weighs 169 lbs/ft\(^3\). What is the weight of a typical slice?
   \[ dF = \]

4. **d.** How much does the pole weigh?
   \[ F = \]

5. **e.** Weight is a downward force, so it creates a moment about the anchor point on the wall (the left end). Write a formula for the magnitude of the moment caused by the weight of a single slice. **Note:** The magnitude of the moment must be positive.
   \[ dM = \]

6. **f.** What is the magnitude of the total moment about the anchor point? **Note:** Your answer should be a positive number.
   \[ M = \]
2. A tapered beam is anchored to a wall, as shown below. Cross-sections of the beam perpendicular to the x-axis are squares. The square against the wall is 0.5 meters by 0.5 meters. Other sections have side lengths that depend on the distance from the wall:

\[ s(x) = 0.5 - 0.1x \text{ meters.} \]

The x-axis is measured in meters. The beam is 3 meters long. It's made of oak that weighs 7200 N/m\(^3\).

Compute the magnitude of the moment about the anchor point caused by the weight of the beam. **Note:**

Your answer should be a positive number.

\[ M = \underline{\phantom{000}} \]

3. A tapered cedar beam is 4.5 feet long and has circular cross sections. The diameter of the larger end is 0.5 feet and the radius of the cross sections is given by

\[ r(x) = \frac{1}{4} - mx \text{ feet} \]

where \( m \) is an unknown constant and \( x \) is the distance from the larger end of the beam.

Given the density of cedar is 23 lbs/ft\(^3\) and the beam weighs 16 pounds, find the diameter of the smaller end of the beam. Be accurate to three decimal digits.

**Warning!** You only get 5 submits, don't guess.
An oak column is 3.5 meters long with circular cross-sections that slope from a 0.4 meter diameter down to a 0.2 meter diameter. Given that oak weighs 7200 N/m³, find the balance point on the x-axis (measured from the left end of column). Be accurate to 3 decimal places.

\[ b = \quad \] 

**Warning!** You only get 10 submits, don't guess.
A cone has a height of $h$ and the radius of the base is $r$ as shown.

Find the volume of this cone in terms of $r$ and $h$.

**Note:** $r$ and $h$ are unknown constants. Do not use either of these letters to label a slice.

1. What is your choice of coordinate system?
   - I choose to place the coordinate system at the base of the cone with the positive $y$-axis pointing up the cone.
   - I choose to place the coordinate system at the tip of the cone with the positive $y$-axis pointing down the cone.

   Changing the choice of coordinate system will affect the correct answers below.

2. What is the volume of a typical slice?
   \[ dV = \]

3. What is the volume of this cone?
   \[ V = \]

When you are done, Google the formula for volume of a cone.