Instructions

- This assignment is similar to Taylor II, but it asks that you imagine what is happening beyond what you can see on the graphs.
- Beyond can mean expanded graph window, or Taylor polynomial degrees that are bigger than the top end of the slider.
- Use your judgement and knowledge from Taylor II, but "think outside the graph".
- All questions have severely limited submissions. DO NOT GUESS!
- All numerical answers must be exact integers.
Here are Taylor polynomials for $f(x) = \frac{1}{x}$ centered at $x = c$. You can move the center point and you can change the Taylor polynomial degree. Click here to view the graph in a separate window.

a. Set the center point at $x = 2$. Previous exploration discovered that there is an $x$-coordinate beyond which the Taylor polynomials fail to approximate $f(x)$. (If you don’t remember, explore again.)

What is the distance from that location to the center point?  

b. Other examples, notably $\sin(x)$ and $e^x$, had Taylor polynomials that did not display this behavior. For comparison, click here to look at $\sin(x)$. Move that slider around to see if there are any problem locations like the one you found above.

Discussion Question: What suspicious feature does the function $\frac{1}{x}$ have that $\sin(x)$ does not have?  

[Hint: It’s “vertical”]

What is the distance from this feature to the center point at $x = 2$?  

c. Move the center point to $x = 5$. Run the degree up to 80.

What is the distance from the center to the point where approximations seem to fail?  

d. Keep the center at $x = 5$ and the degree at 80. What is the distance from the center to the feature that is likely causing the approximations to fail?  

e. Move the center point to $x = 10$. Move the degree slider around and see what happens. On the right side of the center point, do you see any problems with the approximations in the visible graph?  

- Yes
- No

Discussion Question: Describe the behavior on the left side of the center point when you move the degree slider.

f. Suppose you could expand the graph window as much as you wanted, keeping the center point at $x = 10$. In the expanded graph window, would $T_{80}(x)$ always appear to be a good approximation for points to the right of the center?  

- Yes
- No

g. Suppose you could extend the slider as high as you wanted (unlimited degree), keeping the center point at $x = 10$. Could you get a good approximation on the interval $[10, 100]$?  

- Yes
- No
h. With the center point at $x = 10$, suppose you had unlimited degrees available. What is the largest possible interval on which you could expect good approximations? Write your answer below by entering the left and right endpoints of the interval.

$$\_\_\_\_ \leq x \leq \_\_\_\_$$

2. Question Details

Here are Taylor polynomials for $f(x) = \sin(x)$ centered at $x = \pi$. Click here to view the graph in a separate window.

a. Move the slider all the way to 61. True or False: $T_{61}(x)$ seems to perfectly match $\sin(x)$ on the entire visible graph.

- True
- False

b. Suppose you could expand the graph window as much as you wanted. Would $T_{61}(x)$ be a near perfect match for $\sin(x)$ forever?

- Yes
- No

c. Suppose you could extend the slider as high as you wanted (unlimited degree). Could you get a near perfect approximation on the interval $[-100\pi, 100\pi]$?

- Yes
- No

d. With unlimited degree, could you get a near perfect approximation on the interval $[-1000\pi, 1000\pi]$?

- Yes
- No

e. With unlimited degree, does there exist an interval where you cannot get near perfect approximation?

- Yes
- No
Here are Taylor polynomials for $f(x) = \sqrt{x}$, centered at $x = c$. You can slide the center point to any location in the graph. Click here to view the graph in a separate window.

Move the center point to $x = 15$. Keep it there throughout this problem.

a. Set the degree to something big.  
On the right side of the center point, do you see any problems with the approximations in the visible graph?

- Yes
- No

Discussion Question: Describe the behavior on the left side of the center point when you move the degree slider.

b. Is there a problem lurking somewhere to the right, off the visible graph? By problem, we mean a location where the Taylor polynomials fail to be good approximations.

- Yes
- No

c. Suppose you could extend the slider as high as you wanted (unlimited degree). Could you get the problem to go away?

- Yes
- No

d. Suppose you had unlimited degrees available. What is the largest possible interval on which you could expect good approximations? Write your answer below by entering the left and right endpoints of the interval.

$\quad < x <$
Here are Taylor polynomials for \( f(x) = \frac{1}{1 + x^2} \), centered at \( x = 0 \). Click here to view the graph in a separate window.

Previous exploration discovered that there is an interval where the Taylor polynomials provide good approximations, but outside that interval they fail to match the graph of \( f(x) \). (If you don’t remember, explore again.)

**Vocabulary:** This interval is called the **interval of convergence**.

**Discussion question:** What would happen to the interval of convergence if you moved the center point to \( x = 0.5 \)?

This is meant to be a wide open question. I may help your discussion if you try to make some predictions. You can test your predictions by selecting an answer. There are no points for this problem. WebAssign answers are just feedback on your predictions.

**Prediction Question 1:** The old interval had a problem at \( x = 1 \). Will the new interval have a problem at \( x = 1 \)?

- Yes
- No

**Prediction Question 2:** The old interval had a problem at \( x = -1 \). Will the new interval have a problem at \( x = -1 \)?

- Yes
- No

**Prediction Question 3:** The old interval ran from \( 1 \) to \( 1 \), so its width was \( 2 \). Will the new interval be wider, same, or narrower?

- Wider
- Same
- Narrower

**Prediction Question 4:** With the new center point, assuming unlimited degree, can you get a good approximation at \( x = 1.5 \)?

- Yes
- No

How about \( x = -0.5 \)?
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