Instructions
Previous problems asked you to convert a function into a Taylor polynomial. Many of these problems will ask you to look at a Taylor polynomial and guess which function it came from. You will need to know the formulas as the end of the Taylor Series Notes and Learning Goals.

1. Question Details

Simplify the following, write it using a single factorial:

\[ \frac{5}{5!} = \] 

Note: WebAssign may not require a simplified answer, but if you can't write it with a single factorial, you didn't do the problem.

Consider the power series for \( e^x \):

\[ e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \ldots \]

Differentiate the power series. The result is a new power series. Enter the new series up to the fifth order term. Write all coefficients using single factorials.

\[ \ldots \]

The new series should be familiar. What function does it represent?

2. Question Details

Write out the Taylor series for \( \sin(x) \) up to at least the 7-th degree term. Differentiate it. The result is a new power series. Enter the new series up to the 6-th order term. Try to write all coefficients using single factorials.

\[ \ldots \]

The new power series should be familiar. What function does it represent?
3. Question Details

An unknown function, \( f(x) \), has Taylor series:

\[ f(x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5 + \cdots \]

Differentiate the power series. Write the result out to at least the fifth degree:

\[ \frac{df}{dx} = \quad + \cdots \]

What was the original unknown function?

\[ f(x) = \]

4. Question Details

Recall that \( i = \sqrt{-1} \), so that \( i^2 = -1 \). Simplify each of the following as far as possible:

\[ i^2 = \]

\[ i^3 = \]

\[ i^4 = \]

\[ i^5 = \]

Use substitution to write the 7-th degree Taylor series for \( e^{ix} \) centered at \( x = 0 \). Simplify each term of the series so that there is at most one appearance of \( i \).

\[ e^{ix} = \quad + \cdots \]
This problem refers to your answer to the previous problem. Make sure you have access to that answer, preferably written out on paper.

From your 7-th degree approximation of $e^{ix}$, collect all the terms that do not contain $i$. Write them here as a standard form Taylor polynomial.

**Terms with no $i$:**

If this pattern were extended to infinity, what function would be represented by this series?

From your 7-th degree approximation of $e^{ix}$, collect all the terms that include $i$. Factor out $i$. Write the result as a standard form Taylor polynomial. Do not include $i$ in your answer.

**Terms with $i$ factored out:** $i \cdot \left( \right)$

If this pattern were extended to infinity, what function would be represented by this series? Do not include $i$ in your answer.

Combine both answers, including the multiple of $i$, to obtain a famous formula for $e^{ix}$.

$e^{ix} =$

Google "Euler's formula".