Instructions
Read today’s Notes and Learning Goals.

1. Question Details
A steel rod is 80 cm long and has constant density
\[ \rho = 23.5 \, \text{g/cm} \]

Find the total mass, \( m \), of the rod. Since density is constant, you do not need an integral.

\[ m = \]

2. Question Details
A 1.5 meter steel rod is manufactured such that it has a variable linear density given by
\[ \rho(x) = 1168 \sin(1.12x + 1.019) \]

Where \( \rho \) is measured in grams per meter and \( x \) is the distance in meters from the left end of the rod as shown.

Let \( m \) be the mass of the rod.

1. Find the infinitesimal amount of mass, \( dm \), of a segment of length \( dx \) at position \( x \).

\[ dm = \]

2. Find the bounds of integration
   lower bound: \( a = \)
   upper bound: \( b = \)

3. On your own paper, write an integral for the mass of the rod.
   Compute your integral to find the mass. Round your answer to the nearest gram and include units.

\[ m = \]
3. A steel rod of length 50 cm is manufactured such that it has a variable density given by
\[ \rho(x) = 36 - kx \text{ g/cm} \]
Here \( x \) is the distance in cm from the left end of the rod, and \( k \) is an unknown constant measured in g/cm².

1. A typical slice is shown in the figure. Write a formula for the mass of the typical slice. Do not include units.
\[ dm = \]

2. On your own paper, write an integral for the mass of the rod. Then compute your integral to find the mass. Give an exact symbolic answer that involves \( k \).
\[ m = \]

3. Find \( k \) so that the mass of the rod is exactly 1500 g. Include units.
\[ k = \]

4. A steel rod of unknown length, \( L \), is manufactured such that it has a variable density given by
\[ \rho(x) = 54.8 - 0.247x \]

Here \( \rho \) is measured in g/cm and \( x \) is the distance in cm from the left end of the rod.

1. A typical slice is shown in the figure. Write a formula for the mass of the typical slice. Do not include units.
\[ dm = \]

2. On your own paper, write an integral for the mass of the rod. Then compute your integral to find the mass. Give an exact answer involving \( L \).
\[ m = \]

3. Find \( L \) so that the mass of the rod is exactly 3500 g. Include units. Be accurate to one decimal place.
\[ L = \]
A thin rod of length \( L = 12 \text{ cm} \) is shown below. It carries an electrical charge, distributed along the rod, so that

\[ \lambda(x) = 2 - 2e^{-0.3x} \]

Here \( \lambda \) is measured in nanocoulombs per centimeter \((\text{nC/cm})\), and \( x \) is the distance in cm from the left end of the rod.

1. A typical slice is shown in the figure. Write a formula for the charge on a typical slice. Do not include units.

\[ dC = \]

2. On your own paper, write an integral for the total charge on the rod. Then compute your integral to find the charge. Be accurate to one decimal place and include units.

\[ C = \]

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6. Question Details

The figure below represents a highway leading out of a city. The traffic density on this highway is modeled by

\[ k(x) = \frac{9860}{(x+5)^2} \text{ vehicles/mile} \]

where \( x \) is the distance in miles from the left end of the highway.

1. A typical slice is shown in the figure. Write a formula for the number of vehicles, \( dN \), in that slice of highway. Do not include units.

\[ dN = \]

2. On your own paper, write an integral for the total number of vehicles on the highway. Compute your integral. Round to the nearest vehicle. Do not include units in the answer box.

\[ N = \text{vehicles} \]
A six foot beam carries a distributed load of 150 lbs/ft, as shown below.

Find the total load, also called force, \( F \), on the beam. Since the distributed load is constant, you do not need an integral.

\[ F = \] 

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A four meter long beam carries a distributed load given by

\[ w(x) = 200 - 12.5x^2 \]

where \( w \) is measured in Newtons per meter (N/m), and \( x \) is the distance in meters from the left end of the beam as shown.

1. A typical slice is shown in the figure. Write a formula for force on this slice. Do not include units.
   \[ dF = \]

2. On your own paper, write an integral for the total force on the beam. Compute your integral. Be accurate to one decimal place and include units.
   \[ F = \]
A free standing tower is 80 feet tall. It experiences a wind loading of

\[ w(y) = 2.2y^{1/3} \text{ lbs/ft} \]

where \( y \) is the distance in feet from the bottom of the tower.

1. A typical slice is shown in the figure. Write a formula for force on this slice. Do not include units.

\[ dF = \]

2. On your own paper, write an integral for the total force on the tower. Compute your integral. Be accurate to one decimal place and include units.

\[ F = \]
Consider a circular slice, which is a ring of radius $r$ and thickness $dr$ as shown.

Find the circumference of the original slice. That is, find the length of the dotted center line.

$$\text{circumference} =$$

Find the area of the circular slice. Use the unrolled form, since this is just like a rectangle.

$$dA =$$
The population density of foxes decreases with the distance $r$ from the center of their territory, as given by

$$ \rho(r) = 7.3 - \frac{r}{2} \text{ foxes per square kilometer.} $$

Let $P$ be the number of foxes. Follow the steps to find the number of foxes within 5 km of the center.

1. Find the area of the circular slice a distance $r$ from the center with thickness $dr$, as shown in the figure.

$$ dA = \boxed{} $$

2. Find the number of foxes, $dP$, that live in the slice.

$$ dP = \boxed{} $$

3. Find the bounds of integration.

   lower bound: $a = \boxed{}$

   upper bound: $b = \boxed{}$

4. On your own paper, write an integral for the total number of foxes. Compute your integral. Round to the nearest fox. Do not include units.

   Population = $\boxed{\text{foxes}}$
The density of a circular ring varies with the distance $r$ from the center of the ring:

$$\rho(r) = \frac{162}{r^2} \text{ g/cm}^2$$

Follow the steps to find the mass, $m$, of the ring.

1. Find the mass, $dm$, of a circular slice a distance $r$ from the center, with thickness $dr$.

   $$dm = \ldots$$

2. Find the bounds of integration.

   lower bound: $a = \ldots$

   upper bound: $b = \ldots$

3. On your own paper, write an integral for the total mass of the ring. Compute your integral. Be accurate to the nearest gram.

   $$m = \ldots$$
An elk population has a variable population density

\[ \rho(r) = 2 - ar \text{ elk/m}^2 \]

where \( r \) is the distance in miles from the center of the territory and \( a \) is an unknown positive constant. Suppose there are 850 elk within 12 miles of the center.

1. Find the number of elk, \( dP \), in a circular slice a distance \( r \) from the center, with thickness \( dr \).

\[ dP = \]

2. On your own paper, write an integral for the total number of elk that live within 12 miles of the center. Compute your integral. Your answer will include the unknown constant \( a \).

\[ P = \]

3. What is the value of the unknown constant \( a \) such that there are 850 elk in a 12 mile radius? Be accurate to at least four decimal digits. Do not include units.

\[ a = \text{elk/mi}^3 \]
A quarter circle of radius 5 is located in the first quadrant, as shown below. The equation for this circle is $x^2 + y^2 = 25$. We have covered three different ways to slice up this area. For each of the following methods, write the area of a typical slice.

a. Thin vertical slices, using the $x$-axis as the axis of integration.

\[ dA = \]

b. Thin circular slices, using the radial distance, $r$ the axis of integration.

\[ dA = \]

c. Thin horizontal slices, using the $y$-axis as the axis of integration.

\[ dA = \]
A quarter circle of radius 5 is located in the first quadrant, as shown below. The equation for this circle is $x^2 + y^2 = 25$. We have covered three different ways to slice up this area. Vertical slices, horizontal slices and radial slices as shown.

Suppose this region has a variable density as described below. Choose the correct slicing strategy to use for that particular density function, then find the mass of a typical slice.

a. The density varies with the radial distance from the center of the circle, $\rho(r) = 7 - r$.
   - Use thin vertical slices with the axis of integration as the $x$-axis.
   - Use thin horizontal slices with the axis of integration as the $y$-axis.
   - Use thin circular slices with the axis of integration as the $r$-axis.
   - None of the above.

What is the mass of a typical slice in this case?

$b m = \boxed{}$

b. The density varies with the distance from the $y$-axis, $\rho(x) = x(5-x)$.
   - Use thin vertical slices with the axis of integration as the $x$-axis.
   - Use thin horizontal slices with the axis of integration as the $y$-axis.
   - Use thin circular slices with the axis of integration as the $r$-axis.
   - None of the above.

What is the mass of a typical slice in this case?

$b m = \boxed{}$

c. The density varies with the distance from the $x$-axis, $\rho(y) = 25 - y^2$.
   - Use thin vertical slices with the axis of integration as the $x$-axis.
   - Use thin horizontal slices with the axis of integration as the $y$-axis.
   - Use thin circular slices with the axis of integration as the $r$-axis.
   - None of the above.

What is the mass of a typical slice in this case?

$b m = \boxed{}$