Instructions

- Read today’s Notes and Learning Goals
- In this assignment you are NOT ALLOWED to use the exponential function on your calculator or on any equivalent device
- Many questions in this assignment ask for decimal answers. All such answers must be rounded to 4 decimal places. Anything less accurate or more accurate will be marked incorrect.

1. Question Details

True or False? For this assignment you are NOT ALLOWED to use the e^x button on your calculator (or any equivalent device).
If you are unsure, read the assignment instructions.

- True
- False

2. Question Details

True or False? All the numerical non-integer calculations and/or estimates on these homework problems must be rounded to four (4) decimal places.

- False
- True

3. Question Details

Find the 2nd degree Taylor Polynomial for e^x centered at x = 0.

\[ e^x = \] Use exact coefficients. Decimals will not be accepted.

Use the Taylor polynomial to estimate e^1. That is, plug in \( x = 1 \).

\[ e^1 = \]

Find the 5th degree Taylor Polynomial for e^x centered at x = 0. All coefficients must be exact.

\[ e^x = \]

Use this Taylor polynomial to estimate e^1. Round to 4 decimal places.

\[ e^1 = \]

Note: WebAssign tolerances are set very tightly. You will probably have to enter the full expression into your calculator so that you do not accumulate roundoff error.
4. The exercise you just completed, using two different degrees of Taylor polynomial to estimate $e^1$, can be repeated for every degree. The table below shows the results for degrees 0 through 10. Complete the table by entering your answers from above in the correct cells.

<table>
<thead>
<tr>
<th>degree =</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>1.0000</td>
<td>2.0000</td>
<td>2.6667</td>
<td>2.7083</td>
<td>2.7181</td>
<td>2.7183</td>
<td>2.7183</td>
<td>2.7183</td>
<td>2.7183</td>
<td>2.7183</td>
<td>2.7183</td>
</tr>
</tbody>
</table>

Higher degrees give better approximations, but eventually they stop getting any better, at least when rounded to 4 decimal places.

To 4 decimal places, what is the best possible approximation?

$e = \underline{\phantom{1.0000}}$

What is the minimum degree that achieves this approximation?

degree = \underline{\phantom{0}}

5. Using Taylor polynomials of degree 0 through 6, centered at 0, to estimate $e^{0.5}$. Enter your answers in the table below. Round to 4 decimal places.

<table>
<thead>
<tr>
<th>degree =</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{0.5}$</td>
<td>\underline{\phantom{1.0000}}</td>
<td>\underline{\phantom{2.0000}}</td>
<td>\underline{\phantom{2.6667}}</td>
<td>\underline{\phantom{2.7083}}</td>
<td>\underline{\phantom{2.7181}}</td>
<td>\underline{\phantom{2.7183}}</td>
<td>\underline{\phantom{2.7183}}</td>
</tr>
</tbody>
</table>

To 4 decimal places, what is the best possible approximation?

$e^{0.5} = \underline{\phantom{1.0000}}$

What is the minimum degree that achieves this approximation?

degree = \underline{\phantom{0}}

6. The 8th degree Taylor polynomial approximation for the function $\cos(x)$ centered at 0 is

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8$$

Make a table like the one in the previous problem. Use it to find the best possible approximation of $\cos(1)$ to 4 decimal places. Do it without touching the "cos" button on your calculator.

$\cos(1) = \underline{\phantom{1.0000}}$

What is the minimum degree that achieves this approximation?

degree = \underline{\phantom{0}}
Download and print Worksheet #1. If you are working in class, get a copy from your instructor. As you fill out the table in the worksheet, you will be asked to return here and answer the questions below.

In the row for $e^{0.5}$ ...

a. What degree resulted in "No Change"?

b. How many cells did you leave blank?

c. What is the stepsize?

In the row for $e^{0.2}$ ...

d. What degree resulted in "No Change"?

e. What is the best approximation for $e^{0.2}$?

f. What degree resulted in the best approximation?

In the full table...

g. How many blank cells are there in the degree 7 column?

h. What is the best approximation for $e^{0.1}$?

i. What is the quadratic approximation $e^{0.05}$?

j. What is your table entry for stepsize = 0.2 and degree = 3?

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8. Question Details

All questions refer to your clean and complete Worksheet #1. If you don't have one, complete the worksheet before you start this problem.

a. Suppose that you are required to use degree 3 polynomials. What is the largest allowable stepsize if you need 4 decimal places of accuracy? Choose from this list:

- 1.0
- 0.5
- 0.2
- 0.1
- 0.05

b. Suppose that you are required to use stepsize = 0.2. What is the smallest degree that will give you 4 decimal places of accuracy?

c. Suppose that you are required to use stepsize = 0.5, but you are allowed to select any degree ≤ 4. What's the best approximation you can get?

d. What's the error in that approximation? That is, what is the difference between your answer above and the best possible 4 decimal place approximation of $e^{0.5}$?

e. You are required to use cubic polynomials and stepsize = 0.5. What is the error in your approximation?