The graph of the functions $y = x^2$ and $y = \frac{1}{x^2}$ are shown below.

a. Which of the following limit expressions best describes the graph of $y = x^2$ as $x \to \infty$?

- $\lim_{x \to \infty} x^2 = 2x$
- $\lim_{x \to \infty} x^2 = 0$
- $\lim_{x \to \infty} x^2 = 2$
- $\lim_{x \to \infty} x^2 = \infty$

b. Which of the following limit expressions best describes the graph of $y = \frac{1}{x^2}$ as $x \to \infty$?

- $\lim_{x \to \infty} \frac{1}{x^2} = \frac{-2}{x^3}$
- $\lim_{x \to \infty} \frac{1}{x^2} = 0$
- $\lim_{x \to \infty} \frac{1}{x^2} = 2$
- $\lim_{x \to \infty} \frac{1}{x^2} = \infty$
The graph of the functions $y = e^x$ and $y = e^{-x} = \frac{1}{e^x}$ are shown below.

Find the following limits

a. $\lim_{x \to \infty} e^{-x} = \phantom{0}$

b. $\lim_{x \to \infty} e^x = \phantom{0}$

The graph of the functions $y = \sqrt{x}$ and $y = \ln(x)$ are shown below.

Find the following limits

a. $\lim_{x \to \infty} \sqrt{x} = \phantom{0}$

b. $\lim_{x \to \infty} \ln(x) = \phantom{0}$

Suppose $f(x)$ is a function such that $\lim_{x \to \infty} f(x) = \infty$. Find the following limit

$\lim_{x \to \infty} \frac{1}{f(x)} = \phantom{0}$
5. **Question Details**

**fa16 limit at inf 4 [3654406]**

a. Find the following limit

\[ \lim_{{x \to \infty}} \frac{x^2}{x^3} = \]

**Hint:** Simplify the fraction first.

b. Which of the following statements best describes the relationship the above limit gives between \( x^2 \) and \( x^3 \) as \( x \to \infty \)?

- \( x^2 \) goes to infinity slower than \( x^3 \).
- \( x^2 \) goes to infinity about the same speed as \( x^3 \).
- \( x^2 \) goes to infinity faster than \( x^3 \).

6. **Question Details**

**fa16 limit at inf 5 [3654407]**

a. Find the following limit

\[ \lim_{{x \to \infty}} \frac{e^x}{x^2} = \]

**Hint:** Use L'Hopital's Rule or think about which function is dominant as \( x \to \infty \).

b. Which of the following statements best describes the relationship the above limit gives between \( e^x \) and \( x^2 \) as \( x \to \infty \)?

- \( e^x \) goes to infinity slower than \( x^2 \).
- \( e^x \) goes to infinity about the same speed as \( x^2 \).
- \( e^x \) goes to infinity faster than \( x^2 \).

7. **Question Details**

**fa16 limit at inf 6 [3654408]**

a. Find the following limit

\[ \lim_{{x \to \infty}} \frac{5x^2}{3x^2} = \]

**Hint:** Simplify the fraction first.

b. Which of the following statements best describes the relationship the above limit gives between \( 5x^2 \) and \( 3x^2 \) as \( x \to \infty \)?

- \( 5x^2 \) goes to infinity slower than \( 3x^2 \).
- \( 5x^2 \) goes to infinity faster than \( 3x^2 \).
- \( 5x^2 \) goes to infinity at about the same speed as \( 3x^2 \).
All of the following functions go to infinity as $x \to \infty$, $\lim_{x \to \infty} f(x) = \infty$.

Order these by the speed at which they go to infinity. Enter the functions in the boxes below using the notation "Slower Function $\ll$ Faster Function". Only 3 submits per answer box.

$\lim_{x \to \infty}$

All of the following functions go to infinity as $x \to \infty$, $\lim_{x \to \infty} f(x) = \infty$.

Order these by the speed at which they go to infinity. Enter the functions in the boxes below using the notation "Slower Function $\ll$ Faster Function". Only 3 submits per answer box.
10. All the following families of functions go to infinity as \( x \to \infty \), \( \lim_{x \to \infty} f(x) = \infty \).

A: Powers: \( x^n \) where \( n > 1 \)
B: Exponential: \( a^x \) where \( a > 1 \)
C: Logarithms: \( \log_a(x) \) where \( a > 1 \)
D: Roots: \( \sqrt[n]{x} = x^{1/n} \) where \( n > 1 \)

Order these families by the speed at which they go to infinity.

---Select--- \(<\)<---Select--- \(<\)<---Select--- \(<\)<---Select--- \(<\<\>

11. All of the following functions go to infinity as \( x \to \infty \), \( \lim_{x \to \infty} f(x) = \infty \).

Order these by the speed at which they go to infinity. 1 is the slowest, 8 is the fastest.

\( x^5 \) ---Select--- \(<\>
\( x^{1/5} \) ---Select--- \(<\>
\( e^x \) ---Select--- \(<\>
\( 5^x \) ---Select--- \(<\>
\( x^2 \) ---Select--- \(<\>
\( x^{1/2} \) ---Select--- \(<\>
\( \ln(x) \) ---Select--- \(<\>
\( 2^x \) ---Select--- \(<\>

12. Consider the limit
\[
\lim_{x \to \infty} \frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8}
\]

a. For large values of \( x \) this fraction is approximately the ratio of the dominant terms (the ones that go to infinity the fastest). Select which is the correct approximation for large values of \( x \).

\[
\frac{2x^2}{2x^3 + x + 8} \approx \frac{-5x^3}{2x^3} \]
\[
\frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8} \approx \frac{6}{8} \]
\[
\frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8} \approx \frac{2x^2}{2x^3} \]
\[
\frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8} \approx \frac{-5x^3}{x}
\]

b. Use the above approximation to find the limit.
\[
\lim_{x \to \infty} \frac{2x^2 - 5x^3 + 6}{2x^3 + x + 8} = \]
Consider the limit
\[
\lim_{x \to \infty} \frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5}
\]

a. For large values of \(x\), the above ratio is approximately:
- \(\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{12}{5}\)
- \(\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{-5}{3}\)
- \(\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{5x^{3/2}}{3x^2}\)
- \(\frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} \approx \frac{-4x^{5/2}}{3x^2}\)

b. Use the above approximation to find the limit.
\[
\lim_{x \to \infty} \frac{5x^{3/2} - 4x^{5/2} + 12}{3x^2 - 6x + 5} = \ldots
\]

Consider the limit
\[
\lim_{x \to \infty} \frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}}
\]

a. For large values of \(x\), the above fraction is approximately:
- \(\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{2x^2}{3x^2}\)
- \(\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{2x^2}{9x^4}\)
- \(\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{2x^2}{9x^2}\)
- \(\frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} \approx \frac{5}{2}\)

b. Use the above approximation to find the limit.
\[
\lim_{x \to \infty} \frac{2x^2 - 6x + 5}{\sqrt{4 - 6x^2 + 9x^4}} = \ldots
\]
Consider the limit
\[ \lim_{x \to \infty} \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \]

a. For large values of \( x \), the above fraction is approximately:
- \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx \frac{3}{5} \)
- \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx \frac{3}{x} \)
- \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx \frac{-6}{5} \)
- \( \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} \approx \frac{3}{5} \)

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{3 - 6 \cdot (2^x)}{5 + x + 5 \cdot (3^x)} = \]

---

Consider the limit
\[ \lim_{x \to \infty} \frac{\sqrt{25x^4 + 15x^2 + 6}}{7x^2 + 9x + 10} \]

a. For large values of \( x \), the above fraction is approximately:
\[ \frac{\sqrt{25x^4 + 15x^2 + 6}}{7x^2 + 9x + 10} \approx \]

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{\sqrt{25x^4 + 15x^2 + 6}}{7x^2 + 9x + 10} = \]

---

Consider the limit
\[ \lim_{x \to \infty} \frac{3x^3 - 5x^5 + 1}{5x^5 - 7 \cdot (2^x)} \]

a. For large values of \( x \), the above fraction is approximately:
\[ \frac{3x^3 - 5x^5 + 1}{5x^5 - 7 \cdot (2^x)} \approx \]

b. Use the above approximation to find the limit.
\[ \lim_{x \to \infty} \frac{3x^3 - 5x^5 + 1}{5x^5 - 7 \cdot (2^x)} = \]
18. Question Details

Consider the limit
\[ \lim_{x \to \infty} \frac{3x^5 + 9x^4 - 31x}{2x^4 - 31x^2 + 12} \]

a. For large values of \( x \), the above fraction is approximately:
\[ \frac{3x^5 + 9x^4 - 31x}{2x^4 - 31x^2 + 12} \approx \]

b. Use the above approximation to find the limit.

\[ \lim_{x \to \infty} \frac{3x^5 + 9x^4 - 31x}{2x^4 - 31x^2 + 12} = \]

19. Question Details

Evaluate the following limit
\[ \lim_{x \to \infty} \frac{\sqrt{9x^4 - 3x^2 + 5x + 1}}{5x^2 - 6x + 1} = \]

20. Question Details

Evaluate the following limit
\[ \lim_{x \to \infty} \frac{3x^2 + 5^x}{4x^2 + 6^{-x} + 3^x} = \]
Answer the following questions. **You only get two submits per answer box.**

a. Order the functions, \( x^2, x^3 \) and \( \sqrt{x} \), by the speed they go to infinity.

\[
\begin{array}{c}
\boxed{\quad < < \quad}
\end{array}
\]

b. Order the functions, \( \frac{1}{x^2}, \frac{1}{x^3} \) and \( \frac{1}{\sqrt{x}} \), by their tail thickness, as shown in the graph below. Use the notation "Thinner Tail \( \ll \) Thicker Tail".

\[
\begin{array}{c}
\boxed{\quad < < \quad}
\end{array}
\]

22. **Question Details**

The following functions, \( \frac{1}{x}, \frac{1}{\ln(x)} \) and \( \frac{1}{2^x} \), all have a horizontal asymptote at 0, \( \lim_{x \to \infty} f(x) = 0 \)

Order these functions by their tail thickness. **You only get two submits per answer box.**

\[
\begin{array}{c}
\boxed{\quad < < \quad}
\end{array}
\]
All of the following functions have a horizontal asymptote at 0, \( \lim_{x \to \infty} f(x) = 0 \).

Order these by the thickness of their tail. 1 is the thinnest, 6 is the thickest.

\[
\begin{align*}
\frac{1}{x^{1/5}} & \quad \text{---Select---} \\
\frac{1}{e^x} & \quad \text{---Select---} \\
\frac{1}{x^{3/5}} & \quad \text{---Select---} \\
\frac{1}{\ln(x)} & \quad \text{---Select---} \\
\frac{1}{3^x} & \quad \text{---Select---} \\
\frac{1}{x^5} & \quad \text{---Select---}
\end{align*}
\]