In this lesson you must determine whether a Taylor polynomial provides a **good** or **bad** approximation to a given function.

- **Good** and **bad** are subjective judgment calls, not mathematically defined terms.

- All questions will be answered by looking at graphs. Although **good** and **bad** are not precisely defined, the guidelines for this assignment are:
  - **Good** means the graph of $T_n(x)$ appears to match the graph of $f(x)$.
  - **Bad** means the graphs do not appear to match.

- **WARNING!** All WebAssign questions will have severely limited submissions. **DO NOT GUESS.** **DO NOT WASTE** your limited submissions.
  **Work in teams.** It is probably best for one person on a team to have a computer to show the graphs, and another person to record answers on paper.

- The good/bad question can be asked in three different ways.

  1. It will be asked about a single location that is not the center point.

    **Example:** Suppose $f(x) = \sqrt{25 - x^2}$ and $T_1(x)$ is the linear Taylor polynomial centered at $x = 2$, as shown below.

    ![Graph](image)

    **Question:** At $x = 4$, is $T_1(x)$ a good approximation of $f(x)$ or a bad approximation?

    **Answer:** **Bad.** Clearly the graphs are well separated at $x = 4$. 


2. It will be asked about an interval. The Taylor polynomial center point will be either an endpoint of the interval or exactly in the middle.

**Example:** The figure shows \( f(x) = \sqrt{25 - x^2} \) and \( T_4(x) \) centered at \( x = 2 \).

![Graph showing \( f(x) = \sqrt{25 - x^2} \) and \( T_4(x) \) centered at \( x = 2 \).]

**Question:** On the interval \([0, 4]\), is \( T_4(x) \) a good approximation of \( f(x) \) or a bad approximation?

**Answer:** Good. On this interval the graphs are indistinguishable.

**Question:** How about on the interval \([2, 5]\)? Good or bad?

**Answer:** Bad. The graphs clearly separate before they get to \( x = 5 \).

3. It will be asked by specifying a distance from the center point.

**Example:** The figure shows \( f(x) = \sqrt{25 - x^2} \) and \( T_5(x) \) centered at \( x = 2 \).

![Graph showing \( f(x) = \sqrt{25 - x^2} \) and \( T_5(x) \) centered at \( x = 2 \).]
**Question:** At a distance of 2 from the center point, is $T_5(x)$ a good approximation of $f(x)$ or a bad approximation?

**Answer:** Good. You have to check on $x = 0$ and $x = 4$. Both look good.

**Question:** How about at a distance of 3?

**Answer:** Bad. If either $x = -1$ or $x = 5$ is bad, then the answer is bad. Although $x = -1$ is a close call, $x = 5$ is clearly bad.

**Question:** How far from the center can you go and still have $T_5(x)$ be a good approximation?

**Answer:** Probably the best you can say is

“Somewhere between 2 and 3.”

This is too subtle and open-ended for a WebAssign question. However, it is likely to occur on worksheets, quizzes, and exams, so you should be thinking about it. WebAssign will have multiple-choice versions of this question.

- By the end of this assignment you should be able to answer these questions:

1. For a target approximation location (or interval, or distance from center), how big does $n$ have to be to get a good approximation?
2. For a target approximation location (or interval, or distance from center), is there always a big enough $n$?
3. For a given $n$, how wide an interval (or what locations, or what distances) can I use and still have a good approximation?