1. Know what a **Power Series** is.

   - A power series is a series that has coefficients and powers of \( x \), instead of just numbers.

   **Expanded Example:**
   
   \[
   1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \frac{x^4}{16} + \cdots
   \]

   **Sigma Notation Example:**
   
   \[
   \sum_{n=0}^{\infty} \frac{x^n}{n^2}
   \]

   - Think of a power series as an infinitely long polynomial.
   - A power series can also be shifted, or centered at \( x = c \).

   **General Formula:**
   
   \[
   \sum_{n=0}^{\infty} a_n(x - c)^n = a_0 + a_1(x - c) + a_2(x - c)^2 + a_3(x - c)^3 + a_4(x - c)^4 + \cdots
   \]

2. Know what a **Taylor series** is.

   - A Taylor series is an infinitely long Taylor polynomial.

   **Example:** The Taylor series for \( \frac{1}{x} \) centered at \( x = 2 \) is
   
   \[
   \frac{1}{x} = \frac{1}{2} - \frac{1}{4}(x - 2) + \frac{1}{8}(x - 2)^2 - \frac{1}{16}(x - 2)^3 + \frac{1}{32}(x - 2)^4 + \cdots
   \]

   **Note:** Compare this to your answer to [Taylor I, Problem 12](#).

   **Using sigma notation:**
   
   \[
   \frac{1}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x - 2)^n
   \]
3. Given \( f(x) \) and a center point, \( x = c \), be able to compute the first few terms of a Taylor series, one coefficient at a time. Here are two methods:

- Equate derivatives, as in [Taylor I].
  - Compute the symbolic derivative of \( f(x) \). Plug in \( x = c \).
  - Compute the symbolic derivative of the general Taylor polynomial. Plug in \( x = c \).
  - Set equal and solve for the coefficient.

- Compute one coefficient at a time, using this formula:
  \[
  a_n = \frac{f^{(n)}(c)}{n!}
  \]

4. Be able to use the following four formulas. Usage is explored in your homework.

  **Note:** These formulas will appear on the formula sheet for Exam 2. Be ready to use them.

  \[
  e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots
  \]

  \[
  \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{7!} + \cdots
  \]

  \[
  \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{6!} + \cdots
  \]

  \[
  \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots
  \]

5. [This video](#) is a good overview of Taylor Polynomials and Taylor Series.