1. Learn to transform an antiderivative, $\int f(x) \, dx$, via substitution.

The process begins with a formula:

$$u = \text{formula with } x$$

After that you are locked into the following steps.

**Step 1.** Compute $du$. Write it as $du = u' \, dx$.

**Step 2.** Use the equations for $u$ and $du$ to substitute all the $x$’s and the $dx$ in the original problem. You will have to recognize two types of substitution opportunities:

- Things that look like the formula for $u$. Replace with $u$.
- Things that look like the formula for $du$. Replace with $du$.

**Note:** Sometimes you have to do a bit of algebra to get the $du$ to work out.

- If the previous steps leave some $x$’s in the problem, solve $u = \text{formula with } x$ for the variable $x$. Use your answer to replace remaining $x$’s.

The result is a transformed antiderivative problem written only in terms of $u$.

2. Know that substitution does not solve the original problem; it just replaces the original antiderivative problem with a new one.

Expect homework and exam questions where you are asked to write down the transformed antiderivative problem as your final answer. You don’t actually find the antiderivative.

3. In order to solve the original problem you have to complete 2 more steps:

**Step 3.** Guess an answer to the new antiderivative problem. Your answer will have $u$’s in it.

**Step 4.** Change all the $u$’s back into formula with $x$.

Video examples:

- Khan Academy Excellent example, and the follow-up videos are also very good.
- Khan Academy Shows what happens when there are extra $x$’s, in Step 2.
- Patrick JMT Also a very good example.
4. Transform an integral, \( \int_{a}^{b} f(x) \, dx \), via substitution. The process is similar, but you also have to transform the limits of integration:

- Steps 1 and 2 are the same to transform the integrand.
- The bounds \( x = a \) and \( x = b \) are values of the original variable. Use the formula with \( x \) to get the new limits of integration in terms of \( u \).
- The integral is transformed when the integrand and bounds are in terms of \( u \).

5. Know that this does not solve the original problem. It just replaces the original integral with a new integral.

   Expect homework and exam questions where you are asked to write down the new integral, with new limits, as your final answer. You don’t actually compute the integral.

6. In order to compute the original integral, use the Fundamental Theorem, by either:

   - Use the antiderivative in terms of \( u \) along with the \( u \) values of the bounds.
   - Rewrite the antiderivative in terms of \( x \) and use the \( x \) values of the bounds.

Here are some worked examples:

- Khan Academy
- Patrick JMT