Moment of a Concentrated Force

The figure below shows a beam anchored to a wall at a point $B$. A force of 100 lbs pushes down on the beam at a point 4 feet from the wall.

This force creates a physical quantity called **torque** or **moment**. The numerical value of the moment in this example is

$$(4 \text{ ft})(100 \text{ lbs}) = 400 \text{ ft-lbs}$$

The Pivot Point is Important

There is no such thing as a moment without a specified pivot point. In the example pictured above the pivot point is $B$. The correct vocabulary is the *moment about* $B$. The notation is $M_B$. The formula is

$$M_B = rF$$

where $F$ is the force and $r$ is the distance from the force point to the pivot point.

The Distance is Critical!

Every time you compute a moment you should execute the following three step process:

- Find the **distance** from the force to the pivot.
- Find the **force**.
- Compute: Moment = (Distance) × (Force).

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1Physics books usually use the word “torque.” Engineering books prefer the word “moment.” Calculus books use the word “moment,” but in a more abstract way: they use mass instead of force, they ignore units, and they sometimes reverse the counter-clockwise-is-positive rule.
Sign Conventions

Force, distance and moment are signed quantities, meaning they might be positive or negative depending on context. For this lesson we will adhere to the following sign conventions:

- Treat all forces as positive.
- Treat all distances as positive.
- All counter-clockwise moments are positive.
- All clockwise moments are negative.

Distributed Loads

Suppose that a beam carries a distributed load as shown below.

- The distributed load is described by a function, \( w(x) = 8x - x^2 \) lbs/ft.
- The function only makes sense if there is an \( x \)-axis. In this and many similar situations the beam itself is the \( x \)-axis.
- \( O \) is the origin on the \( x \)-axis.
- The figure shows a slice of the beam.
- The slice is located at \( x \).

\(^2\)Actually, they are vector quantities, but that’s for a different class.
Example: For the beam and load shown below, write an integral for $M_B$.

![Diagram of a beam and load](image)

Solution: First compute the (tiny) moment caused by the (tiny) force on just one slice:

![Diagram of a beam and load](image)

Use the three step process:

- The distance from the slice to pivot point is
  \[ \text{distance} = (4 - x) \]

- The (tiny) force on the slice is
  \[ dF = (8x - x^2) \, dx \]

- The (tiny) moment caused by the (tiny) force on this slice is
  \[ dM_B = \text{(distance)} \times \text{(force)} = (4 - x)(8x - x^2) \, dx \]

Finish by writing an integral for the total moment. The bounds of integration run from the left end of the load to the right end of the load.

\[ M_B = \int_0^4 (4 - x)(8x - x^2) \, dx \]