Suppose that a particle travels along a path with vector valued position function $\mathbf{r}'(t)$.

This lesson is concerned with the distance traveled along that path, also known as **arc length**.

Here are things you should know.

1. For constant speed in a straight line,

   $$\text{distance} = \text{(speed)} \times \text{(time)}$$

2. If either speed is variable OR the path is curved, then you must use a tiny amount of time, $dt$, and you must compute a tiny amount of distance, $ds$. The formula is

   $$ds = (\text{speed})\, dt$$

   In vector notation this is

   $$ds = |\mathbf{v}|\, dt$$

   **Note:** You will not be asked to communicate slicing strategy for this. However, if you were asked....

   - The axis of integration is **time**, or $t$.
   - A tiny slice of time is $dt$. But since there is no time axis visible you can’t label this.
   - A tiny slice of distance is pictured and labeled at right.

3. Total distance traveled, or total arc length, is computed using an integral.

   $$\text{distance} = \int_a^b |\mathbf{v}|\, dt$$

   Since the axis of integration is time, the bounds are time values:

   $$a = \text{Starting time}$$
   $$b = \text{Ending time}$$
**Example:** An object is launched from a height of 192 feet. It’s position as a function of time is

$$\mathbf{r}(t) = (80t, 192 + 64t - 16t^2)$$

with $t$ in seconds. Its trajectory is shown below. At lands on the ground at the instant $t = 6$ seconds. Compute the total distance it travels from launch to landing.

![Diagram showing the trajectory of the object and landings at t=0 and t=6 seconds.](image)

**Solution:**

1. Compute tiny distance.

   $$\mathbf{r}'(t) = (80, 192 + 64 - 32t)$$ (given)
   $$\mathbf{v}(t) = (80, 64 - 32t)$$ (derivative of $\mathbf{r}$)
   $$|\mathbf{v}(t)| = \sqrt{80^2 + (64 - 32t)^2}$$ (length of $\mathbf{v}$)
   $$ds = \sqrt{80^2 + (64 - 32t)^2} dt$$ (speed $\times$ time)

2. Write an integral for total distance.

   $$distance = \int_{0}^{6} \sqrt{80^2 + (64 - 32t)^2} dt$$

3. Use a calculator or a computer to finish the computation

   $$distance \approx 602.5 \text{ m}$$

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$^1$A fanatical Calculus II student might attempt this by hand, starting with the trig substitution $t = 2 + 2.5 \sec u$. 