Show all your work.

IN PROBLEM 1 ONLY: You must use limit methods and the definition of derivative. You cannot use any Chapter 3 methods and you cannot use L’Hospital’s rule.

1. (10 pts.) Find \( f'(3) \) for the function \( f(x) = 3x^2 + \frac{2}{x} \)

FOR THE REST OF THE EXAM: You may use any methods from this semester, although all work must be shown and any information obtained from your calculator must be so indicated.

2. The population of a colony of flour beetles is given by the function 
\[
P(t) = \frac{200}{1 + 99e^{-0.1t}}
\]
The units on \( t \) are days and the units on \( P \) are beetles.

(a) (5 pts.) Find the number of beetles at time \( t = 25 \) days.
(b) (5 pts.) Find the change in the number of beetles on the interval \( 0 \leq t \leq 25 \) days.
(c) (5 pts.) Find the rate of change of the number of beetles on the interval \( 0 \leq t \leq 25 \) days.
(d) (10 pts.) The rate of change of the number of beetles at the instant \( t = 25 \) days. You may use any methods.

3. (25 pts.) Find \( \frac{dy}{dx} \) for each of the following. Use any methods. Assume that \( f \) and \( g \) are abstract functions of \( x \) and that \( a, b \) and \( c \) are constants.

(a) \( y = 13(\cos x)^4 + 5 \tan x \)
(b) \( y = \sec x \sin x^2 \)
(c) \( y = e^{\cot 3x} \)
(d) \( y = \frac{ax^2 + b}{cx - x^3} \)
(e) \( y = (x^{-3} + 1) \left( \frac{f}{g} \right) \)
4. (10 pts.) Find all locations in $[0, 2\pi]$ where $y = 2\sin x + 3x$ has a tangent line with slope 1.

5. (15 pts.) An airplane flies over a spotter at point $A$ on the ground as shown at right. The airplane flies straight and level at an altitude of 30,000 feet with velocity 600 ft/sec. How fast is the angle $\theta$ changing 25 seconds after the plane passes directly over the spotter?

6. (10 pts.) Use differential approximation to estimate the value of $\tan 46^\circ$.

7. Suppose that $f(x) = x^3 + 4x^2 - 3x$.
   (a) (10 pts.) Where is $f$ increasing? Decreasing?
   (b) (10 pts.) Where is $f$ concave up? Down?
   (c) (10 pts.) Sketch a very good graph of $f$.

8. (15 pts.) A box as shown at right has a fixed volume of 1000 cubic inches. What is the smallest possible value for the quantity $4x + w$?
9. The following table gives the rate of change of temperature of a certain object, in units of $\degree C/s$, measured at 2 minute intervals.

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dT/dt$ ($\degree C/min$)</td>
<td>10</td>
<td>5.5</td>
<td>2.1</td>
<td>-0.6</td>
<td>-2.6</td>
<td>-4.2</td>
</tr>
</tbody>
</table>

(a) (5 pts.) Graph the rate of change data.
(b) (5 pts.) Estimate the total change in temperature from time $t = 0$ to $t = 10$ minutes.
(c) (5 pts.) Shade an area in your graph that exactly matches your computation of total change.
(d) (5 pts.) If the object’s temperature was 60$\degree C$ at time 0, at what time (approximately) was its temperature 75$\degree C$?

10. (15 pts.) The graph of a function $f$ is shown at right. Let

$$g(x) = \int_{0}^{x} f(t) \, dt$$

Compute the following exactly:

(a) $g(0)$
(b) $g(2)$
(c) $g(6)$
(d) $g(-2)$

11. (10 pts.) Compute

$$\int_{2}^{4} x^4 + 3x^{-2} + 2x^{-1} \, dx$$

Show all your work. I will not accept an unsupported answer obtained from your calculator.

12. (10 pts.) Compute

$$\int_{1}^{4} x^2 \sqrt{2x-1} \, dx$$

Show all your work. I will not accept an unsupported answer obtained from your calculator.