Review for Exam 1

Warning!
There will be at least one problem on this exam that requires you to compute a derivative using a limit of secant slopes. Any such problems will include instructions to this effect — often phrased as “Use the definition of derivative.” If you have doubts, ask during the exam.

Overview of Exam Conditions

- Unless specified as above, any techniques from Chapters 1 and 2 or Sections 3.1 and 3.2 are legal on any problem.

- Your exam is not in a regular class period. It will take place on Thursday, Feb 22, from 3:15 pm to 5:55 pm, in the Business Bldg, Room 105.

- This will be a closed-book, no-notes exam. You are NOT allowed
  - any printed materials
  - any written materials
  - a laptop or other computer
  - a cell phone or other communication device
  - headphones or earphones
  - anything that can connect to the internet

- You are allowed pens, pencils, erasers, a ruler or straightedge, and any calculator that meets the restrictions above.

- If you need blank scratch paper or graph paper, it will be provided.

- You are expected to bring a scientific calculator — one that can handle arithmetic, sine, cosine, square roots, logs and exponents.

- Expect to see about ten to twelve problems. Many of these will remind you of homework problems, but some may not. Any problems that do not seem like homework problems are doable using the techniques that you have been practicing in your homework.

- The exam will cover all of Chapter 2 and Sections 3.1 and 3.2. Chapter 1 material will naturally appear as part of other problems. Section 3.2 problems are the ONLY problems in which you will be allowed to use quick rules for derivatives. These problems will be clearly indicated on the test.

- The exam is designed to take about 1 hour if you recognize how to do each problem as soon as you see it. If you have done all the assigned homework, then this should happen for nearly all the problems. The exam is scheduled for 2 hours and 40 minutes in order to eliminate any time pressure.
Study Tips

- Do lots of homework!

**Warning!**
The graded problems alone are not sufficient. Be sure you can work all the homework problems.

- You goal should be to do so much homework that you can look at a problem and immediately know what to do with it.

- Once you know what to do, you should be able to do it quickly.

- The only way to get to this point is to work a large number of homework problems.

- **After you have worked all the assigned homework**, you might want to test yourself on the following review problems.
  
  
  Chapter 3, p 228: 1, 3, 5, 9, 51, 52, 85a, 85c, 86a, 93, 94, 95, 97, 99-103, 107, 109.

Chapter 1

- Be able to graph a function.

- Know how to evaluate a function given any input (number or letter).

- Be able to work the reverse problem. That is, given a target output, determine the input(s) that caused it.

- Know how to find the change in a function on an interval. Know the notation for this.

- Be able to do all of this for a function given by a formula, a graph, a table of values, or a description.

- You should be comfortable with the inputs, outputs, and graphs of the following basic functions, as well as shifted or stretched versions:

  \[
  f(x) = x \\
  f(x) = x^2 \\
  f(x) = \frac{1}{x} \\
  f(x) = \sin x \\
  f(x) = \cos x \\
  f(x) = \tan x \\
  f(x) = e^x \\
  f(x) = \ln x
  \]
Section 2.1

• Be able to compute secant slope on an interval.
• Be able to compute rate of change of a function on an interval. (Also called *average* rate of change.)
• If a function measures position, be able to compute (average) velocity on an interval.
• Know that these are ALL THE SAME THING.
• Be able to use numerical computations of secant slopes to guess a tangent slope.
• Be able to use algebraic computations of secant slopes to get an exact tangent slope.
• Know that this is also called “rate of change at a point”, “instantaneous rate of change”, or just “rate of change”.
• Know that if a function measures position, this can be called “velocity at an instant”, “instantaneous velocity”, or just “velocity”.
• Be able to do this if the function is given by a formula, a graph, a table of values, or a description.

Section 2.2

• Be able to compute limits by looking at a graph.
• Be able to compute limits by plugging in nearby inputs and then making a good guess.
• Be able to compute limits by factoring and canceling.
• Be able to compute limits by squeezing.
• Know which method applies in what situation.

Sections 2.3

• Given a function and a target interval on the y-axis, be able to locate an interval on the x-axis so that every x value has output in the target interval.
• Understand that
  “Find $\delta$ so that $0 < |x - x_0| < \delta \implies |f(x) - L| < \epsilon$”
  is the same question.
• Be able to express your answer as an interval or as a value of $\delta$.
• Be able to do this if the function is given as a graph or a formula.
• Be able to do this if $\epsilon$ is a number or a letter.
Sections 2.4

• Be able to compute left and right limits using any of the Section 2.2 methods.

• Be able to compute left and right limits involving absolute value.

• Know the following infinite limits

\[ \lim_{x \to -\infty} e^x = 0 \]

\[ \lim_{x \to \pm\infty} \frac{1}{x} = 0 \]

And more generally,

\[ \frac{\text{anything finite}}{\text{anything infinite}} \to 0 \]

• Know how to compute other limits at infinity by reducing to these. Usually this means dividing out the highest power in the problem.

• Be able to locate discontinuities or ranges of continuity by looking at a graph.

• Be able to compute other limits by algebraically reducing to known limits. In particular, be able to reduce to and recognize

\[ \lim_{t \to 0} \frac{\sin t}{t} = 1 \]

Section 2.5

• Know how to locate vertical asymptotes.

• Know how to express the right and left hand limits at a vertical asymptote.

Section 2.6

• Know how to locate the points where a function is discontinuous.

• Know how to write the intervals on which a function is continuous.

• Be able to do this for functions given as graphs or formulas.

• Be able to solve for an unknown constant that makes a function continuous.

Section 2.7

• Be able to compute tangent slope at a point.

• Be able to compute rate of change at a point.

• Be able to compute velocity at an instant.

• Be able to compute derivative at a point.
• Know that these are ALL THE SAME THING.

• Be able to do all these computations if the function is given by a formula, a graph, a table of values, or a description.

• Know that this will require you to compute a limit using one of the techniques from Section 2.2. Mostly you will use algebra, but be aware that sometimes you might have to guess based on nearby inputs.

• Be able to do this if the point is a letter, not a number.

• Be able to do the reverse problem. That is, given a tangent slope (or rate of change, or velocity) find out what input location it came from.

• Be able to find equations of tangent lines.

Section 3.1

• Given a function \( f(x) \) (or \( y \), or whatever), know all of the notations for derivative at a point. This includes if the point is a number, a letter, or even the letter that serves as input to the original function.

• Be able to compute \( f' \) at a point. Be able to do this if the point is a number, a letter, or even the letter that serves as input to the original function.

• Understand that \( f' \) is really a new function; one that measures tangent slopes at all possible points on the original function.

• Be able to graph \( f' \). You should be able to do this just by looking at a graph of the original \( f \).

Section 3.2

• Know the rules for derivative of \( x^r \), \( e^x \), constants, constant multiples, sums, products and quotients.

• Be able to use these rules to compute derivatives quickly and accurately.

• Understand that these rules produce the derivative at a location that is named for the letter that is also input to the function.

• Know how to use these rules to compute a derivative when at a location other than this (perhaps a number, perhaps some other letter.)

• Be able to do all of this even for abstract functions and unknown constants.