Homework 4/27

1. Use the velocity data in Problem 9, Section 5.1 for the following problems.
   
   (a) Graph the velocity data. Be sure to label your graph properly.
   (b) Work problem 9(a).
   (c) In your velocity graph, shade an area that exactly matches your computation from 9(a).
   (d) Repeat parts (a)-(c) for problem 9(b). You will have to redraw your velocity graph.

2. Use the velocity data in Problem 11, Section 5.1 for the following problems.
   
   (a) Graph the velocity data. Be sure to label your graph properly.
   (b) Work problem 11(a).
   (c) In your velocity graph, shade an area that exactly matches your computation from 11(a).
   (d) Repeat parts (a)-(c) for problem 11(b). You will have to redraw your velocity graph.

3. Refer to Section 5.1; Problem 12.
   
   (a) Work Problem 12(a). You must assume constant velocity on each 0.001 hour time interval, but you get to choose the constant.
   (b) Shade an area the matches your computation for 12(a). If you don’t like to write in your book you will have to recreate the velocity graph.
   (c) Is your answer to 12(a) an underestimate or an overestimate of the true distance traveled? Why?
   (d) Work Problem 12(b).
   (e) Is your answer to 12(b) an underestimate or an overestimate of the true time? Why?

4. Suppose that the rate of change of the charge stored in a capacitor is given by

\[ f(t) = 1 + e^{-0.5t} \sin(2\pi t) \]

Time \( t \) is measured in seconds and rate of change of capacitance is measured in micro-Farads per second (\( \mu \text{F/s} \)).

   (a) Graph the rate of change data on the domain \( 0 \leq t \leq 2 \) seconds.
   (b) Find total change in capacitance, \( \Delta C \), on the time interval \( 0 \leq t \leq 2 \) seconds. Use four equal sub-intervals. Assume constant capacitance on each sub-interval. You choose the constant.
(c) In your graph of rate of change data, shade an area that matches your computation of total change.

(d) Recompute $\Delta C$ on $[0, 2]$ using 8 equal sub-intervals. Draw another graph of rate of change data and shade an area that matches this computation.

(e) Recompute $\Delta C$ on $[0, 2]$ using 12 equal sub-intervals. Draw another graph of rate of change data and shade an area that matches this computation.

(f) Recompute $\Delta C$ on $[0, 2]$ using 16 equal sub-intervals. Draw another graph of rate of change data and shade an area that matches this computation.

(g) Use your 16 interval approximation to find when $\Delta C$ reaches $1.2 \, \mu F$.

---

**Selected Answers**

3. All answers will depend on your choices of time intervals and velocities within each interval. I used 10 equal intervals and left endpoint data.

   (a) 0.898 miles; (b) Between 0.006 and 0.007 hours, closer to 0.006.

4. All answers will depend on your choices of rate of change data in each interval. I used left endpoint data in all cases.

   (a) 2.0000 $\mu F$; (b) 2.0784 $\mu F$; (c) 2.0906 $\mu F$; (d) 2.0947 $\mu F$;

   (e) Between 1.125 and 1.250 seconds. Linear interpolation gives $t = 1.136$ seconds.

NOTE: If you worked this problem before the correction, then your answers with left endpoints should have been:

   (a) 2.1532 $\mu F$; (b) 2.1858 $\mu F$; (c) 2.1916 $\mu F$; (d) 2.1936 $\mu F$;

   (e) Between 0.750 and 0.825 seconds.
Graph for Problem 4(f) with left endpoints
Graph for Problem 4 with 60 sub-intervals.