NOTE 1: Regardless of your prior experience with calculus, you must use limit methods for all derivatives on this exam.

NOTE 2: You must show all of your work and indicate units (when applicable) for full credit.

1. (15 pts) Let \( D(t) \) be the US national debt at time \( t \). The table below gives approximate values of this function by providing end of year estimates, in billions of dollars, from 1980 to 2000.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( D(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>930.2</td>
</tr>
<tr>
<td>1985</td>
<td>1945.9</td>
</tr>
<tr>
<td>1990</td>
<td>3233.3</td>
</tr>
<tr>
<td>1995</td>
<td>4974.0</td>
</tr>
<tr>
<td>2000</td>
<td>5674.2</td>
</tr>
</tbody>
</table>

(a) What was the change in \( D \) on the interval \( 1985 \leq t \leq 1990 \)?

Solution:

\[
3233.3 - 1945.9 = 1287.4 \text{ billions of } \$
\]

(b) What was the rate of change of \( D \) on the interval \( 1990 \leq t \leq 1995 \)?

Solution:

\[
\frac{4974 - 3233.3}{1995 - 1990} = \frac{1740.7}{5} = 348.14 \text{ billions of } \$/\text{year}
\]

(b) Guess the value of \( D'(1990) \).

Solution:

<table>
<thead>
<tr>
<th>fixed</th>
<th>free</th>
<th>( \Delta t )</th>
<th>( \Delta D )</th>
<th>( \Delta D/\Delta t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>1985</td>
<td>5</td>
<td>1287.4</td>
<td>257.48</td>
</tr>
<tr>
<td>1990</td>
<td>1995</td>
<td>-5</td>
<td>1740.7</td>
<td>348.14</td>
</tr>
</tbody>
</table>

\[
\Rightarrow D'(t) \approx \frac{348.14 + 257.48}{2} = 302.81 \text{ billions of } \$/\text{year}
\]
2.
(a) (10 pts) Using the graph of $f$ below compute at least two secant slopes with fixed end $x = 0$. You choose the free ends.

Solution:

<table>
<thead>
<tr>
<th>fixed</th>
<th>free</th>
<th>$\Delta t$</th>
<th>$\Delta D$</th>
<th>$\Delta D/\Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>1/4</td>
<td>-1/4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1/4</td>
<td>-1/4</td>
</tr>
</tbody>
</table>

(b) (5 pts) Guess the tangent slope at $(0,0)$.

Solution: Based on part (a) it looks like the slope $\approx -1/4$. 

![Graph of function](image_url)
3. (10 pts) A particle moves along a straight line with position equation \( s(t) = \sin\left(\frac{\pi}{t}\right) + 1 \) with \( t \) measured in seconds and \( s \) measured in feet. Find the average velocity for \( \frac{2}{3} \leq t \leq 2 \).

Solution:

\[
\text{average velocity on } \left[\frac{2}{3}, 2\right] = \frac{s(2) - s\left(\frac{2}{3}\right)}{2 - \frac{2}{3}}
\]

\[
= \frac{\sin\left(\frac{\pi}{2}\right) + 1 - \left(\sin\left(\frac{\pi}{2/3}\right) + 1\right)}{4/3}
\]

\[
= \frac{2}{4/3}
\]

\[
= \frac{3}{2} \text{ ft/s}
\]
4. (15 pts) Use the graph of $f$ below to answer $(a) - (c)$.

\[ \begin{array}{c}
\text{(a) Find } \lim_{x \to -2^-} f(x). \\
\text{Solution:} \quad 2
\end{array} \]

\[ \begin{array}{c}
\text{(b) Find } f(-1). \\
\text{Solution:} \quad -2
\end{array} \]

\[ \begin{array}{c}
\text{(c) Find } c \text{ such that } \lim_{x \to c} f(x) \text{ does not exist but } f(c) \text{ does exist.} \\
\text{Solution:} \quad 2
\end{array} \]
5. (10 pts) If a rock is thrown upward on the planet Mars with an initial velocity of 10 m/s, its height (in meters) after $t$ seconds is given by $h(t) = 10t - 1.86t^2$. Find it’s height when it’s velocity is $-8$ m/s.

Solution:

$$dq = \frac{10z - 1.86z^2 - (10t - 1.86t^2)}{z - t}$$

$$= \frac{10(z - t) - 1.86(z^2 - t^2)}{z - t}$$

$$= 10 - 1.86(z + t)$$

$$\Rightarrow h'(t) = \lim_{z \to t} [10 - 1.86(z + t)] = 10 - 3.72t$$

$$10 - 3.72t = -8 \Rightarrow t = 4.8387 \text{ s}$$

$$h(4.8387) = 4.8387 \text{ m}$$
6. (15 pts) Find the equation of the tangent line to the curve \( w = g(z) = z^3 - z^2 \) at \( z = 1 \).

Solution:

\[
dq = \frac{t^3 - t^2 - (z^3 - z^2)}{t - z}
\]

\[
= \frac{t^3 - z^3 - (t^2 - z^2)}{t - z}
\]

\[
= t^2 + tz + z^2 - (t + z)
\]

\[
\Rightarrow g'(z) = \lim_{t \to z} (t^2 + tz + z^2 - (t + z)) = 3z^3 - 2z
\]

\[
\Rightarrow g'(1) = 1
\]

\[
\Rightarrow w = z - 1
\]
7. (10 pts) Find \( \frac{d}{dx} f(x) \) for \( f(x) = x - \frac{1}{\sqrt{x}} \).

Solution:

\[
dq = \frac{z - \frac{1}{\sqrt{z}} - \left( x - \frac{1}{\sqrt{x}} \right)}{z - x}
\]

\[
= \frac{z - x + \left( \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{z}} \right)}{z - x}
\]

\[
= 1 + \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{z}}}{z - x}
\]

\[
= 1 + \frac{\sqrt{z} - \sqrt{x}}{(z - x) \sqrt{x} \sqrt{z}}
\]

\[
= 1 + \frac{z - x}{(z - x) \sqrt{x} \sqrt{z} (\sqrt{z} + \sqrt{x})}
\]

\[
= 1 + \frac{1}{\sqrt{x} \sqrt{z} (\sqrt{z} + \sqrt{x})}
\]

\[
\Rightarrow \frac{d}{dx} f(x) = \lim_{z \to x} 1 + \frac{1}{\sqrt{x} \sqrt{z} (\sqrt{z} + \sqrt{x})} = 1 + \frac{2}{x \sqrt{x}}
\]
8. (10 pts) If the tangent line to \( y = f(x) \) at \((4, 3)\) passes through the point \((0, 2)\) find \( f(4) \) and \( \frac{dy}{dx}_{x=4} \).

Solution:

\[(4, 3) \text{ on the graph } \Rightarrow f(4) = 3\]

through \((4, 3)\) and \((0, 2)\) \(\Rightarrow \frac{dy}{dx}_{x=4} = m = \frac{3 - 2}{4 - 0} = \frac{1}{4}\)