Method IV — Binomial Series. All of the series in these problems are centered at 0.

1. §8.10: 1, 3, 5, 7, 9, 15.

2. Compute the fourth order approximation of

\[ f(x) = \frac{1}{\sqrt{1 + x^2}} \]

centered at 0. Use any methods.

Applications. In each of these problems you will need to replace a function with a power series approximation. Sometimes I will specify center point and order. Other times it will be up to you to pick something sensible. In every problem it will be you job do chose a method of generating the power series approximation.

3. Use the fifth order approximation of \( \sin^{-1} x \) at 0 to estimate

\[ \int_0^{1/2} \sin^{-1} x \, dx \]

4. Use at least a fifth order approximation to estimate

\[ \int_0^{1/2} [\sin^{-1} x]^2 \, dx \]

5. Consider the region bounded by \( y = 0.5, y = \sin x, \) and the \( y \)-axis. Rotate this region about the \( y \)-axis to form a solid. Suppose that the axes are measured in feet and the solid is filled with water (62.5 lbs/ft³).

(a) Write an integral for the energy released if the water drains out a hole in the bottom of the tank.

(b) Compute your integral using at least a fifth order polynomial approximation.

6. Find the length of the curve

\[ y = \frac{1}{7} x^{7/2}; \quad 0 \leq x \leq 1 \]

Use at least three non-zero terms of a polynomial approximation to compute your integral.
7. Use a quadratic approximation of $e^{-x}$ at 0 to estimate the solution to

$$e^{-x} = x$$

8. Use a quadratic approximation at an appropriate center point to estimate the solution to

$$x + \ln x = 2$$

Hints and Answers

2. $1 - \frac{1}{2}x^2 + \frac{3}{8}x^4$

3. $1963/15360 \approx 0.1278$.

4. $7/160 = 0.04375$.

5. (b) $2375\pi/2304 \approx 3.24$ ft-lbs.

6. $4309/4224 \approx 1.020$.

7. $2 - \sqrt{2} \approx 0.586$.

8. $3 - \sqrt{2} \approx 1.586$. 

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