Show all your work.

7. The conical tank shown at right is filled with water (62.5 lbs/ft³). There is no water in the pipe below the valve. The valve is opened and all the water drains down to the ground, one foot below the valve, releasing 4,000 ft-lbs of potential energy. How tall is the cone?

**NOTES:**

If you compute any integrals you must show work according to Type I Integration Instructions.

Potential Energy = (Weight) × (Height above Ground)

\[ dV = \pi x^2 dy \quad y = \frac{1}{2} x \implies x = \frac{2}{L} y \]

\[ = \pi \left( \frac{2}{L} y \right)^2 dy \]

\[ dW = (62.5)(\pi)\left( \frac{y}{L^2} \right) y dy = 250 \frac{\pi}{L^2} y^2 dy \]

\[ dU = 250 \frac{\pi}{L^2} y^2 (y - (-1)) dy \]

\[ U = \int_0^L \frac{250\pi}{L^2} \left( y^3 + y^2 \right) dy \]

\[ = \frac{250\pi}{L^2} \left[ \frac{y^4}{4} + \frac{y^3}{3} \right]_0^L = \frac{250\pi}{L^2} \left[ \frac{L^4}{4} + \frac{L^3}{3} \right] \]

\[ U = \frac{125\pi}{2} L^2 + \frac{250\pi}{3} L \]
\[ U = \frac{125 \pi}{2} L^2 + \frac{250 \pi}{3} L = 4000 \]

\[ \frac{2}{125 \pi} \left( \frac{125 \pi}{2} L^2 + \frac{250 \pi}{3} L - 4000 \right) = 0 \]

\[ L^2 + \frac{4}{3} L - \frac{64}{\pi} = 0 \]

\[ L = -\frac{4}{3} \pm \sqrt{\frac{16}{9} + 4 \left( \frac{64}{\pi} \right)} \]

\[ L = -\frac{2}{3} \pm \sqrt{\frac{16}{9} + \frac{64}{\pi}} \text{ ft} \]

\[ \frac{2}{125 \pi} \]

\[ L \text{ can't be negative, so } \]

\[ L = -\frac{2}{3} + \sqrt{\frac{4}{9} + \frac{64}{\pi}} \text{ ft} \]

\[ L \approx 3.90 \text{ ft} \]
8. A thin, solid tube of aluminum is bent into a semicircle and attached to a wall as shown at right. (The view in the picture is straight down.)

- The semicircle has a radius of 10 inches.
- The aluminum tube has a diameter of 0.5 inches.
- Aluminum weighs 0.1 lbs/in³.

Imagine an axis connecting the ends of the tube. Compute the moment of the tube about that axis. Show all work as specified in Type 1 Integration Instructions.

NOTE:

Moment = (Weight) × (Distance to Axis)

\[
\begin{align*}
\int 20 \, ds & = \pi \left( \frac{1}{4} \right)^2 ds = \frac{\pi}{16} \, ds, \\
\int 20 \, ds & = 0.12V = \frac{\pi}{160} \, ds,
\end{align*}
\]

\[x = \sqrt{100 - y^2}\]

\[x' = \frac{1}{2\sqrt{100 - y^2}} (-2y) = \frac{-y}{\sqrt{100 - y^2}}\]

\[ds = \sqrt{\frac{y^2}{100 - y^2} + 1} \, dy\]

\[dM_y = x \cdot \frac{\pi}{160} \, ds\]

\[= \sqrt{100 - y^2} \left( \frac{\pi}{160} \right) \sqrt{\frac{y^2}{100 - y^2} + 1} \, dy\]
Simplify

c/My = \frac{\pi}{160} \sqrt{(100-y^2) \frac{y^2}{100-y^2} + (100-y^2)} \ dy

= \frac{\pi}{160} \sqrt{y^2 + 100 - y^2} \ dy

= \frac{\pi}{16} \ dy

M_y = \int_{-10}^{10} \frac{\pi}{16} \ dy = \left[ \frac{\pi}{16} y \right]_{-10}^{10} = \frac{10\pi}{16} - \left( \frac{-10\pi}{16} \right)

= \frac{20\pi}{16} = \frac{5\pi}{4} \approx 3.93 \text{ in-lbs}