• Show all your work.

• If you use your calculator to do anything beyond arithmetic or graphing, be sure to indicate “Here I used my calculator” somehow.

• If a problem says “Write an integral for...,” you must express everything in terms of your variable of integration.

• If you compute an integral, show your work and don’t skip any big steps. I have to know that you did it, not your calculator.

1. (10 pts.) Let $R$ be the region enclosed by the curves $y = 1 - x^3$, $y = 0$, $x = 0$

   (a) Chose an axis of integration.
   (b) Draw a typical slice of area.
   (c) Compute the area of that slice.
   (d) Write an integral for the total area of $R$.

2. (15 pts.) Suppose that the region from Problem 1 is a sheet of material with variable density: $\rho = (x + 1)$ g/cm$^2$. The axes are measured in cm. Find the total mass.

   • You ARE allowed to reuse anything from Problem 1 without showing work.
   • You are NOT allowed to use your calculator to evaluate the integral. Do it yourself and show all the steps.

3. (15 pts.) Suppose the region bounded by

   $y = 0; \quad x = \pi/2; \quad$ and $\quad y = \sin x, \quad 0 \leq x \leq \pi/2$

   is rotated about the axis $x = \pi$.

   (a) Chose an axis of integration.
   (b) Draw a typical slice. (2-D or 3-D, your choice.)
   (c) Compute the volume of that slice.
   (d) Write an integral for the total volume.
4. (25 pts.) Suppose there is a tank full of water (density 1000 kg/m$^3$) in the shape shown at right.

(a) Write an integral for the work needed to pump all the water to a point 1 meter above the top of the tank.

\[
\text{Work} = \text{Force} \times \text{Distance}
\]

(b) Write an integral for the amount of force exerted on one triangular end of the tank.

\[
\text{Force} = \text{Pressure} \times \text{Area}
\]

\[
\text{Pressure} = (1000) \times (9.81) \times \text{Depth}
\]

5. (10 pts.) Write an integral for the length of the curve

\[x = y^2, \quad 0 \leq y \leq 2\]

6. (15 pts.) Suppose the curve

\[x = y^2, \quad 0 \leq y \leq c\]

is rotated about the x-axis. Axes are measured in feet, and the area of the resulting surface is $7\pi/6 \text{ ft}^2$. What is $c$?

- You ARE allowed to reuse anything from Problem 5 without showing work.
- You are NOT allowed to use your calculator to evaluate the integral. Do it yourself and show all the steps.
- Hint: Some axes are nicer than others.

7. (10 pts.) If a concentrated weight $F$ lies $x$ feet from the y axis, its second moment about the y-axis is

\[M_{yy} = Fx^2\]

Another way to say this is

\[(\text{second moment}) = (\text{force}) \times (\text{distance to axis})^2\]

Write an integral for $M_{yy}$ for the distributed mass shown at right. Assume density varies as $(2 - x)$ lb/ft$^2$. 

\[\text{Diagram of tank}\]

\[\text{Diagram of concentrated weight}\]