Show all your work!

1. (15pts.) The growth of an investment is shown in the table below, where $t$ is measured in years and $A$ is measured in dollars.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>5000</td>
<td>5120</td>
<td>5250</td>
<td>5390</td>
<td>5540</td>
</tr>
</tbody>
</table>

(a) Find the average rate of change of $A$ on the interval $0.5 \leq t \leq 1$ year.
(b) Find the average rate of change of $A$ on the interval $1 \leq t \leq 1.5$ years.
(c) Estimate the instantaneous rate of change of $A$ at time $t = 1$ year.

2. (15 pts.) Suppose that $f(x) = 3x^2 - 2x + 4$.

(a) Find the secant slope on the interval $[2, 3]$.
(b) Use a limits to compute the tangent slope at 3. DO NOT USE chapter three derivative methods.

3. (15 pts.) Find $dy/dx$ for each of the following:

(a) $y = \ln \left( \frac{x + 1}{x} \right)$
(b) $y = \frac{\sin x + x}{x^2 + \cos x}$
(c) $y = \tan(e^{kx})$, where $k$ is a constant.

4. (20 pts.) An object moves along a straight path so that its position (in meters) after $t$ seconds is

$$s(t) = t^2 + \cos(2t)$$
(a) Find one instant in time when the particle’s acceleration is zero.
(b) Find its velocity at that instant.

5. (15 pts.) Two cars are approaching an intersection as shown at right. One is moving at 20 miles per hour, the other at 30 miles per hour. What is the rate of change of $\theta$ when both cars are 100 feet from the intersection?

6. (10 pts.) The graph of $y = ax^3 + bx$ is shown at right, along with a tangent line at the point $(2, 1)$. If the tangent also passes through $(0, -3)$, what are $a$ and $b$?

7. (10 pts.) The volume of an object (in cubic inches) is given by $V(x) = 32\pi r^2(r - 10)$, where $r$ is measured in inches. Use differentials to approximate the change in volume if $r$ increases from 5 inches to 5.5 inches.

8. (20 pts.) For $f(x) = 4x^3 - 3x^2$ find:
   (a) The exact intervals on which $f$ is increasing, and those on which it is decreasing.
   (b) The exact intervals on which $f$ is concave up, and those on which it is concave down.

9. (15 pts.) The box shown at right has square ends, an open top, and a divider in the middle. Its total volume is 0.5 m$^3$. Assume that “surface area” means the total area of sides, ends, bottom and divider. Find the minimum possible surface area. For full credit, you must:
(a) Write a function for surface area.
(b) Graph that function.
(c) Locate the minimum surface area exactly. (Calculator approximation will not suffice.)
(d) Show all your work.

10. (15 pts.) The velocity of a moving object (in m/s) is measured every 5 seconds and recorded in the following table:

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$ (m/s)</td>
<td>8.1</td>
<td>6.8</td>
<td>5.7</td>
<td>5.1</td>
<td>4.9</td>
</tr>
</tbody>
</table>

(a) Graph the velocity data on the graph paper provided below. (Label your graph properly!)
(b) Estimate the total change in the object’s position on the time interval $0 \leq t \leq 20$ seconds.
(c) Shade an area in your graph that matches your computation.
11. (20 pts.) The graph of a function $f$ is shown at right. If $g(x) = \int_0^x f(t) \, dt$, compute the following:
   (a) $g(-2)$
   (b) $g(0)$
   (c) $g(4)$
   (d) $g'(2)$

12. (20 pts.) Compute each of the following exactly:
   
   (a) $\int_0^{\pi/2} (t + 4t^2 - \sin t) \, dt$
   
   (b) $\int_{-2}^0 xe^{x^2-4} \, dx$

13. (10 pts.) The rate of change of a certain quantity is given by $f(t) = 3t + 2/t$. If the quantity is 15 at time $t = 10$, what is the quantity at time $t = 20$?