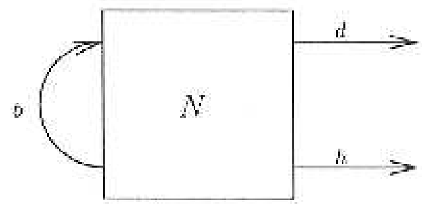
Exam 1
M464-001
Fall, 2008

Name: Key

Part I. Suppose that in a population of N animals we have the following conditions (as represented in the diagram at right.)



- Birth rate is $b = 0.01$ *per month* per animal.
- Death rate is $d = \dfrac{N}{80000}$ *per month* per animal.
- $h = 3$ animals are removed from the population in each *two month* time period.

1. Write a difference equation for N.
2. Find all equilibrium solutions.
3. Sketch the equilibrium solution(s) on a time-N graph. Be sure your graph is properly labeled.
4. On the same graph, sketch the graph of a population for which $N(0) = 300$ animals.
5. On the same graph, sketch the graph of a population for which $N(0) = 700$ animals.

1. $\Delta N = 0.01\,N - \dfrac{N^2}{80000} - 1.5$

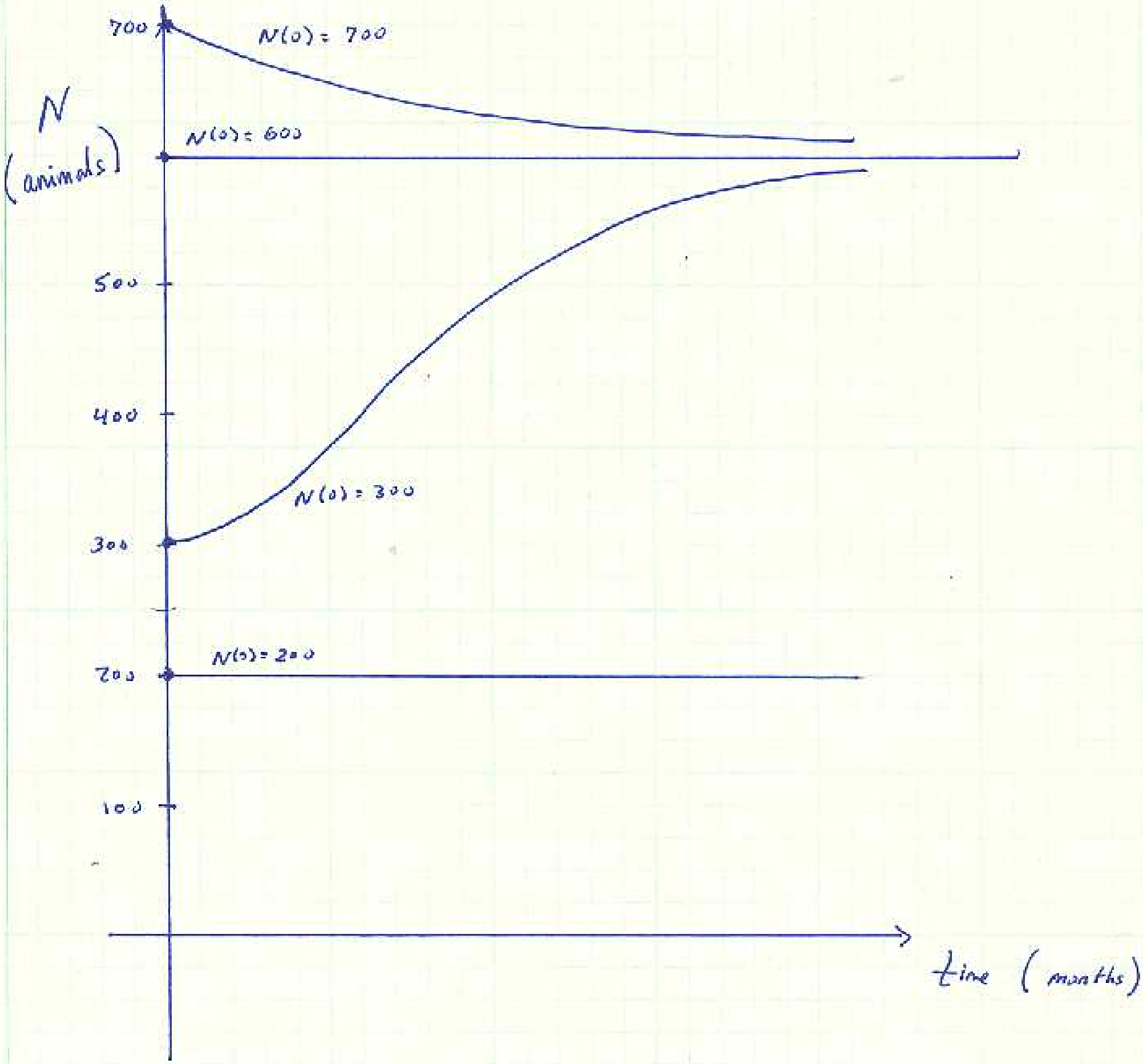
2. $\Delta N = \dfrac{1}{80000}\left[800\,N - N^2 - 120{,}000\right] = 0$

$$N^2 - 800N + 120{,}000 = 0$$

$$(N-200)(N-600) = 0$$

$N = 200$

$N = 600$


N
(animals)
700
N(0) = 700
N(0) = 600
500
400
N(0) = 300
300
N(0) = 200
200
100
time (months)

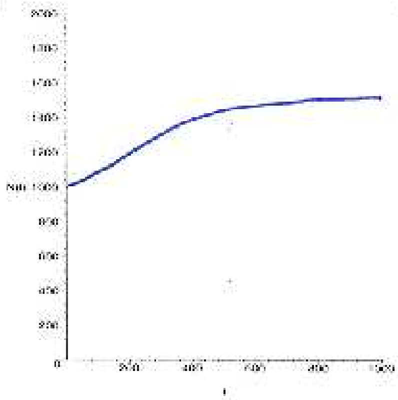
Part II. Below there are three difference equation models. There are also three population graphs, each with $N(0) = 1000$. For each model, indicate which graph, if any, is a possible solution. In the blank after each equation write the letter of the appropriate graph or "None".

1. $\Delta N = \dfrac{1}{80000}(1800N - N^2 - 890000)$ __B__

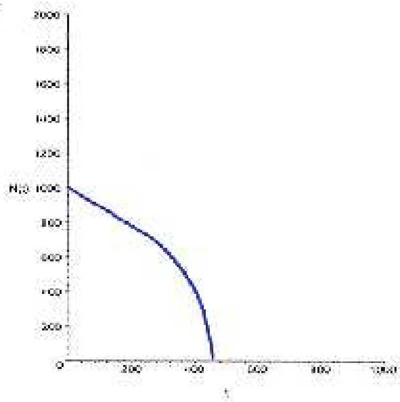
2. $\Delta N = \dfrac{1}{80000}(1800N - N^2 - 810000)$ __None__

3. $\Delta N = \dfrac{1}{80000}(2400N - N^2 - 1350000)$ __A__

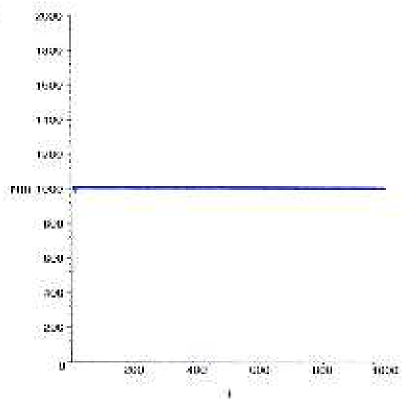
A.)



B.)



C.)



1. $N^2 - 1800N + 890000 = 0$?

$(N^2 - 1800 + 810000) + 8000 = 0$

$(N - 900)^2 + 8000 = 0$ No roots. (OR, use quad. formula.)

No equil. & $N(0) = 1000 \Rightarrow \Delta N < 0 \Rightarrow$ eventuall extinction. $\boxed{B}$

2. $\Delta N = \frac{1}{[illegible]}(N - 900)^2$. Single equilibrium at $N = 900$

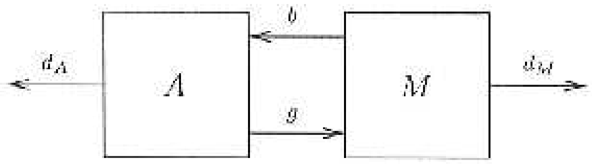
$N(0) = 1000 \Rightarrow \Delta N < 0$, But no extinction. $\boxed{\text{None}}$

3. $\Delta N = \dfrac{-1}{80000}(N - 1500)(N - 900)$; $N(0) = 1000 \Rightarrow \Delta N > 0$

seems like $\boxed{A}$

Part III. Suppose that a population is divided into M mature animals and A adolescents. Assume the following conditions (as represented in the diagram below):

- Birth rate is $b = 0.4$ adolescents per month per mature animal.
- Death rate for adolescents is $d_A = 0.1$ per month per adolescent.
- Death rate for mature animals is $d_M = 0.1$ per month per mature animal.
- Adolescents mature into adults at a rate of $g = 0.2$ per month per adolescent.



1. Write difference equations for A and M.

2. Rewrite your model as a recursive matrix model. In other words, if

$$\begin{bmatrix} A_{n+1} \\ M_{n+1} \end{bmatrix} = B \begin{bmatrix} A_n \\ M_n \end{bmatrix}$$

what is the matrix B?

1. $\Delta A = 0.4\,M - 0.1\,A - 0.2\,A$

$\Delta M = 0.2\,A - 0.1\,M$

2. $A_{n+1} - A_n = -.3A_n + .4M_n \Rightarrow A_{n+1} = .7A_n + .4M_n$

$M_{n+1} - M_n = .2A_n - .1M_n \Rightarrow M_{n+1} = .2A_n + .9M_n$

$$\begin{bmatrix} A_{n+1} \\ M_{n+1} \end{bmatrix} = \begin{bmatrix} .7 & .4 \\ .2 & .9 \end{bmatrix} \begin{bmatrix} A_n \\ M_n \end{bmatrix}$$

Part IV. Suppose that a population of adolescent and mature animals is modeled by the recursive model

$$\begin{bmatrix} A_{n+1} \\ M_{n+1} \end{bmatrix} = \begin{bmatrix} .7 & .4 \\ .2 & .9 \end{bmatrix} \begin{bmatrix} A_n \\ M_n \end{bmatrix}$$

1. Find eigenvectors and eigenvalues for this model.

2. Use your work to plot the graph of a population that starts out with $A(0) = 100$ animals and $M(0) = 0$. Plot this on A-M axes. Be sure your graph is properly labeled.

1. $\det \begin{bmatrix} .7-\lambda & .4 \\ .2 & .9-\lambda \end{bmatrix} = (.7-\lambda)(.9-\lambda) - .08$

$= .63 - 1.6\lambda + \lambda^2 - .08 = 0$

$\lambda^2 - 1.6\lambda + .55 = 0$

$(\lambda - 1.1)(\lambda - 0.5) = 0$

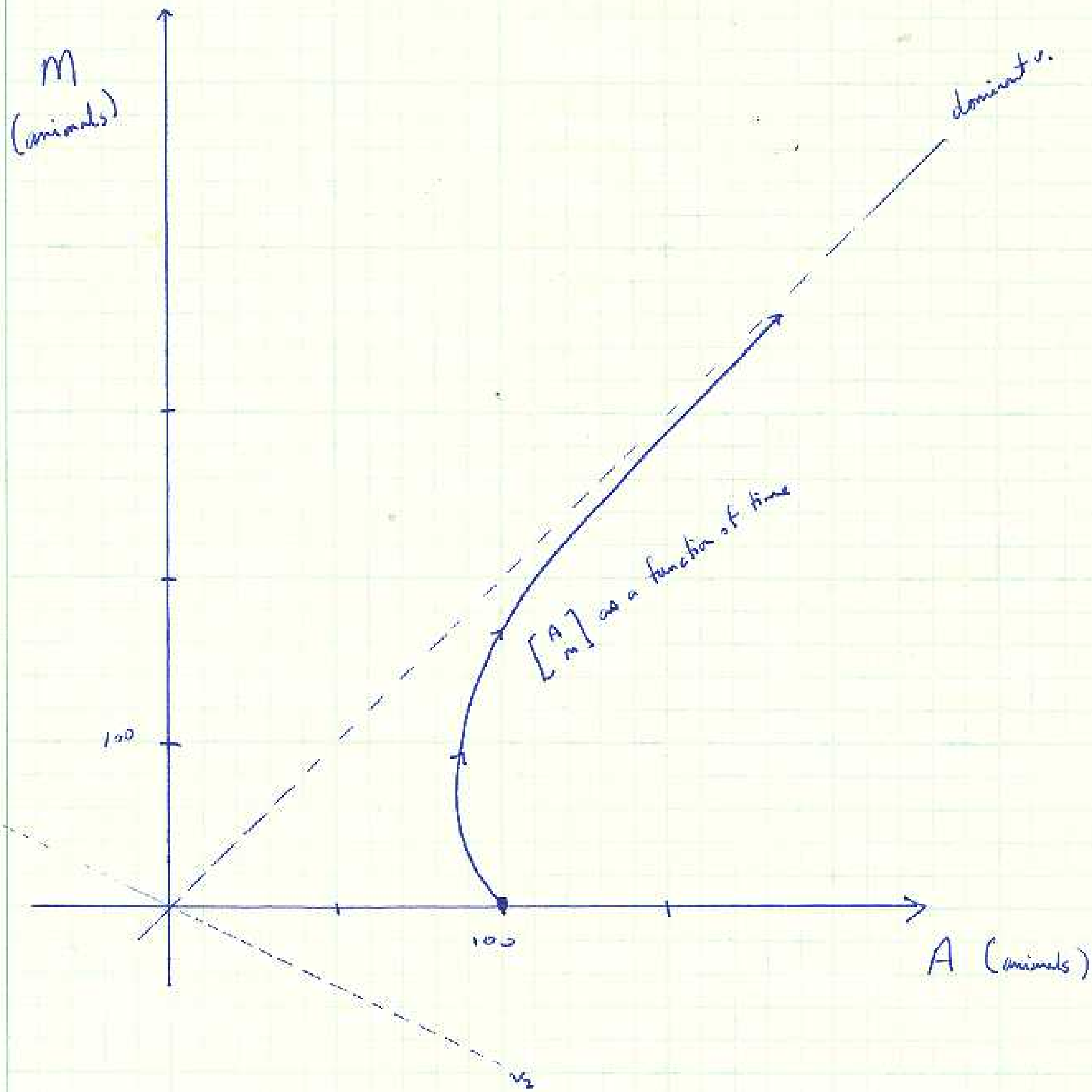
$\lambda_1 = 1.1, \ \lambda_2 = 0.5$

For $\lambda_1 = 1.1$, $\begin{bmatrix} -.4 & .4 \\ .2 & -.2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow x = 1, y = 1$

$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (dominant)

For $\lambda_2 = 1.2$, $\begin{bmatrix} .2 & .4 \\ .2 & .4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow x = -2, y = 1$

$V_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$


M
(animals)
dominant v.
$\begin{bmatrix} A \\ M \end{bmatrix}$ as a function of time
100
100
A (animals)
v_2