A population has **logistic growth** if its difference equation model is

$$\Delta R = kR \left(1 - \frac{R}{N}\right)$$

here $k$ is the growth rate and $N$ is the stable equilibrium population. ($N$ is also called the **carrying capacity**.)

In this worksheet you will build a model of two populations, rabbits $R$ and foxes $F$, such that

- If $F = 0$, then $R$ has logistic growth with $k = 0.02$ and $N = 100$.
- If $F \neq 0$, then $0.002RF$ rabbits are eaten by foxes in each time step.
- Foxes have a constant death rate $d = 0.01$ per fox per time step.
- Foxes have a birth rate of $0.0004R$ per fox per time step.

1. Write difference equations for $R$ and $F$.

2. Assume $F = 0$. Then sketch equilibria and solutions for possible populations of $R$. Do this on the horizontal axis of an $R$-$F$ coordinate system.

3. Assume $R = 0$ and do a similar sketch on the $F$-axis.

4. Find all other equilibria for this model.

5. For each equilibrium, linearize, compute eigenvalues, and if eigenvalues are real, find eigenvectors.

6. Try to sketch the solution curve that starts with initial populations $R(0) = 2$ and $F(0) = 10$.

7. Try again with $R(0) = 150$ and $F(0) = 50$. 