Math 464, Worksheet 18

Alter the population model from Worksheet 17 so that the two species cooperate:
\[
\begin{align*}
\Delta x &= 0.02x - 0.01x^2 + 0.01xy \\
\Delta y &= 0.03y - 0.01y^2 + 0.02xy
\end{align*}
\]

1. Locate all equilibria.

2. For each equilibrium point:
   (a) linearize
   (b) find eigenvectors
   (c) sketch them on one sheet of graph paper.

3. For each initial population shown below, sketch the graph of \((x(t), y(t))\) as a parametric function. Use your eigenvectors and equilibria as a guide. Check yourself using Excel. You can crib my Worksheet 16 solution as a starting point.

   (a) \((0.1, 0.1)\)    (b) \((0.1, 1)\)    (c) \((0.1, 3)\)    (d) \((0.1, 5)\)
   (f) \((1, 5)\)    (g) \((2, 5)\)    (h) \((3, 5)\)    (i) \((4, 5)\)
   (j) \((2, 1)\)    (k) \((3, 1)\)    (l) \((4, 1)\)    (m) \((5, 1)\)
   (j) \((5, 5)\)

4. Adjust the model so that interactions are good for species \(x\) but bad for species \(y\). (Perhaps species \(x\) is foxes and species \(y\) is rabbits. Interaction is good for foxes — dinner — and corresponding bad for rabbits.)
\[
\begin{align*}
\Delta x &= 0.02x - 0.01x^2 + 0.01xy \\
\Delta y &= 0.03y - 0.01y^2 - 0.02xy
\end{align*}
\]

Repeat Problems 1-3 for this model.

5. Switch the good/bad interactions:
\[
\begin{align*}
\Delta x &= 0.02x - 0.01x^2 - 0.01xy \\
\Delta y &= 0.03y - 0.01y^2 + 0.02xy
\end{align*}
\]

Repeat Problems 1-3 for this model.