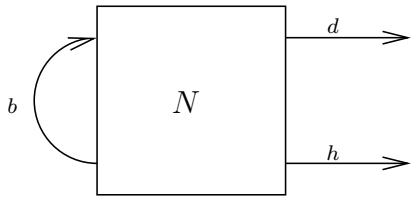
Exam 1
M464-001
Fall, 2008

Name:_____

Part I. Suppose that in a population of N animals we have the following conditions (as represented in the diagram at right.)



- Birth rate is $b = 0.01$ *per month* per animal.
- Death rate is $d = \dfrac{N}{80000}$ *per month* per animal.
- $h = 3$ animals are removed from the population in each *two month* time period.

1. Write a difference equation for N.
2. Find all equilibrium solutions.
3. Sketch the equilibrium solution(s) on a time-N graph. Be sure your graph is properly labeled.
4. On the same graph, sketch the graph of a population for which $N(0) = 300$ animals.
5. On the same graph, sketch the graph of a population for which $N(0) = 700$ animals.

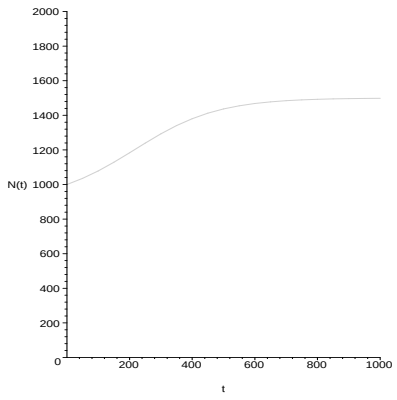
Part II. Below there are three difference equation models. There are also three population graphs, each with $N(0) = 1000$. For each model, indicate which graph, if any, is a possible solution. In the blank after each equation write the letter of the appropriate graph or "None".

1. $\Delta N = \dfrac{1}{80000}(1800N - N^2 - 890000)$ _____

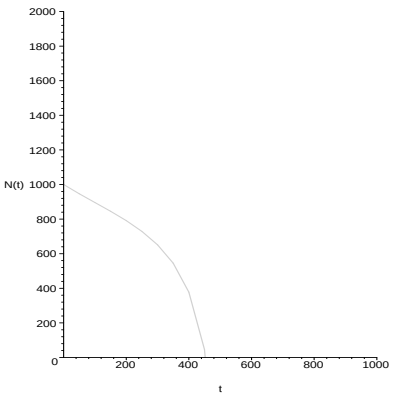
2. $\Delta N = \dfrac{1}{80000}(1800N - N^2 - 810000)$ _____

3. $\Delta N = \dfrac{1}{80000}(2400N - N^2 - 1350000)$ _____

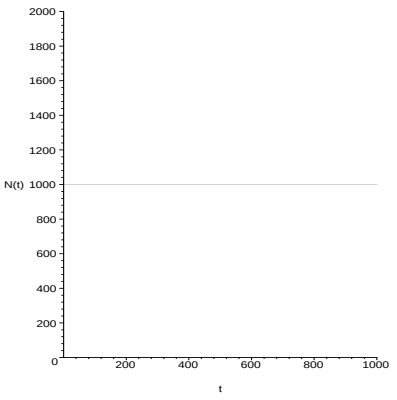
A.)



B.)

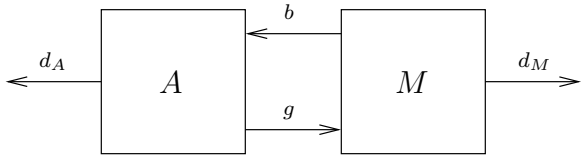


C.)



Part III. Suppose that a population is divided into M mature animals and A adolescents. Assume the following conditions (as represented in the diagram below):

- Birth rate is $b = 0.4$ adolescents per month per mature animal.
- Death rate for adolescents is $d_A = 0.1$ per month per adolescent.
- Death rate for mature animals is $d_M = 0.1$ per month per mature animal.
- Adolescents mature into adults at a rate of $g = 0.2$ per month per adolescent.



1. Write difference equations for A and M.
2. Rewrite your model as a recursive matrix model. In other words, if

$$\begin{bmatrix} A_{n+1} \\ M_{n+1} \end{bmatrix} = B \begin{bmatrix} A_n \\ M_n \end{bmatrix}$$

what is the matrix B?

Part IV. Suppose that a population of adolescent and mature animals is modeled by the recursive model

$$\begin{bmatrix} A_{n+1} \\ M_{n+1} \end{bmatrix} = \begin{bmatrix} .7 & .4 \\ .2 & .9 \end{bmatrix} \begin{bmatrix} A_n \\ M_n \end{bmatrix}$$

1. Find eigenvectors and eigenvalues for this model.

2. Use your work to plot the graph of a population that starts out with $A(0) = 100$ animals and $M(0) = 0$. Plot this on A-M axes. Be sure your graph is properly labeled.