Homework, 10/22

Part 1:

1. Use differentials to approximate \((2.002)^5\).
2. Use differentials to approximate \((1.997)^5\).
3. Use differentials to approximate \((2 + h)^5\).
4. Use differentials to approximate \((x)^5\) near \(x = 2\).
5. Use your approximation to solve \(x^5 = 35\).
6. Graph \(y = x^5\) and your answer to part (4) on the same axes. Your graph must extend at least from \(x = 0\) to \(x = 3\).

Part 2: Warning!. Work this problem in radians!

1. Use differentials to approximate \(\tan 46^\circ\).
   [Hint: Convert \(\Delta x = 1^\circ\) to radians.]
2. Use differentials to approximate \(\tan 44^\circ\).
3. Use differentials to approximate \(\tan(\pi/4 + h)\).
4. Use differentials to approximate \(\tan x\) near \(x = \pi/4\). Simplify your answer.
5. Use your approximation to solve \(\tan x = .98\).
6. Graph \(y = \tan x\) and your answer to part (4) on the same axes. Your graph must extend at least from \(x = 0\) to \(x = \pi/2\).

Part 3:

1. The volume of a cube with side length \(x\) is \(V = x^3\). Use differentials to approximate \(V(x)\) near \(x = 30\).
2. The surface area of the cube is \(A = 6x^2\). Use differentials to approximate \(A(x)\) near \(x = 30\).
3. Use your approximations to find:
   (a) What \(x\) gives a volume of 30000?
   (b) What \(x\) gives an area of 5000?
Part 4: Suppose that \( f(x) = 1 + \sin x + e^{0.1x} \).

1. Guess an answer to the equation \( f(x) = 2 \).

2. Use differentials to approximate \( f(x) \) near your answer to Problem (1).

3. Use your approximation to solve the equation \( f(x) = 2.2 \).

Answers

Part 1:
1) 32.16. 2) 31.76. 3) 32 + 80h. 4) 32 + 80(x - 2). 5) \( x = 2 + \frac{3}{80} = 2.0375 \).

Part 2:
1) \( 1 + \pi /90 \). 2) \( 1 - \pi /90 \). 3) \( 1 + 2h \). 4) \( 1 + 2(x - \pi/4) \). 5) \( x = \pi/4 - 0.01 \approx 0.7754 \).

Part 3:
1) \( V(x) \approx 27000 + 2700(x - 30) \). 2) \( A(x) \approx 5400 + 360(x - 30) \). 3) \( x = 30 + 10/9 \approx 31.11 \). 4) \( x = 30 - 10/9 \approx 28.89 \).

Part 4:
1) \( x = 0 \). 2) \( f(x) \approx 2 + 1.1x \). 3) \( x = \frac{2}{11} \approx .1818 \).