• Show all your work.

• Regardless of your prior experience with calculus, you must use limit methods for all derivatives on this exam.

1. An investment grows according to the formula

   \[ A(t) = 100e^{0.1t} \]

   where \( A \) is measured in dollars and \( t \) is measured in years.

   (a) (5 pts.) Find the average rate of change of \( A \) on the interval \( 0.9 \leq t \leq 1 \) year.
   (b) (5 pts.) Find the average rate of change of \( A \) on the interval \( 1 < t < 1.1 \) years.
   (c) (5 pts.) Guess the instantaneous rate of change of \( A \) at time \( t = 1 \) year.

\[ a) \quad \frac{100e^{0.1(1)} - 100e^{0.1(0.9)}}{1 - 0.9} \approx 10.997 \text{ \$/yr.} \]

\[ b) \quad \frac{100e^{0.1(1.1)} - 100e^{0.1(1)}}{1.1 - 1} \approx 11.107 \text{ \$/yr.} \]

\[ c) \quad \text{Seems like} \approx 11.05 \text{ \$/yr.} \]
2. (15 pts.) If \( f(x) = \sqrt{2x-1} \), find the rate of change of \( f \) at the point \( x = 5 \).

\( f(x) - f(5) \)
\( \frac{\sqrt{2x-1} - \sqrt{2(5)-1}}{x-5} \)
\( = \frac{\sqrt{2x-1} - 3}{x-5} \cdot \frac{\sqrt{2x-1} + 3}{\sqrt{2x-1} + 3} \)
\( = \frac{2x-1 - 9}{(x-5)(\sqrt{2x-1} + 3)} \)
\( = \frac{2x - 10}{(x-5)(\sqrt{2x-1} + 3)} \)
\( = \frac{2(x-5)}{(x-5)(\sqrt{2x-1} + 3)} = \frac{2}{\sqrt{2x-1} + 3} \)

(2) tan slope: \( x \rightarrow 5 \) gives
\( \frac{2}{\sqrt{2(5)-1} + 3} = \frac{2}{5 + 3} = \frac{1}{3} \)
3. (15 pts.) Use the graph of $f$ shown at right to answer the following questions.

(a) $\lim_{x \to 1^+} f(x) = \frac{1}{2}$
(b) $\lim_{x \to 2} f(x) = \frac{1}{2}$
(c) $\lim_{x \to 1^-} f(x) = \frac{2}{2}$
(d) $\lim_{x \to 1^+} f(x) = \frac{1}{2}$
(e) $\lim_{x \to 1^-} f(x) = \text{DNE}$

4. Use the graph of $f$ shown at right to:

(a) (10 pts.) Compute at least two secant slopes with fixed end at $(0,0)$. You choose the free ends.
(b) (5 pts.) Guess the tangent slope at $(0,0)$.

a) I chose intervals $[0, 0.2]$ and $[-0.2, 0]$

(i) $\frac{2}{-2} = 1.5$

(ii) $\frac{2}{-2} = 1.5$

b) I guess 1.5
5. (15 pts.) Suppose that \( y = x + \frac{1}{x} \). Find the slope of the tangent line at the point \((1, 2)\).

\[ \text{Secant slope:} \quad \frac{x + \frac{1}{x} - (1 + \frac{1}{1})}{x - 1} = \frac{(x + \frac{1}{x} - 2)}{(x-1)} \cdot \frac{x}{x} = \frac{x^2 + 1 - 2x}{(x-1)x} = \frac{x^2 - 2x + 1}{(x-1)x} \]

\[= \frac{(x-1)(x-1)}{(x-1)x} = \frac{x-1}{x} \]

\[\text{Tangent slope:} \quad x \to 1 \quad \text{gives} \quad \frac{1-1}{1} = 0 \]
6. (15 pts.) A moving object has position (in meters) given by \( f(t) = 30 + 10t - 2t^2 \), with \( t \) in seconds. Find its position when its velocity is 8 m/s.

(i) Tangent slope. 
\[
\frac{f(z) - f(t)}{z - t} =
\]
\[
= \frac{(30 + 10z - 2z^2) - (30 + 10t - 2t^2)}{z - t}
\]
\[
= \frac{30 - 30 + 10z - 10t - 2z^2 + 2t^2}{z - t}
\]
\[
= \frac{10(z - t) - 2(z^2 - t^2)}{z - t}
\]
\[
= \frac{10(z - t) - 2(z - t)(z + t)}{z - t}
\]
\[
= 10 - 2(z + t)
\]

(ii) From slope, \( z \rightarrow t \) gives \( f'(t) = 10 - 4t \)

(iii) \( 10 - 4t = 8 \Rightarrow -4t = -8 \Rightarrow t = \frac{1}{2} \) s.

(iv) Position:
\[
f(\frac{1}{2}) = 30 + 10(\frac{1}{2}) - 2(\frac{1}{2})^2
\]
\[
= 30 + 5 - .5 = 34.5 \text{ m}
\]
7. (10 pts.) You wish to install a cable running from $P$ to $Q$ to $R$ as shown in the figure at right. It costs you $180 per foot for the segment from $P$ to $Q$ and $100$ per foot for the segment from $Q$ to $R$. Total cost, $C$, is a function of the distance $x$ from $T$ to $Q$.

Compute the average rate of change of $C(x)$ on the interval $0 \leq x \leq 50$ feet.

\[ \text{Ave Cost is} \quad \frac{C(50) - C(0)}{50 - 0}. \]

\[ C(0) = (150 \text{ ft}) (180 \text{ \$/ft}) + (350 \text{ ft}) (100 \text{ \$/ft}) \]

\[ = \$62,000 \]

\[ C(50) = \left( \sqrt{150^2 + 50^2} \text{ ft} \right) (180 \text{ \$/ft}) + (300 \text{ ft}) (100 \text{ \$/ft}) \]

\[ = \$58,460 \]

\[ \text{Ave Cost:} \quad \frac{58,460 - 62,000}{50 \text{ ft}} = -70.8 \text{ \$/ft} \]