2. (12 pts.) Find and sketch (on the next page) the image of \( \{ z : |z - i| = 2 \} \) under each of the following maps:

(a) \( f(z) = 2iz + 2 - i \)
(b) \( f(z) = 1/z \)

NOTE: Algebraic solutions are not required. However, if you use geometric methods your sketches must be very accurate.

b) [points] Note: center does not map to center.

\[
\begin{align*}
f(3i) &= \frac{1}{3i} = -\frac{1}{3} i \\
f(-i) &= \frac{1}{-i} = i \\
f(2+i) &= \frac{2-i}{\sqrt{5}} = \frac{2}{\sqrt{5}} - \frac{i}{\sqrt{5}} \\
f(2-i) &= \frac{-2+i}{\sqrt{5}} = -\frac{2}{\sqrt{5}} + \frac{i}{\sqrt{5}}
\end{align*}
\]

b) [algebra]. inverse map is also \( f \), so:

\[
f(x + yi) = \frac{x}{x^2 + y^2} - i \frac{y}{x^2 + y^2}
\]

Constraint:
\[
\left( \frac{x}{x^2 + y^2} \right)^2 + \left( \frac{y}{x^2 + y^2} - 1 \right)^2 = 2^2
\]

\[
(x^2 + y^2) \left[ \frac{x^2}{(x^2 + y^2)^2} + \frac{y^2}{(x^2 + y^2)^2} - \frac{2}{(x^2 + y^2)} + 1 \right] = \left[ 4 \right] (x^2 + y^2)
\]

\[ 1 - 2y + x^2 + y^2 = 4(x^2 + y^2) \]