Worksheet 2 Solutions

Every series answer must have the form

\[ \sum a_n(z - c)^n \]

where \( a_n \) is given by some formula and \( c \) is a constant shift (often 0). Domain notation must clearly indicate a disk of certain radius and center. (But the center is always the same as \( c \) above.) The most efficient notation is probably \(|z - c| < R\).

1. \[ \sum \frac{3}{2^n} z^n; \quad |z| < 2 \]

2. \[ \sum (-1)^n (z - 1)^n; \quad |z - 1| < 1 \]

3. \[ \sum \frac{-1}{a^{n+1}} z^n; \quad |z| < a \]

4. \[ \sum \frac{n}{a^{n+1}} z^{n-1}; \quad |z| < a \]

   is a correct answer, but it is preferable to reindex so that the power of \( z \) is \( n \). I.e.,

\[ \sum \frac{n+1}{a^{n+2}} z^n \]

5. \[ \sum \frac{(n+2)(n+1)}{-2a^{n+3}} z^n; \quad |z| < a \]

6. Partial fractions expansion is

\[ \frac{z + 2}{(z - 2)^3} = \frac{1}{z - 2} + \frac{4}{(z - 2)^2} + \frac{5}{(z - 2)^3} \]

Add reindexed answers from Problems 3, 4, and 5. Simplify to

\[ \sum \frac{-5n^2 + n - 2}{2^{n+4}} z^n; \quad |z| < 2 \]

7. Answer is part of the problem. Domain is \(|z| < 1\).

8. (a) \(|z - 1| < 1 \)

(b) \[ \sum (-1)^n (z - 1)^n \]

(c) Refer to Problem 2.