This is a take home quest. Work all problems on separate paper. Due date is Monday, April 17.

1. (10 pts.) Find the angle of rotation and the scale factor for \( f(z) = \frac{-2iz + 1}{z + i} \) at the origin.

2. (15 pts.) Suppose that \( z_1, z_2, w_1 \) and \( w_2 \) are complex constants such that \( z_1 \neq z_2 \) and \( w_1 \neq w_2 \). Find the linear fractional transformation that maps

\[
\begin{align*}
0 & \mapsto \infty \\
z_1 & \mapsto w_1 \\
z_2 & \mapsto w_2
\end{align*}
\]

You may use any methods, including the implicit formula from your text. However, you must write your answer in the explicit form \( f(z) = \frac{az + b}{cz + d} \).

3. (15 pts.) Let \( f(z) = \frac{az + b}{z + d} \)

(a) Prove that \( f \) maps the real axis to either a circle or a line.

(b) Prove that it’s a line if and only if \( d \) is a real number.

[Hint: Work this as an inverse mapping problem. You only need to find the coefficients of the highest degree terms.]

4. (10 pts.) Find a linear fractional transformation \( f(z) \) that maps the lower half plane \( \{z : \text{Im} \, z < 0\} \) onto the unit disk \( \{z : |z| < 1\} \). Then sketch the behavior of \( f(\sin(z)) \). You do not need to compute anything, but you do need to include images of several vertical and horizontal lines. Refer to Figure 9.17, p. 382 as a starting point.