1. The cylinder pictured at right has closed top and bottom and a volume of 21 cubic inches. Find the minimum possible surface area.

\[ A = 2 \pi r y + 2 \pi r^2 \]

\[ V = \pi r^2 y = 21 \quad \Rightarrow \quad y = \frac{21}{\pi r^2} \]

\[ A = 2 \pi r \cdot \frac{21}{\pi r^2} + 2 \pi r^2 \]

\[ = 2 \pi r^2 + 42 \quad 0 < r < \infty \]

\[ A' = 4 \pi r - 42 r^{-2} = 0 \]

\[ 4 \pi r = \frac{42}{r^2} \]

\[ r^3 = \frac{42}{4 \pi} = \frac{21}{\pi} \]

\[ r = \sqrt[3]{\frac{21}{\pi}} \approx 1.4951 \text{ in.} \]

\[ A'' = 4 \pi + 84 r^{-3} > 0 \quad \Rightarrow \quad A \]

\[ \min A = 2 \pi \left( \sqrt[3]{\frac{21}{\pi}} \right)^2 + \frac{42}{\sqrt[3]{\frac{21}{\pi}}} \approx 42.1 \text{ in.}^2 \]
2. Assume that the cost of making a closed cylinder is 2 cents per square inch of curved side material plus 3 cents per square inch of top and bottom material. Find the dimensions of the lowest cost cylinder with volume 21 cubic inches.

\[
C = 2 \cdot 2\pi ry + 3 \cdot 2\pi r^2, \quad y = \frac{21}{\pi r^2}
\]

\[
C = 4\pi r \cdot \frac{21}{\pi r^2} + 6\pi r^2
\]

\[
= 6\pi r^2 + 84\pi r^{-1}, \quad 0 < r < \infty
\]

\[
C' = 12\pi r - 84\pi r^{-2} = 0, \quad 0 < r < \infty
\]

\[
12\pi = \frac{84}{r^2}
\]

\[
r^2 = \frac{7}{\pi}
\]

\[
r = \sqrt[3]{\frac{7}{\pi}}
\]

\[
C'' = 12\pi + 168\pi r^{-3} > 0 \Rightarrow \text{global min.}
\]

\[
y = \frac{21}{\pi} \frac{7^{1/3}}{\pi^{1/3}} = \frac{21}{\pi^{1/3} \cdot 7^{1/3}} = y
\]

\[
r \approx 1.3 \text{ in}, \quad y \approx 3.9 \text{ in}
\]

(Also note: \( y = 3r \))