Review for Final Exam

Part I: Previous Exams

Most of the test will look very much like your first four exams, except that there will NOT be any Chapter 1 problems. Chapter 1 material can and will show up inside other problems, so you still have to know it all.

- Your first priority is to make sure that you can work all the problems on the old exams (except for Chapter 1 problems). Blank exams are posted on the final exam web page.

- Note that Chapter 2 requires you to compute derivatives using the limit definition. You MUST know the definition and you WILL see this problem on the final. You should also know that you CANNOT use L’Hospital’s rule on such a problem. This is the one time I will require you to use a specific technique.

- After you make sure you can do all the problems on old exams you should look through the first three review sheets. The final exam covers ALL the material mentioned on those review sheets, even things that did not make it on to the in-class exams.

Part II: Chapter 5.5

There will be a some problems from Chapter 5, perhaps 40-50 points out of the total 200 point final exam.

Section 5.1

- Be able to solve problems that involve adding up a bunch of rectangles.

- Note that these can be actual rectangles, in which case you get area, or abstract rectangles. See problems 11–16 for examples.

Section 5.2

- Know that the word “integrate” means “add up lots of really tiny rectangles”.

- Know that the notation for this is something like

\[ \int_a^b \text{(rectangle height)} \cdot \text{(tiny width)} \]

- Also know that the rectangle height is the output of a function, \( f(x) \), and the tiny width is \( dx \). So the normal notation is

\[ \int_a^b f(x) \, dx \]

- Be able to compute the exact value of an integral if the function has a graph consisting of straight lines and circles.
• Be able to compute an integral by rewriting it in terms of things that you can compute exactly. See problems 35–42 and 47–50 for examples.

Section 5.3

• Know that this is a function:

\[ F(x) = \int_a^x f(t) \, dt \]

• Know that its input is the \( x \) at the top end of the integral, and that its output is generated by adding up all the little rectangles from \( a \) to \( x \).

• Be able to evaluate this sort of function.

• Be able to graph this sort of function.

• Be able to determine where it is increasing, decreasing, concave up, concave down, has local extrema, or has inflection points.

• Know that its derivative is \( F'(x) = f(x) \). This is the Fundamental Theorem of Calculus.

• Be able to use the FTC to compute integrals via guessing an antiderivative.

Section 5.4

• Know the following antiderivatives.

1. \( \int x^r \, dx = \frac{x^{r+1}}{r+1} + C, \quad \text{if } r \neq -1 \)

2. \( \int x^{-1} \, dx = \ln |x| + C \)

3. \( \int e^x \, dx = e^x + C \)

4. \( \int \sin x \, dx = -\cos x + C \)

5. \( \int \cos x \, dx = \sin x + C \)

• Know that addition, subtraction, and constant multiples are no impediment to finding an antiderivative.

• Know that multiplication and division are absolute barriers to finding an antiderivative. You must simplify.

Section 5.5

• Know that you MUST substitute in order to antidifferentiate a function inside a function.

• Know that you MAY substitute to simplify the denominator of a quotient.
• Know that once you write down $u =$ something, you are locked into the following procedure:

1. Find $\frac{du}{dx}$
2. Solve for $dx$
3. Substitute as much as possible in the original problem.
4. Get rid of any extra $x$’s
5. Guess the new antiderivative
6. Change the answer back to $x$’s.