Math 464, Homework 13

As in Homework 12, suppose that we have a model in which:

<table>
<thead>
<tr>
<th></th>
<th>Gen 0 → Offspring → Gen One</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>x → p^2 → ?</td>
</tr>
<tr>
<td>Aa</td>
<td>y → 2pq → ?</td>
</tr>
<tr>
<td>aa</td>
<td>z → q^2 → ?</td>
</tr>
<tr>
<td>A</td>
<td>p → p → ?</td>
</tr>
<tr>
<td>a</td>
<td>q → q → ?</td>
</tr>
</tbody>
</table>

The unknown Gen One values depend on relative survival rates. We now assume that, relative to the Aa genotype,

- AA survives at a rate of \((1 + \alpha)\)
- aa survives at a rate of \((1 + \beta)\)

Note that \(\alpha\) and \(\beta\) could be negative or positive, but that \(\alpha, \beta \geq -1\).

1. Once again, Gen One \(p\) is determined by Gen Zero \(p\). Write this fact as a difference equation with time step equal to one generation.

2. Verify that \(p = 0\) and \(p = 1\) are equilibria. Note that this is true for any values of \(\alpha\) and \(\beta\).

3. Find the other equilibrium.
   
   HINT: If you know that \(p = 1\) is a root, then you know that \(p - 1\) is a factor.

4. Suppose that \(\beta = 0\) and \(\alpha < 0\). What does the phase portrait look like?

5. What if \(\beta = 0\) and \(\alpha > 0\)?

6. If \(\alpha = 0\) and \(\beta < 0\)?

7. If \(\alpha = \) and \(\beta > 0\)?

8. If \(\alpha = \beta\)?
   
   WARNING: Might have to work this problem in subcases.

9. What if \(\alpha + \beta = 0\)? (Possible subcases here, too.)