Worksheet 3

Suppose that \( S \) is the solid shape between the paraboloid \( z = x^2 + y^2 - 100 \) and the plane \( z = 0 \). Assume that the \( x- \), \( y- \) and \( z- \)axes have units of meters.

1. Write an iterated triple integral for the volume of \( S \).

2. Write out the formula for a little bit of volume, \( dV \). What are its units?

3. Suppose that \( dV \) is little bit of water. Density is 1000 kg/m\(^3\). Gravitational acceleration is \( 9.81 \) m/s\(^2\). What does the little bit of water weigh? Give units.
   
   HINTS:
   
   • Weight = Mass \( \times \) Gravitational Acceleration
   
   • Mass = Density \( \times \) Volume

4. Suppose you lift the little bit of water up to height \( z = 0 \). How much work did you do? Give units.
   
   HINT: Work = Weight \( \times \) Distance

5. Suppose you lift ALL the water up to height \( z = 0 \). How much work did you do? Give units.

6. Suppose that you want to dig a hole in the shape of \( S \). If the density of the soil you excavate is \( \rho \) kg/m\(^3\), how much work does it take to dig the hole?

7. Suppose that you are digging up ore that is worth more as you dig deeper: at \( h \) meters below ground the ore is worth \( 2\sqrt{h} \) \$/m\(^3\). What is the total value of the ore excavated from \( S \).