Homework 3, Graded Problems.

1. Suppose that
\[ \mathbf{r}(t) = \cos \left( \frac{\pi(t - 1)}{2} \right) \mathbf{i} + \left( 1 + \sin \left( \frac{\pi(t - 1)}{2} \right) \right) \mathbf{j} + t \mathbf{k} \]

(a) Sketch the portion of the curve for \( 0 \leq t \leq 2 \).
(b) Compute \( \mathbf{r}''(1) \).
(c) Sketch \( \mathbf{r}''(1) \), based at the point \( \mathbf{r}(1) \), in the same picture as part (a).
(d) Compute that tangential and normal components of \( \mathbf{r}''(1) \).

2. Suppose that \( \mathbf{r}(t) = \langle t^3 - 1, e^t, 1 - t \rangle \).

(a) Where does this intersect the y-axis?
(b) At that point, what is the angle between this curve and the y-axis?

3. A particle travels along the curve \( \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} - 3t\mathbf{k} \). Suppose that the tangent line at \( \mathbf{r}(t_0) \)
intersects the line
\[ \langle -1, 0, 0 \rangle + u \langle 0, 0, 1 \rangle \]
What is \( t_0 \) and where does the tangent line cross the other line?

4. Suppose that \( f(x, y) = \sqrt{4 - x^2 - 4y^2} \).

(a) Find and sketch the domain of \( f \).
(b) Sketch the level curves \( f(x, y) = 0, 1, 2 \). Your sketch should have all these curves on one set of axes, and each curve should be labeled with its level.