Homework 12, Graded Problems.

1. Use the vector field pictured below to answer estimate whether the given integrals are positive, negative, or zero. Then determine whether or not the field is conservative.

(a) $\int_C \mathbf{F} \cdot \mathbf{d} \mathbf{r}$, where $C$ is the segment from $(0, 0)$ to $(2, 2)$.
(b) $\int_C \mathbf{F} \cdot \mathbf{d} \mathbf{r}$, where $C$ is the segment from $(-2, -2)$ to $(2, 2)$.
(c) $\int_C \mathbf{F} \cdot \mathbf{d} \mathbf{r}$, where $C$ is the circle $x^2 + y^2 = 1$, traversed counterclockwise.
(d) $\int_C \mathbf{F} \cdot \mathbf{d} \mathbf{r}$, where $C$ is the segment from $(0, -2)$ to $(2, 0)$.
2. Suppose that $C$ is the boundary of the rectangle $[1, 5] \times [-1, 2]$, and that $C$ is oriented counterclockwise around the rectangle.

(a) If $\mathbf{F} = e^{x^2+y^2}\mathbf{i} + x \cos y \mathbf{j}$, write $\int_C \mathbf{F} \cdot d\mathbf{r}$ as a double integral. Do not evaluate.

(b) Sketch the domain of integration for your double integral.

3. Suppose that $C$ is the boundary of the triangle with vertices $(1, 0)$, $(3, 0)$ and $(2, 4)$. If $C$ is oriented clockwise, write the following line integral as an *iterated* double integral.

$$\int_C x^2 y \, dx + (2x + y^2) \, dy$$

4. If $\mathbf{F} = -x^2 \mathbf{i} + xy^2 \mathbf{j}$ and $C$ is the curve pictured at right, compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

Use any methods.