SHOW ALL YOUR WORK

1. (10 pts.) Solve the equation

\[ \frac{1}{x + 1} + \frac{2}{x} = 0 \]

2. (10 pts.) A function \( f \) has the graph shown at right. Sketch the graph of

\[ g(x) = f(2x + 6) \]

on the axes provided below.
3. (10 pts.) Use the graph of \( f(x) \) shown at right to compute the following:

(a) \( (f \circ f)(-1) \)
(b) \( (f^{-1} \circ f^{-1})(5) \)

4. (10 pts.) A rectangle has a perimeter of 20 inches. Express the area of the rectangle a function of the length of one side. Then find the maximum possible area.

5. (10 pts.) Find the quotient and remainder:

\[
\frac{x^3 - 2x^2 + 1}{x + 5}
\]

6. (10 pts.) Find a cubic polynomial with integer coefficients that has roots \( x = 3 \) and \( x = 3 - 2i \).

7. (10 pts.) Find all real and complex roots of the given polynomial. Be sure to show all your work. Calculator solutions will not suffice.

\[ p(x) = x^3 - x^2 + 2 \]

8. (10 pts.) Write the following expression so that there are no products, quotients, or exponents inside the logarithm.

\[ \ln \left( \frac{(x + 1)^{-2}(x - 2)}{x} \right) \]

9. (10 pts.) Solve

\[ \ln(x - 1) + \ln(x + 1) = 0 \]
10. (10 pts.) Suppose that a population of foxes grows according the model

\[ p(t) = 120e^{kt} \]

where \( t \) is the number of years from now and \( p \) is the number of foxes. If there are 150 foxes 5 years from now, how many foxes will there be 10 years from now?

11. (10 pts.) Suppose that \( \tan t = -2 \) and \( t \) is in the second quadrant. Compute

(a) \( \sin t \)
(b) \( \cos t \)

12. (10 pts.) Find the period, amplitude and phase shift for the transformed sine function shown below. Then write a formula for it in the form

\[ f(x) = a \sin k(x - b) \]

13. (10 pts.) The triangle at right has sides of length 8 and 7 and an angle of 45°, as shown. Find the other two angles.
14. (10 pts.) A surveyor wishes to determine the distance across a lake as shown at right. Use the distances and angle given in the figure to find the distance across the lake.

15. (10 pts.) Find all solutions to the equation

\[ \tan(2x) - \sqrt{3} = 0 \]

16. (10 pts.) Find all solutions to the equation

\[ \cos(2x) = \sin x \]

17. (10 pts.) Suppose a point has rectangular coordinates \((-8\sqrt{3}, 8)\). Convert to polar coordinates.

18. (10 pts.) Find all fourth roots of the complex number \(-8\sqrt{3} + 8i\). HINT: your answer to Problem 17 might be useful.

19. (10 pts.) Solve the system of equations

\[
\begin{align*}
2x + 3y^2 &= 17 \\
3x - y^2 &= 9
\end{align*}
\]

20. (10 pts.) Verify the identity

\[ \sin(2\sin^{-1} x) = 2x\sqrt{1 - x^2} \]