Review for Exam 2

What follows is intended as a guide to focus your study for Exam 2. Read the Overview and the Study Tips. After that there is list, arranged by section, of the ideas and techniques that you must know for the exam.

Overview

• Expect to see somewhere from 10 to 12 problems that look very much like homework problems.

• About 50% to 60% of the exam will be problems that should remind you of the easy to moderate homework problems.

• Another 30% to 40% will look like the more involved homework problems.

• The last 10% will be a new problem that requires an original application of the ideas and/or techniques covered on this exam.

• VERY IMPORTANT NOTE: A graphing calculator is not required for this test, but there are several places where one could be used. If you use your calculator to draw a graph, please “show your work” by sketching a copy of the calculator’s graph on your test paper.

Study Tips

• Do lots of homework!

• Your goal should be to do so much homework that you can look at a problem and immediately know what to do with it.

• Once you know what to do, you should be able to do it quickly.

• The only way to get to this point is to work a large number of homework problems. The absolute minimum is what I have assigned. If there is any type of problem that you do not feel confident about, find that problem type in your textbook and work all of the similar unassigned problems. If you need more than that, let me know. I can supply you with as many as you could possibly want.

Section 2.8

• Given functions $f$ and $g$, be able to evaluate, graph, and/or find the domain of $f + g$, $f - g$, $fg$, $f/g$, and any other algebraic combination of these.

• Be able to do this if the functions are given as formulas, graphs, or tables.

• Be able to evaluate, graph, and/or find the domain of the composition of two functions.

• Be able to do this if the functions are given as formulas, graphs, or tables.
Section 2.9

- Be able to determine if a function is one-to-one.
- Be able to do this if the function is given by a formula, a graph, or a table of values.
- Know the basic rule that relates a function and its inverse:

  \[ f(x) = y \iff f^{-1}(y) = x \]

- Know that this means "Inputs and outputs trade places."

- Given a function \( f \) (either by formula, graph, or table) be able to compute \( f^{-1} \) of a single input.

- Given a function described by a formula, be able to construct a formula for \( f^{-1} \).

- Given a graph of a function, be able to graph the inverse function.

Section 3.1

- Know the "fundamental theorem"

  \( (x - c) \text{ is a factor if and only if } c \text{ is a root.} \)

- Be able to graph a polynomial with the following information exactly displayed.
  1. Exact roots.
  2. Correct behavior near each root.
  3. Correct end behavior.

- You must show work that leads to roots. (I.e., factoring). If you use a graphing calculator, you must include a sketch of your calculator screen.

Section 3.2

- Be able to do long division.

- Be smart enough NOT to do long division when the divisor is linear.

- For linear divisors, be able to:
  1. Find quotient and remainder.
  2. Find output of a polynomial. (It's just the remainder.)
  3. Determine if a divisor is a factor.
  4. Determine if a number is a root.

- Given roots of a polynomial, be able to find the polynomial.
• Be able to solve such a problem with an additional condition, such as integer coefficients, constant term 1, etc.

Section 3.3
• Know the basic routine for finding the roots of a high degree polynomial:
  1. Guess a root.
  2. Check by division (the short way).
  3. Repeat until you reach a quadratic.

NOTE: On these problems you must show work in which you check you guesses. You must also show me a picture of anything you get from a graphing calculator.

• Know the basic routine for factoring a high degree polynomial:
  1. Find the roots.
  2. Turn them into factors.

Section 3.4
• Know what a complex number is.
• Know that \( i^2 = -1 \).

• Be able to add, subtract, multiply and divide complex numbers. Note that your answers must be in the form \( a + bi \).

NOTE: Most calculators will do this. I need you to do it by hand and show your work.

• Be able to find complex roots of quadratic equations.

Section 3.5
• Be able to do the Section 3.3 problems, but with complex roots.

• Know that a real coefficient polynomial can only have complex roots in conjugate pairs.

Section 3.6
You will have an exam problem that asks you to graph a rational function. Your calculator might be of some help, but you need to show your work as well, and you need some specific details in the graph.

• You must show work that finds the roots. Factoring is sufficient.

• You must show work that finds the vertical asymptotes. Factoring is sufficient.

• Show as much work as is needed to find out if the end behavior involves an asymptote, either horizontal or otherwise.
• Your graph must display exact roots, exactly positioned asymptotes of all types, correct behavior near each root, correct behavior near each asymptote, and correct end behavior.

• If you use a graphing calculator to help with any of this, show your work by sketching a picture of the calculator’s graph on your exam paper.

Section 8.1-8.3
Note that these problems will not show up on the exam segregated by section, nor will you have instructions like “solve by substitution”. It will be your job to recognize the problem type and to choose an appropriate solution method. I will accept any valid technique other than calculator solutions.

• Know the universal technique for all equation systems:
  1. Pick one equation; solve it for one variable.
  2. Substitute into all other equations.
  3. Repeat until you have one equation in one variable.
  4. Solve for that variable.
  5. Go back to find the other variables.

• Know that a single solution to a system is more than one number. This is critical when you are writing the answers to non-linear systems.

• Be able to recognize if a system is linear or not.

• It is not required (since the universal method will work), but it is useful to know a quick, reliable method for solving small linear systems.

• It is not required (since the universal method will work), but it is useful to know the following trick for non-linear systems:
  1. Identify a common term in two equations.
  2. Eliminate that term (either by solve and substitute or by adding equations).